Newsletter No. 75 **July 2008**

What do Roman numerals, fractions, Euclidean geometry, the astrolabe and the abacus all have in common? Well, they were all ways earlier people attempted to describe and understand their world. Two were systems for representing numbers, one was a system for representing plane shapes and drawing inferences about them, two were simple machines to aid particular calculations. They have now all been superseded, of course. That's not to say they're not useful or interesting, it's just that like chariots for transport, charcoal for cooking and flints for cutting we have better and more efficient ways of doing things. A knowledge of Roman numerals is still useful if you want to date old statues or books. Fractions would be of value if you thought you might like to tackle some of those old maths matriculation papers. Astrolabes make wonderful ornaments today, like the ones auntie brought back from her summer holidays made of seashells. Apart from a few results like the angles of a plane triangle summing to π radians (180° in another system) no one uses Euclidean geometry today. Like Boolean algebra in the 1950s it will soon be reserved for university study.
Where does that leave us? Well, I guess all I'm saying, maybe with a bit of tongue in cheek, is that knowledge advances, habits change, theories evolve. It doesn't make the old ones less interesting, it just replaces them with more convenient, efficient, up-to-date alternatives.

> *The history of maths for a classroom teacher is not a list of names, a chronological order of events and a stockpile of anecdotes. It should mean the evolution of mathematical ideas and knowledge, the personalities who were responsible for them, the times and climate which nurtured or perhaps stifled them and the impact and influence they had upon contemporary society.*
>
> **M.K.S.**

Catholic Bishops must retire at age 75. Mathematicians Jerome Cardan and Leonhard Euler died at that age, as did Archimedes, Alexander Graham Bell, Leonard Brezhnev, Duke Ellington, Edward Lear and both the Marx brothers Chico and Harpo.

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Diary Dates

Another reminder that Maths Week 2008 is the week of the 11th to 15th August. The site is still not available on the NZAMT site, but keep an eye out and we'll remind you each month.

New Additions to the Family

We are pleased to announce the arrival of a number of new activities to the Families section. If you are unaware of this part of the site you will find its link from the homepage. Each activity is designed for parents and caregivers to do at home with their child. The section has a numeracy focus and is arranged by stages of the Number Framework. The activities are designed to be short, fun and easy for families to do. Children can practice their doubles as they pair socks together, use food packaging to round numbers, practice fractions as they cut fruit and share pizza, and play variations on the games memory, snap, bingo and go fish to practice their number knowledge. The games in the Family section could also become part of games students play in maths at school.

Booke Review

The Art of Mensuration by Albrecht Dürer

Albrecht Dürer, perhaps the greatest of German artists, wrote on a range of subjects that reflected the interests of this Renaissance genius. He's probably most well known for his artistic prints - a number come to immediate mind; the rabbit, the hands praying and 'Melancholia' (next page) which incorporates a four-by-four magic square with its date along the bottom row.

Instruction in the Art Of Mensuration with Compass and Rule, as it is more correctly titled, was Dürer's first book and published in 1525. It is a treatise on descriptive geometry founded largely on Euclid. It contains numerous geometrical figures and unusual curves devised by Dürer. It also contains his original paper-folding methods for constructing solids, many of which are well known today and were first to appear in the book.

He wrote the book in German for students to improve their artistic methods and practices. It also had the effect of spreading the German language, as well as science generally, among those to whom Latin was not available. It was very quickly translated into other European languages. Early in the book he wrote:

> Seeing that it is useful for stonemasons, painters and joiners to know how to set up a common sundial on towers, houses and walls, I will write somewhat thereof …. Builders, painters and others sometimes have to show writing on high walls and so it is necessary for them to know how to form their letters correctly.

The book assumed a good knowledge of plane and spherical trigonometry. Dürer paid attention to many of the classical problems of geometry as well as to plane curves of his own devising. He shows, for example, how to construct a heart-shaped curve with a double ruler. So, if you fancy a message on the side of your house you don't need to look for a tagger.

Tickle Tackle

Last month I told you about Brian's new book Tickle Tackle. Brian's publishers even offered a copy of the book for answering the problem below. But I had no answers. I'll leave the prize on offer for just one more month. Here is the problem again.

> *$2000 is divided into ten purses, each labelled with the sum of money it contains ($1, $4, $8, $16, $32, $64, $125, $250, $500, $1000), which are to be shared between two players with the rule that each player in turn may take 1, 2 or 3 purses of the lowest value. If both players use the best strategy, how much does each player get?*

Does it help to say that the first player should take the two smallest value bags to start with? Why?

Please send your solutions to derek@nzmaths.co.nz. Make sure that you add your name and address. If you are a school student, please let us know the name of the school and your teacher's name.

Maybe They Got It Right

Following on from a point in our May issue about adding fractions where we wrote:

> It appears that many students add 3/4 and 1/2 by adding the numerators together to get 4 and the denominators to get 6 and then giving the answer as 4/6. A quick check shows that it isn't correct.

Reg Alteo once commented (in the now defunct *Mathsmatters* magazine) that 7/8 + 5/7 was a terrible problem. He added that even put in context the problem seemed contrived. For example, Gibb's Coal Company delivered 7/8 of a tonne of coal on Monday and 5/7 of a tonne on Tuesday. How much coal is that altogether? Uggh! Or how about, Mary ate 7/8 of her own pie and 5/7 of Melissa's, how much pie is that altogether? Ughh again! I mean, whoever needs to add fractions like that these days?

He wrote, is it any wonder that our students resolutely refuse to give the expected answer 89/56. It has very little meaning for them and, if the truth be told, very little for me. What answer do they usually give? In my experience students use a much simpler algorithm for adding fractions, namely a/b + c/d = (a+c)/(b+d). For years I've ranted and raged, cajoled and bribed - anything, to force the concept of common denominator and the correct method of adding fractions. And for over a quarter of a century my students have smiled and, for a while at least to please me, gone along with a/b + c/d = (ad+bc)/bd. After a week or so though they are invariably back to using their simpler algorithm.

Why do they persist in doing this? They must know they are wrong. But why has that nagging doubt crept into my mind? My students have often proved right in the past (like the time I told them emphatically that there was only one solution to a problem and within minutes they had found another). But they can't be right in this case …. Can they? Can they?

> *"Er, John. Can you explain why you put* 7/8 + 5/7 = 12/15?"
>
> *"Please sir ... (*aren't those old-fashioned conventions delightful?) *"... I got 7 out of 8 for my maths homework last week and 5 out of 7 this week. Altogether that's 12 out of 15."*

The nagging doubt becomes a rushing torrent. Oh dear! We are encouraged to put our teaching of mathematics in context. The context given by John is much more familiar than the Gibb's Coal problem. Maybe they got it right!! I mean, if the a/b + c/d = (a+c)/(b+d) algorithm is more familiar to my students and the problems are such that the method is correct, maybe I should be teaching it. What a delightful thought. No more hassles when teaching fractions. But does the method apply widely enough to be useful?

<u>Example</u>: June drives 80 km in 2 hours, then 150 km in 3 hours. What is her average speed for the whole journey?

$$80/2 \oplus 150/3 = 230/5 = 46 \text{ km.p.h.}$$

I use ⊕ instead of + in these cases to avoid confusion with the conventional method of adding fractions.

Example: Richard gains 3 wickets for 20 runs in the first innings and 5 wickets for 130 runs in the second. What are his figures for the match?

$3/20 \oplus 5/130 = 8/150$ (8 wickets for 150 runs)

Well, there are certainly enough applications and there is absolutely no doubt that the contexts are more familiar and more pleasing to students.

Reg Alteo concluded: From now on I will only teach the $a/b + c/d = (ad+bc)/bd$ algorithm to the most able students. The less able will never understand it and will never need to use it. For them the $a/b \oplus c/d = (a+c)/(b+d)$ makes much more sense.

Of course, it was all tongue in cheek but the item did elicit a reply from John C. Turner then in the Department of Mathematics and Statistics at the University of Waikato who in writing about some new ways of looking at Pythagoras' Theorem added:

'By one of life's remarkable coincidences Reg Alteo's article *Maybe They Got It Right* deals with the rules for adding fractions:

Rule 1: $a/b + c/d = (ad+bc)/bd$

and Rule 2: $a/b \oplus c/d = (a+c)/(b+d)$

He makes a good case for not chastising students if they use Rule 2 rather than Rule 1. You should gently advise them (if they are adding ordinary fractions) of the rules of the game YOU wish them to play at that particular time. (Of course, it gets very frustrating for a teacher if his/her students will *never* play the right game).

The coincidence I referred to is that Rule 2 is precisely the correct one to use in our theories of Pythagorean triples and simple continued fractions. In number theory $(a+c)/(b+d)$ is called the mediant of a/b and c/d. There is a fascinating topic known as Farey sequences which is important in number theory and uses mediant addition. We also use it in our researches into braiding theory.

Solution to June's Endeavour

The smallest solution is $x = 15$, $y = 20$ and $z = 12$. There is a general formula for this problem which can be obtained with a bit of persistent algebra. It is: $x = m^4 - n^4$, $y = 2mn(m^2 + n^2)$, $z = 2mn(m^2 - n^2)$. You might like to check that these are indeed solutions to:

$$\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{z^2}$$

This Month's Endeavour

24 has the property that it is one less than a square number and that it's double is also one less than a square number. I wonder if you could find the next smallest number with that property?

The answer is in Afterthoughts … DON'T LOOK!

Solution to last month's Junior Problem

Last month we tried codes for the first time and got many students sending in their response. First the code with preliminaries.

Just so that the code's not impossibly difficult I'll tell you two things and give you a hint. First I have written out (but not where you can see it), all the letters of the alphabet in order and put the numbers from 0 to 25 underneath them. So A is 0, B is 1, and so on up to Z which is 25. Second, I'm using a shift cipher or Caesar cipher. (At worst now you can look this up on the web.) Then the hint is that there is one letter in the alphabet that gets used more than any other.

So here is the message.

G L Z K X E   U A N G B K   J K I U J K   J Z N O Y S

K Y Y G M K   K S G O R Z   N K C U X J   K K F K E V

K K F K E Z   U J K X K Q   G Z T F S G   Z N Y . I U . T F

If you can decode this you will know what to do next.

You might wonder now why I got so many students sending me an email that simply said "EEZEY PEEZEY". If you haven't done it already, I'll leave it to you to work out but there is an additional hint that the two dots in the last group of letters here are part of an email address. The rest is up to you.

But I have to say that one of the reasons that I had so many replies last month was that two teachers gave it to their class to try. There must be more teachers out there that can do this. But this time tell the poor students that there is a prize for getting the correct answer!

Anyway, this month's winner is James Ayres of Room 11, Raumati Beach School. Well done James and all of the others too.

This Month's Junior Problem

Last month's coding worked so well I thought that I'd try it again. All the hints this month are in last month's problem. Follow the directions and don't forget in your answer to say who you are and what school you go to. In case you win the month's prize you might want to give an address – your school address will do.

DRSCWYXD	RNOBOUSC	KGKICYZV	OKCOCOXN
DROKXCGO	BDYDRSCA	EOCDSYXD	YKXNBOGR
SCOWKSVK	NNBOCCSC	DROCKWOK	CNOBOUCT
ECDBOZVK	MONOBOUL	IKXNBOGG	RKDSCKXN
BOGCNKEQ	RDOBCXKW	OIYEMKXC	OKBMRYXD
ROXJWKDR	CGOLCSDO		

Afterthoughts

The number you were looking for is 840.