# NZmar.. 

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Reading about Brian Stokes's new book Tickle Tackle in our June issue put me in mind of the well-known game Noughts and Crosses which is also known as Tic-tac-toe, or Ticktacktoe as the Americans call it. Of all the board games ever devised it and its near relatives have undoubtedly been the most popular. I remember seeing two examples in the British Museum in London, one of pottery pieces on a limestone slab dated 850 BCE, the other a New Zealand version made of paua. It is a game of position related to others like Pong hau k'i played in China, the Maori game Mu Torere and Achi played by Ghanian children. Games like Nine Men's Morris and Go-moku are derived from it. More complicated versions of the basic game are played in three dimensions or on boards of unlimited size. I guess the rules of Noughts and Crosses are well-known to everyone. Less well-known, perhaps, is that there are a number of simple alternative versions to the game like, for example, the loser being the first to get three of his pieces in a line (vertically, horizontally or diagonally). This version of the game is sometimes called Toetacktick! In Wild Ticktacktoe, invented by Solomon W. Golomb, players may use either symbol, O or X . The first to complete a line of three in either symbol is the winner. There is also the misère version where the first player to make three in a line loses. Since the game and most of its versions are essentially quite straightforward, strategies for playing them are readily worked out. For example, the standard version of the game will always end in a draw if the opponents play intelligently. Some other versions of the game are included in Afterthoughts below.

At evening, when with pencil, and smooth slate
In square divisions parcelled out and all
With crosses and with cyphers scribbled o'er
We schemed and puzzled, head opposed to head In strife too humble to be named in verse.

William Wordsworth
76 is an automorphic number in that its square ends in itself, this $76^{2}=5776$. Among the famous people who died at age 76 are included Albert Einstein, Edward Elgar, T.S. Eliot and Livy.

## INDEX

What's new on nzmaths.co.nz
Diary dates
Crop Circle
Booke Review
Bill's Number Plates Problem
Solution to July's Endeavour
Endeavour of the month
Solution to last month's Junior Problem
This Month's Junior Problem
Afterthoughts

## What's new on nzmaths.co.nz

There are three new objects available in the Learning Objects section. These Learning Objects have been developed specifically for the nzmaths site and are not password protected so feel free to have a look. The new objects are:

1. Angles: which helps users measure or draw angles using other angles as units of measurement
http://www.nzmaths.co.nz/LearningObjects/angles/angles.htm
2. Number Line: which helps users to record how they solve addition and subtraction problems using a number line http://www.nzmaths.co.nz/LearningObjects/NumberLine/index.swf
3. Mix Up: which helps users find averages of percentages
http://www.nzmaths.co.nz/LearningObjects/MixUp2/index.swf
These Learning Objects are also available in Maori from this page: http://www.nzmaths.co.nz/maori/lo/default.aspx

A handbook to support the Home-School Partnership: Numeracy has been designed as a guide for schools and communities to plan ways to work together to support children's numeracy achievement. It is available from this page:
http://www.nzmaths.co.nz/numeracy/hspn/index.aspx

## Diary Dates

A final reminder that Maths Week 2008 is this month - the week of the 11th to 15 th August. There is an assurance on the front page of the NZAMT site that "This year is again bigger, better and brighter."
http://www.nzamt.org.nz/sites/cms/

## Crop circle

My son-in-law came across this picture in a recent issue of the Melbourne Herald Sun newspaper (http://www.news.com.au/heraldsun/story/0,21985,23886370$5012749,00 . \mathrm{html}$; June 19, 2008). It shows a recent crop circle in Wiltshire. Your mission is to find the mathematical connection here. As a hint I should note that $\pi$ has a lot to do with circles. We all know about $2 \pi r$ for the circumference and $\pi r^{2}$ for the area. But this circle goes a little bit further and tells you what $\pi$ is, or at least the first 10 significant figures of it. The decimal point might help.


Picture: APEX

If you need any more hints, you should look at the Afterthoughts.
But this is just a reminder that $22 / 7$ is only an approximation to $\pi$. A little play on a calculator gives $22 / 7$ as 3.14285721428572142857214285721428572 . So there are two things to note here straight away. First, 3.142857214 is a long way away from 3.141592654 , the first 10 significant figures of $\pi$. So $22 / 7$ is only a useful approximation to $\pi$. If you want to you can get better approximations. Archimedes did. For instance, $333 / 106$ is not so bad. That weighs in at 3.141509434 . There are many more, just do a web search on 'pi approximations'.

And second, $3.14285721428572142857214285721428572 \ldots$ keeps cycling round with ' 1428572 ' going on for ever. It has to do this because it's a fraction. Every fraction either comes to a halt (that is, at some stage it continues as $0000 \ldots$ ) or repeats. The fact that it has 7 numbers in its repeating bit ought not to come as a surprise. What would you expect the periodic piece of $40 / 41$ to be? Now you can make repeating decimals to order. (There is a unit on this you might like to look at Try Number and Algebra, Level 6, Babylonian Mathematics 2.) And in contrast, $\pi$ never repeats. It always meanders on never quite repeating anything that has gone before.

## Booke Review: A Treatise on the Astrolobe by Geoffrey Chaucer

Geoffrey Chaucer was the first great poet to write extensively in English. His works, in their original form, continue to be studied at universities today. His prose writings include only one scientific work and that is A Treatise on the Astrolobe which was first published in 1391.

The book was originally intended as a text book for Chaucer's son Lewis whose attainments in Latin had not reached the proficiency of what Chaucer called 'plain English words'. The prologue is a fine example of Chaucer's graceful and flowing style. Here's a part:

> Little Lewis my son, I have perceived well by certain signs thy ability to learn sciences touching numbers and proportions; and I also consider thy earnest prayer specially to learn the Treatise of the Astrolobe. Then forasmuch as a philosopher saith, 'he wrappeth him in his friend, who condescendeth to the rightful prayers of his friend', therefore I have given thee an astrolobe for our horizon, composed for the latitude of Oxford, upon which, by means of this little treatise, I purpose to teach thee a certain number of conclusions appertaining to the same instrument.

Chaucer stated for the book that he was not claiming originality but only to provide a reliable authority for the subject under study and for rendering it accessible in the English language. The book begins with a full description of the astrolobe, with diagrams, and shows how it may be used.

In a supplement to the book Chaucer shows how the astrolobe may be used to solve other problems of plane geometry like finding the height of acccessible objects such as towers or poles the feet of which can be reached, or others, inaccessible objects like hills, where it is impossible to reach ground level beneath the highest point.

astrolabe

A French mariner using an astrolabe to fix the position of a star, from a vellum manuscript of Jacques Devaulx (1583; Bibliothèque Nationale, Paris, France). The mariner's astrolabe was introduced in the mid-15th century, but did not see general use until the beginning of the 16th century. It was supplanted by the sextant in the 18th century. (Image © The Art Archive/Dagli Orti) (See
http://encyclopedia.farlex.com/ /viewer.aspx? path=hut\&name=aa334222.jpg.)

## Bill's Number Plates Problem

If you go to the Problem Solving section of the web site and look at Bill's Number Plates, Number Level 5 (6), you'll see the need to create number plates that show a given number. Room 1 of Greenmeadows School has come up with this list for the number 4. I have to say that I don't think that all of them are quite legal, for instance 444, but I admire their ingenuity. Well done Room 1. Can someone else do better?

| FOUR | 4 | 04 | 004 | 0004 | 00004 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 000004 | 04 | 004 | 0004 | 00004 | 000004 |
| FOURS | FOURZ | FOUR4 | 4 FOUR | 4FOUR4 | 2 PLUS 2 |
| 1PLUS3 | 3PLUS1 | 6B4TEN | 6B410 | 5B49 | 4B48 |
| 3B47 | 2B46 | 1B45 | 0B44 | ONEB45 | TWOB4 6 |
| 2B4SIX | 7B411 | 8B412 | 9B413 | 10B414 | 11B415 |
| 12B416 | 13 B 417 | $14 \mathrm{B418}$ | 15B419 | 16B420 | 17B421 |
| 18B422 | 19B423 | 20B424 | 21B425 | 22B426 | 23B427 |
| 24B428 | 25B429 | 26B430 | 27B431 | 28B432 | 29B433 |
| 30B434 | 31B435 | 32B436 | 33B437 | 34B438 | 35B439 |
| 36B440 | 37B441 | 38B442 | 39B443 | 40B444 | 41B445 |
| 42B446 | 43B447 | 44B448 | 45B449 | 46B450 | 47B451 |
| 48B452 | 49B453 | 50B454 | 51B455 | 52B456 | 53B457 |
| 54B458 | 55B459 | 56B460 | 57B461 | 58B462 | 59B463 |
| 60B4 64 | 61B465 | 62B466 | 63B467 | 64B468 | 65B469 |
| 66B470 | 67B471 | 68B472 | 69B473 | $70 \mathrm{B4} 74$ | 71B475 |
| 72B476 | 73B477 | $74 \mathrm{B478}$ | 75B479 | 76B480 | 77B481 |
| 78B482 | 79B483 | 80B484 | 81B485 | 82B486 | 83B487 |
| 84B488 | 85B489 | 86B490 | 87B491 | 88B492 | 89B493 |
| 90B494 | 91B495 | 92B496 | 93B497 | 94B498 | 95B499 |
| 40VER1 | 40VA1 | 2TIME2 | 4 TIME1 | Q4 | Q24 |
| QQQ 4 | QQQQ 4 | QQQQQ 4 | 8MNUS 4 | 7MNUS 3 | 6MNUS2 |
| 5MNUS1 | 4MNUS 0 | 9MNUS 5 | 8MNU54 | 7MNU53 | 6MNU52 |
| 5MNU51 | 4MNU50 | 9MNU55 | $4^{\text {TH }}$ | FOURTH | IV |
| 4AFTR0 | 3AFTR1 | 2AFTR2 | 1AFTR3 | 0AFTR4 | 8DVD2 |
| 12 DVD 3 | 16 DVD 4 | 4DVD1 | 20 DVD5 | 24 DVD6 | 28DVD7 |
| 32DVD8 | 36DVD9 | 2 X 2 | 4X1 | FOURSS | FOURZZ |
| 444444 | 44 | 444 | 4444 | 44444 | FOR |
| FORS | FORZ | FORSS | FORZZ | FORSSS | FORZZZ |
| 3ADD1 | 1ADD3 | 2ADD2 | FO_UR | F_OUR | F_OU_R |
| FO_U_R | F_O_UR | FOUR | FOUR | FOUR | F_O_R |
| F_O_R | F_O_R | FO_R | $\bar{F}$ _OR | FOR | FOR |
| F FOR | FŌUR | FOR | 4FOUR4 | $\overline{4}$ FOR4 | 4FOŪR4 |
| 4 FOR | F0R4 | FOUR4 | 4 FOUR |  |  |

## Solution to July's Endeavour

Last month we pointed out that 24 has the property that it is one less than a square number and that it's double is also one less than a square number. The next smallest number with that property is 840 .

## This Month's Endeavour

In the introduction we mentioned that 76 is an automorphic number because its square ends in itself. There is only one other 2-digit number with this property, can you find it?

## Solution to last month's Junior Problem

Last month's coding worked so well I thought that I'd try it again. All the hints this month are in last month's problem. Follow the directions and don't forget in your answer to say who you are and what school you go to. In case you win the month's prize you might want to give an address - your school address will do.

| DRSCWYXD | RNOBOUSC | KGKICYZV | OKCOCOXN |
| :--- | :--- | :--- | :--- |
| DROKXCGO | BDYDRSCA | EOCDSYXD | YKXNBOGR |
| SCOWKSVK | NNBOCCSC | DROCKWOK | CNOBOUCT |
| ECDBOZVK | MONOBOUL | IKXNBOGG | RKDSCKXN |
| BOGCNKEQ | RDOBCXKW | OIYEMKXC | OKBMRYXD |
| ROXJWKDR | CGOLCSDO |  |  |

So the message said "This month Derek is away so please send the answer to this question to Andrew his email address is the same as Derek's just replace Derek by Andrew. What is Andrew's daughter's name You can search the nzmaths web site."

When you do the search you find "Elizabeth". This month's winner was Baxter Robb of Seatoun Primary School.

## This Month's Junior Problem

To get this month's $\$ 20$ I've gone to counters on a chess board. Put four counters on the board as I have done on the board on the next page.


The counters can move by jumping over another counter to a vacant cell on the other side. So, at the moment, three of the counters can move but the one at the top right can't because it has no other counter to jump over.

The first problem is to see whether you can move the four counters into another place on the board so that they are again in the form of a square. (The four counters are in the bottom left 2 by 2 square of cells. Can you move them to some other 2 by 2 square of cells.)

The second problem is to look at nine counters in the bottom left 3 by 3 square of cells and see if you can move them to another 3 by 3 square of cells.
Then finally, look at 16 counters in the bottom left 4 by 4 square of cells and see if they can be moved to some other 4 by 4 square of cells.

## Afterthoughts

(1) A couple of less interesting versions of the game Tic-tac-toe might be called Noughts and Noughts (or Crosses and Crosses) where both players play with the same symbol. The winner, or the loser in the misère version, is the first to make three in a line.

Two other versions were devised in New Zealand. In Smatick both players in turn first play a O , then both play a X , then both O and so on, The first to complete three in a line is the winner (or the loser in the misère version).

Yet another version of the game, attributed to David Silverman, is called 'Your Move'. The rules are similar to the standard game except that one player tries to achieve a draw while the other attempts the usual three in a line.

Another game just called Noughts, can be found by clicking on our Problem Solving jigsaw piece. It's a Geometry Level 4 problem and I'd really like to see a complete solution.
(2) If you look carefully, maybe using the odd geometric instrument or two, you might be able to see the first 10 significant figures of $\pi$ in the arcs from notch to notch.


But first find the decimal point: it's to the left of the ' 1 ' on the right. The arc between the notch to the left of the ' 3 ' and the notch just above the rightmost ' 1 ', represents the ' 3 ' of $\pi$. An angle of about $108^{\circ}$ is at the centre of that arc. If you then turn through a further anticlockwise angle of $36^{\circ}$ you get to the next notch and you've turned out the ' 0.1 ' of $\pi$. After another anti-clockwise angle of $144^{\circ}$ you reach the next notch and you've turned out the ' 0.004 ' of $\pi$. I'll leave the rest of the 3.141592654 to you. As retired astrophysicist Mike Reed, the discoverer of the encoded value of $\pi$ said in the paper, "The tenth digit has even been correctly rounded up".

Now we've got hold of all of that it wouldn't be difficult for your class to design their own crop circles. Given that ' 1 ' in the picture is represented by $36^{\circ}$, it might provide an interesting exercise in using a protractor. On the other hand why stick with $\pi$ ? There are other interesting numbers that they might like to use for a design.

