Newsletter No. 77 **September 2008**

I was with a group of retired maths teachers recently chatting about the subject and their careers. The consensus was that the content of the syllabus was not much changed from when they were pupils at school - it is still taught under broad headings like algebra, geometry and arithmetic (now called 'number') as it had been all those years ago. One, we'll call her Jenny, had noticed something odd. "When I was at school as a pupil", she said, "we never heard of things like topology, multi-base arithmetic, set theory and vectors. I came across those for the first time when I was studying towards my degree. Then I went teaching. Hadn't been at it for more than a few years when the syllabus changed. The so-called 'modern maths' reached New Zealand and we were into all the new stuff. Set theory - Boolean algebra with intersections and unions, universal and complementary sets, all that, which we were informed was important for children to know. It would help them understand the structure of mathematics. We were told that teaching about numbers in different bases would help kids understand the processes of arithmetic. We even did practical work on simple topology - Klein bottles, Möbius strips and so on. At senior levels some of the vector geometry required for the Bursary exams was quite challenging. Not only did the content change radically but," Jenny added, "some of us had a go at new approaches to our teaching. I even had a maths laboratory for a while." The rest of the group had similar experiences. They'd all had to learn new content and new skills, none of which, they felt, are required these days - all that so-called new stuff has gone.

If you'd blinked you might have missed 'New Maths' altogether.

Anon

R.L. Graham noted in the *Journal of the Australian Mathematical Society*, 1963 that every number greater than 77 is the sum of natural numbers, the sum of whose reciprocals is 1. For example, $78 = 2 + 6 + 8 + 10 + 12 + 40$ and the sum of the reciprocals of these six numbers is one.

Anatole France won the Nobel Prize for Literature when he was 77, David Ben-Gurion, the first Prime Minister of Israel, resigned at that age. Le Corbusier died aged 77 as did Chou En-lai, Galileo, Arthur Koestler, Paul Robeson and Jules Verne.

Booke Review

Discourse on Method by René Descartes

This book opens delightfully with the statement that:

> Good sense is, of all things among men, the most equally distributed; for every one thinks himself so abundantly provided with it, that those even who are the most difficult to satisfy in everything else, do not usually desire a larger measure of this quality than they already possess.

Much of the book is more philosophy than mathematics including rules of morality, the reasons by which Descartes' proves the existence of God and the human soul and aspects of the science of medicine. It is one of the appendices to the book that is of interest to mathematicians as it provides the improved notation with which we are all familiar, that is, the method of restating a geometrical problem in algebraic form and then solving it algebraically - the so-called Cartesian Geometry. This earned Descartes the title of founder of analytic geometry.

The book, although written nearly 400 years ago is eminently readable (in translation from the French) with a chatty style. The battered condition of my copy, a Penguin edition published in 1960, is testament to that. Here's how he begins a section on the nature of curved lines:

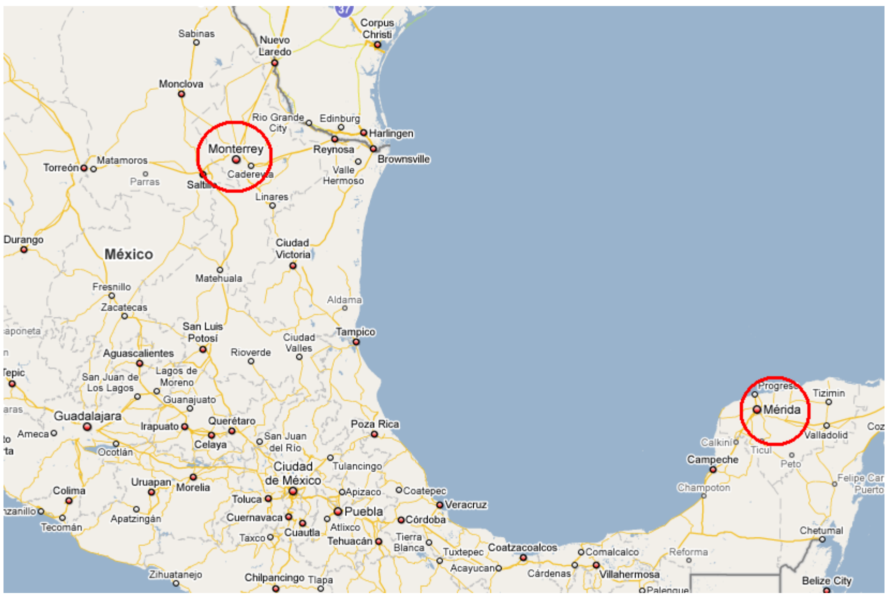
> The ancients were familiar with the fact that the problems of geometry may be divided into three classes, namely, plane, solid and linear problems. This is equivalent to saying that some problems require only circles and straight lines for their construction, while others require a conic section and still others require more complex curves. I am surprised, however, that they did not go further, and distinguish between different degrees of the more complex curves, nor do I see why they called the latter mechanical, rather than geometrical.

In the original work, Descartes is a little inconsistent with his notation. On one occasion he may use aabb while in another a^2b^2. These problems are invariably overcome in translations of the book. It is interesting to see that he avoids the can of worms represented by complex roots of equations when he writes, 'Every equation can have as many distinct roots as the number of dimensions of the unknown quantity of the equation' (that is, the degree of the equation). 'Can have' notice not 'must have'.

Discourse On Method is historically one of the great books.

ICME 11

This was the year of ICME. The International Congress of Mathematical Education is held every four years. Something else is held every four years but what it is just escapes me. But I think that that and the ICME attract vast crowds. I'm not sure what the final score was but something like 3,000 participants went to Monterrey, Mexico (see the map below) for this year's ICME. If you would like to find out more details than I am going to go into here, then, like most things these days, you can look it up on the web at http://icme11.org/. (That's a surprisingly short URL.)



Anyway, ICME is THE big maths education conference and attracts the big and the small names from all over the world and from all across the spectrum. It caters for researchers and teachers; and primary maths, secondary maths and tertiary maths. Whatever you are interested in, there's almost certainly going to be someone there who wants to speak about it.

I don't plan to say too much about the things that I saw and enjoyed but I will bore you with the Holton wanderings after the conference. I'm happy to answer any questions that you have though. Just write to me at the usual place: derek@nzmaths.co.nz.

But before I finish this little sidetrack, I should tell you that ICMI (the International Commission on Mathematical Instruction) actually is in charge of ICME as well as various Studies (on technology and its impact on maths teaching; on challenging maths; on teacher training; etc.). And you should know that from 2010, Dr Bill Barton of Auckland University is going to be President of ICMI. This is a singular honour for Bill and a reflection of glory for New Zealand. I think that it is the first time that someone from the Southern Hemisphere has been President.

So now on to the Yucatan.

The Other Pyramids

I knew that the Mayans had made their own pyramids and that they were somewhere in the jungles of Mexico, so it seemed appropriate after the Monterrey ICME to go and have a look at them. It turns out that the Mayans lived happily in the Yucatan Peninsula (the bump around Mérida on the map above) until they were ousted by the Spaniards in the late Seventeenth Century. But that's a strange story like many of the stories that the guides like to tell about the Mayans. And maybe you shouldn't believe that they all lived as happily as I have suggested. So let's take a deep breath and mix history and fable together and make a good story. If you want to know the truth you should look Mayans up on the web – Wikipedia is always a good place to start.

The Mayans really seem to have got started about 2000 years BC but their peak period was from about 200 to 900 AD. During that time they became the most densely populated part of the world. But like all cultures (probably) they had a ruling class and a ruled class and of course it was better to be ruling. Now I went to two pyramid sites: one at Uxmal and the other at Chichen Itza. The first of these didn't have human sacrifices and the second did. So let's do the nice ones first. In the picture below I've shown the restored Pyramid of the Magician at Uxmal. I won't bother you with the Wizard's story but exceptional conditions were put up for the new Mayan ruler and the Wizard made sure that her boy satisfied all of those conditions. Suffice to say that the ruins at Uxmal were pretty impressive. When I first saw the pyramid there I let out a 'Wow!' but that may have been because it was 36 degrees Celsius, very humid and the guide insisted on us all standing in the sun while he told us the various legends. (I suggest that you go there in our summer not theirs.)

But perhaps the best place of the two for a pyramid and for intrigue is Chichen Itza. The picture below shows a face of the pyramid that has only partially been restored but it gives you some idea of how awe-inspiring it might have been if you were a humble worker.

It might have been awe-inspiring too if you were just about to be sacrificed. But the story there seems not to be too clear. It may be that they just sacrificed their foes – there are certainly some carvings on the walls of buildings with Mayans holding hearts – but it is possible that the people who were sacrificed wanted to be in order to get a running start into heaven. We know that trend in some current cultures but apparently it goes back to Mayan times. It seems that they had a ball game, there are hoops at both Uxmal and Chichen Itza, and at the height of their powers, the Mayans ran the World Series of the sport. Teams came from all around to take part. And the best bit is, that, because of his achievement, the captain of the series' winning team was sacrificed. Think of that as you watch the Bledisloe Cup.

Now I won't expect you to do the counting, this face hasn't been restored sufficiently to be sure anyway, but there are 91 steps on the face in the picture and there were 91 steps on each of the four faces. Since 4 x 91 = 364. you might suspect that the Mayans didn't know how many days there were in a year. In fact their astronomy was pretty advanced and they clearly did know. But they gave you one day free, a holiday, to make up the full 365.

What's more they did know that 91 = 7 x 13. Clearly 13 is the number of weeks in a season (of which there were four) and 7 the number of days in a week. Then 4 x 7 = 28, the number of days in a lunar month, the menstrual cycle and, coincidentally, the number of bones in your fingers and thumbs. You've no idea what you can do with that if you sit around on pyramids all day in the sun.

But you may be wondering why on earth you would want to build a pyramid in the first place. Well, if you are clever at it, it's a great way to keep the populace under control. Imagine the scene. It's Midsummer Day and you've been called to the north face of the pyramid; all the city is there. Then a personage in an enormous head dress appears in the hole in the building at the top of the pyramid. There is all sorts of mumbo jumbo and at the right moment the sun shines from behind the figure and lights him up. A good trick but you had to know something about the movement of the sun to get that right. But then just to make sure of the people's allegiance, a shadow of a snake starts wriggling down the 91 steps. And the snake is one of the big Mayan gods. What a way to introduce a new king! Was the opening ceremony of the Games any more impressive?

But maybe that day was cloudy. No worries! There is a back up pyramid down the road. You can always herd the population down there for the day after next.

You have to ask how this culture that was clearly on top of its game was beaten by the Spanish. Obviously there were hordes of Spanish soldiers. No, there appear to have been only a 100 or so. Presumably there was dissension among the ranks. There were probably political divisions and somehow the Spaniards were able to exploit that. In the end they knocked over the Mayans in less than two years!

So that just goes to show that just because you have power over numbers you don't have power over hearts.

The Streets of Mérida

Mérida is the capital of the Yucatan State. It was established by the Spaniards in the early days of their conquest. It's quite a pleasant city with some very beautiful old houses. A lot of them are being restored at the moment. But this is leading to political problems. It turns out that they are being restored by rich Americans, that this is driving property prices up, and so the local Mexicans can't affords to buy them. But that's not what I wanted to tell you about Mérida.

You know that a lot of cities are built on a rectangular grid. This is certainly true for New York and many American cities. The CBD of Melbourne is the same. It turns out that Mérida is too, pretty well. Now in Melbourne the different streets have different names. In Manhattan the Streets run East/West and the Avenues run North/South. In fact the East Streets are to the East of Fifth Avenue and the West Streets are to the West of it.

That is a perfectly sensible way to organise things so that navigation is easy. So what does Mérida do? In the historical part of town the even numbered roads run North/South and the odd numbered roads run East/West. What a perfect way to be able to find your way around.

Now before this system every corner was named. So you would have pig corner or corn corner. They came with a suitably decorated tile that was put on a building at the corner. So there are a few ways to find your way around. What others can your students think up?

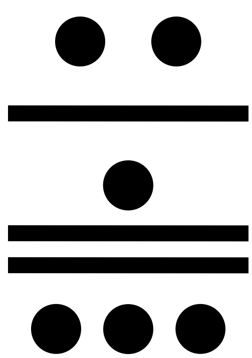
Mayan Numbers

But as I've said above, Mayans had power because they knew how numbers worked and how to apply that knowledge to the Universe. In fact the Mayans had a 'decimal' number system before it was known in Europe. Actually they worked on a base 20 system. And so they even had a zero!

A shell stood for 0, a dot for 1 and a line for 5. I was told that the dot was actually the view of the end of a finger and the line the end view of a hand with its five digits.

To represent numbers from 1 to 19, they first had one to four dots lined horizontally to give the numbers 1 to 4. Then came the line for 5. On top of this they sat one to four dots to give 6 to 9. Obviously two lines made 10 and you sat one to four dots on top to get up to 14. The next trick was to use three lines and enough dots to get you to 19.

At this point the zero came in. 20 would be one dot with a shell underneath. The shell had the property of multiplying what went above by 20. So a line with a shell underneath was 5 x 20 = 100. To get 106 you put down your line (5 x 20) and underneath you put a dot on a line. Because you had that 6 now you had no need for a shell. The position implied that you had 20 times the first line. So the number below had to be 3023.

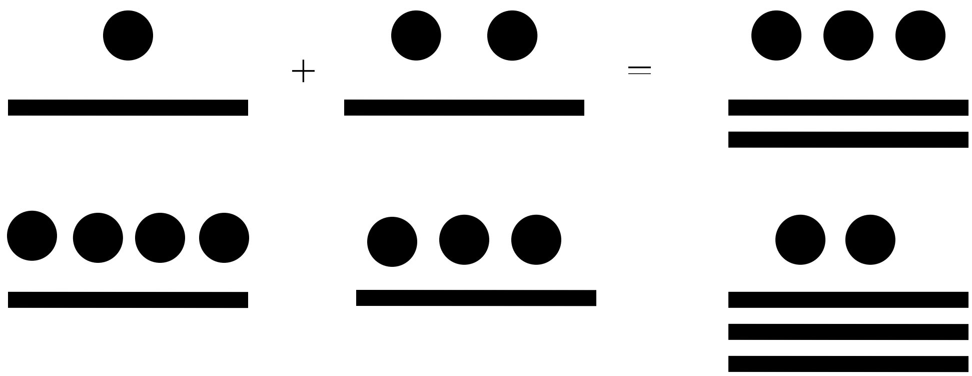


7 x 20 x 20 = 2800

11 x 20 = 220

3

That explains how the Mayans recorded numbers but how did they do arithmetic? Well addition isn't too hard with small numbers. If your dots add up to less than five you just add the dots and the bars – see 6 + 7. On the other hand if the dots add up to more than 5 you change every new 5 to a line. You clearly get 'carry overs'. The example below is 9 + 8. Here the 4 + 3 dots go to 2 dots plus a line.



Of course similar things can be done with subtraction. Really they were ahead of us in the standard addition and subtraction algorithms. I'm not sure what they would do with multiplication and division though but presumably that goes the same way that our algorithms go. Why don't you get your students to look it up on the web.

More on Astrolabes

Oddly enough, following our review last month of Chaucer's dissertation on the astrolabe, there was a recent news item from BBC under the heading, *Medieval 'calculator' stays in UK* about a 14th century astrolabe dubbed the 'pocket calculator of its age' used in the time of Geoffrey Chaucer. This particular brass instrument was found during excavations for an extension to a restaurant, the site of which is known as the House of Agnes, a 17th Century inn on the main road out of Canterbury to London. Apparently funding has been made available for purchase of the find by the British Museum.

The news item described the astrolabe as a device used for telling the time, mapping the stars and making height and depth measurements. David Barrie, involved with providing the funding, said: "The Canterbury Astrolabe Quadrant offers an extraordinary insight into the scientific and technological capabilities of Chaucer's England. Chaucer himself was an expert on astrolabes and wrote in the Canterbury Tales about men's love of 'newfangleness'."

The Canterbury Astrolabe Quadrant

The British Museum will shortly display the tool, thought to have been made in 1388. A spokeswoman for the museum said: "Astrolabe quadrants are among the most sophisticated calculation tools ever made before the invention of the modern computer. They were an extremely handy tool for their owners, enabling them to carry out timekeeping and other calculations. This particular example was made for use with the Sun with the help of the two sighting vanes attached to one side and a now lost plumb-bob. Besides enabling the user to determine the date of Easter he could use it to determine the times of sunrise and sunset, the time in equal and unequal hours or the geographical latitude - to name but a few of the many functions."

Solution to August's Endeavour

Last month we mentioned that 76 is an automorphic number because its square ends in itself and that there is only one other 2-digit number with this property. You had to find it. Not such a difficult task and I'm sure you all quickly realised it was 25.

This Month's Endeavour

List all the prime numbers between 1 and 100 that are one more or one less than a multiple of 6. Call this list 'LIST 1'. How many of them are there?

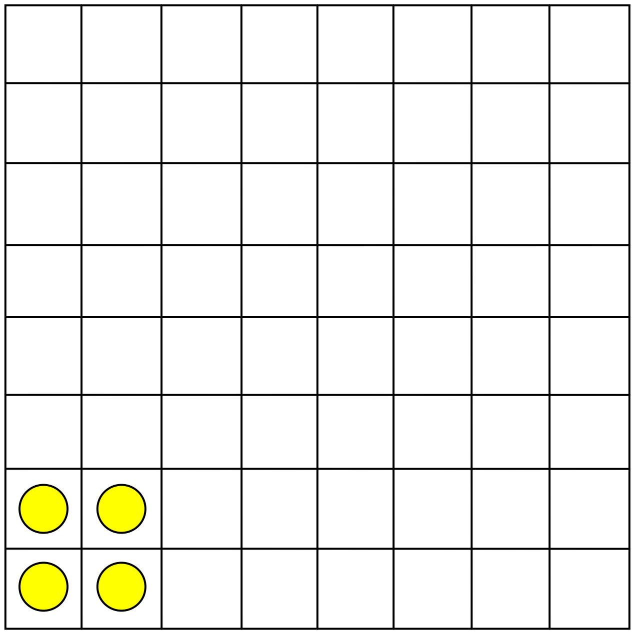
List all the prime numbers between 1 and 100. Call this 'LIST 2". How many numbers are in both LIST 1 and LIST 2? (Remember that 1 is not considered prime)

Is it true that almost all prime numbers are one more or one less than a multiple of six? Explain what you mean by 'almost all'.

Someone once conjectured that every even number is the difference between two consecutive prime numbers. Which even numbers below 100 are <u>not</u> the difference between pairs of numbers in LIST 2? Does your result support or refute the conjecture?

Solution to last month's Junior Problem

The counters can move by jumping over another counter to a vacant cell on the other side. So, at the moment, three of the counters can move but the one at the top right can't because it has no other counter to jump over.



The first problem was to see whether you can move the four counters into another place on the board so that they are again in the form of a square. (The four counters are in the bottom left 2 by 2 square of cells. Can you move them to some other 2 by 2 square of cells?)

This is not so difficult. You can move the left two counters over the right two counters. As a result the group of four moves to the next 2 by 2 square. If you keep doing that or its equivalent vertically, you can move the group of counters to any other 2 by 2 square on the board.

The second problem was to look at nine counters in the bottom left 3 by 3 square of cells and see if you can move them to another 3 by 3 square of cells. Yu should be able to do that but this time you move the counters two 3 by 3 squares across. As I didn't have any answers to this, I'm going to leave it open till next month and for last month's $20 prize.

Then finally, look at 16 counters in the bottom left 4 by 4 square of cells and see if they can be moved to some other 4 by 4 square of cells. What can you do here?

This Month's Junior Problem

To get this month's $20 I'm staying with chessboards. How many squares are there on a chessboard if you don't just consider the 1 by 1 or 8 by 8 squares?