Newsletter No. 74 June 2008

Each newsletter we give a little information about the number of its issue. This month, for example, is our 74th newsletter. 74 is a bit of a challenge to write about, at least, in mathematical terms and if anyone knows any further interesting bits of information about it we'd be pleased to hear them. Each issue we also write a little about other facts relating to its number, including the names of a few people who quit this life at that age. Mathematicians generally seem to live long lives - their average life-span, according to a quick check, is higher than that of the general population, or was in earlier centuries. This could be put down to the fact that mathematicians then usually led more comfortable lives, gaining reasonable rewards including good nutrition from their labours. This led me to thinking about those who died young and there have been a few. The Norwegian Neils Abel died aged 26. Remember the quadratic $ax^2 + bx + c = 0$ has the formulaic solution $\frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$. It turns out that general cubic and quartic equations (with third and fourth powers of x, respectively), have similar kinds of formulae. Among other things, Abel proved that unlike these last three, it is impossible to find a simple formula for the solution of the general quintic. Although recognised as a genius in his field, albeit only towards the end of his life, Abel struggled to make ends meet on the miniscule stipends he received for the teaching jobs he was forced to take. He eventually died of tuberculosis brought on by lack of a good diet and warm environment. Evariste Galois did some good work in the group theory and all sorts of other things and promised to be one of the most original mathematicians of the nineteenth century until he was unfortunately killed in a duel at the age of 20.

As I mentioned, there's not a lot you can say about 74, apart from the fact that 0.74 recurring is 74/99 when represented as a fraction and one or two other trivial things. Maybe you know more and would like to share some of your knowledge. It is true that both Daniel Malan became Prime Minister of South Africa and Jack Wingfield ran his last marathon aged 74. Guiseppe Garibaldi, Frederick the Great, Stan Laurel, Jawaharlal Nehru and Mark Twain all died at that age.

Mr. Smith died. Now he's teaching.

Desktop graffiti.

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What's new on nzmaths.co.nz

Have a look in the Families section of the website. We have added 30 new activities to this section this month. In the next month you can expect to see new material added to the SNP section of the site as well as to the Learning Objects section and to the Families section.

Diary Dates

A reminder for those that are thinking about their term 3 plans that Maths Week 2008 is the week of the 11th to 15th August. The site is not yet available on the NZAMT site, but keep an eye out and we'll remind you each month.

Hidden Treasures in nzmaths

Our thanks to Frances Neill for writing this.

In the process of updating the Achievement Objectives on the units on the nzmaths website we found some hidden treasures. The unit Breakfast Biscuits is a cross strand level four unit. The humble weetbix is tested for its strength with astounding results as we discover how many bricks it can withstand. Had you noticed how two weetbixs fit just nicely into a circular bowl? Can you design a bowl for four weetbix? Ever wondered how much milk those thirsty weetbix really do soak up? Investigate and find out. Go to http://www.nzmaths.co.nz/number/CrossStrand/biscuits.aspx and gain a new appreciation for your cereal.

Other cross strand units can be found on this page: http://www.nzmaths.co.nz/number/crossstrand.aspx .

Remember too that the Figure it Out Theme books provide opportunities for you to integrate the different strands of mathematics into a theme. The Figure It Out theme titles include Levels 2 – 3 Gala and Under the Sea, Level 3 Time Travel and At Camp, Levels 3 – 4 Sport and Moving House, Level 4 – 4+ Getting Around and Disaster Strikes!

New Review: Tickle Tackle by Brian Stokes

And now we move on to a real live book.

Those of you who enjoy problem solving and/or have read and worked through Brian's other book, Stretch, Bend and Boggle, will probably also like his new book Tickle Tackle (published by Kanuka Grove Press). There you will find 100 problems to keep you going for some little while, along with 10 small essays and 10 so-called 'classic perennials'. If you need more persuasion you might like to know that Stretch, Bend and Boggle won a Children's Book Award in 1994 and has sold around 5000 copies.

The latest book is divided into ten chapters each of which is based on a theme. Let's have a look at a couple of these and see something of what they contain to get some idea of the flavour of the book. Chapter 3 is on 'Sport in the Open Air'. There you will find problems such as 'A Weighty Matter'. This problem asks:

> *Five batches of cricket balls were sent along for a Test Match, each batch clearly identified. The manufacturers then sent a fax stating that one batch had each ball 5g below the specified weight of 150 g. The Secretary said "No problem; I have a spring balance which can pick a 5g discrepancy, and I can find the offending batch in just one weighing". How did she do it?*

Now that requires a lot of thinking and not very much real formal school mathematical knowledge. This is the case for many of the problems but some you will have to know about factorisation and Pythagoras' Theorem. Only a few really need an extensive knowledge of secondary mathematics.

Anyway, the above chapter then has a little 'essay' of a page or so that takes up the idea of divisibility by numbers above 10. This is followed by the Classic Perennial (old chestnut, a problem that's been around for a while) of The River Crossing. As a result of old age and experience you may know this one.

> *A man arrives at a river bank with a fox, a rabbit and a giant lettuce. His boat will only carry him and one of the others, but the fox and the rabbit cannot be left by themselves, nor can the rabbit and the lettuce. How many crossings are needed to ferry them both across the river?*

This Perennial is completed by looking at a similar problem with a slightly different answer.

The chapter continues with some Hints, the answer to the similar problem to the perennial, solutions to the 10 problems and the answer to the Perennial. There is a Hint for each problem and these could be critical for helping students to get started.

And that format follows for each chapter. Let's look at the chapter "Trivial- Or Are they?". There we have a problem called Holy Hundred.

> *When will the first day of a new century next be a Sunday?*

These 10 questions are followed by the essay “A Bit About Groups” which essentially thinks about some abstract algebra but keeps it down to earth by looking at multiplication modulo certain numbers as examples.

The Classic Perennial here is the Seven Bridges of Königsberg, which you may already know about. Then we get the usual hints and solutions.

I would think that almost all of the problems in the book would be accessible to very bright Year 8 or 9 students and although they will tickle university students and secondary teachers, there are enough problems that don’t require any formal mathematics that they could be given to more senior and able primary students, especially if they worked in groups and were given the odd hint by their teacher. But I can understand if primary teachers wanted to first fatten their charges up on the problems from Stretch, Bend and Boggle. Secondary teachers though should have no reason not to jump straight into Tickle and Tackle.

Oh and I’ll let you discover who Tickle and Tackle are. But I should say that they are contained in the illustrations that form a delightful feature of the book. It’s likely too that Brian himself has been drawn on several of the pages but don’t let that put you off too much.

This month we will give away one of Brian’s books to the best solution to Problem 4.7. This problem is called the Money Purses.

> *$2000 is divided into ten purses, each labelled with the sum of money it contains ($1, $4, $8, $16, $32, $64, $125, $250, $500, $1000), which are to be shared between two players with the rule that each player in turn may take 1, 2 or 3 purses of the lowest value. If both players use the best strategy, how much does each player get?*

Please send your solutions to derek@nzmaths.co.nz before 28th June. Make sure that you add your name and address. If you are a school student, please let us know the name of the school and your teacher’s name. All the best.

Booke Review

The Whetstone of Witte by Robert Recorde

Robert Recorde is considered to be the founder of the English school of mathematics. He lived in the first half of the sixteenth century at a time of social change and rose to a position of great trust and responsibility. He taught at the University of Oxford before studying medicine at Cambridge and becoming physician to Edward VI and Mary I.

He published four important mathematical text-books. His *The Grounde of Artes*, on the elements of arithmetic, gained an immediate popularity lasting through twenty-eight editions from 1542 to 1699. His *Whetstone of Whitte*, published in 1557, was the first book in English to be devoted to the principles of algebra. It introduced the sign = for equality. Robert Recorde says he selected that particular symbol because than two straight parallel lines "noe 2 thynges can be moare equalle". The book includes a method of finding the square root of an algebraic expression.

The book in its original form is difficult to read both because of the irregular spelling in use at the time and the, what we would call, font. The preface, which he dedicates ‘to the gentle Reader’ (and I have kept the original spelling and punctuation) begins:

> Although nomber be infinite in increasyng: so that there is not in all the worlde, anything that can excede the quantitie of it: nother the grasse on the ground, nother the droppes of water in the sea, no not the small graines of sand through the whole masse of the yearth: yet maie it seme by good reason, that noe man is so experte in Arithmetike, that can nober the commodities of it.

The book represented both the knock on the door for algebra in England and the end of Robert Recorde’s career. He died soon after its publication.

[For more on Robert Recorde and his individual way of multiplication see our issue of November 2002]

Solution to May’s Endeavour

You were asked to find the smallest positive number that can be expressed as the sum of two cubes in two different ways. This problem is connected to the Indian mathematician Ramanujan. You can find out more about him by visiting http://www-groups.dcs.st-and.ac.uk/~history/Biographies/Ramanujan.html.

Ramanujan was a mathematical prodigy who lived near Madras. In view of what we were talking about above, it is known that as a child Ramanujan was shown how to solve cubic equations and he went on to find his own method to solve the quartic.

When he left school Ramanujan really wanted to do research and he eventually contacted G.H. Hardy, an eminent professor at Cambridge (see http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Hardy.html). On seeing what Ramanujan had done up to that time, Hardy immediately invited Ramanujan to Trinity College to work on his formal mathematical development, he earned the equivalent of a PhD there, and also to continue with his research.

It was during this period after Ramanujan completed his degree that he became ill and was taken to hospital. Hardy tells the following story in his book on Ramanujan.

> *I remember once going to see him when he was lying ill at Putney. I had ridden in taxi cab number 1729 and remarked that the number seemed to me rather a dull one, and that I hoped it was not an unfavourable omen. "No," he replied, "it is a very interesting number; it is the smallest number expressible as the sum of two cubes in two different ways."*

Of course that is the problem that we posed last month. . However, as the problem is worded there is a smaller solution, $728 = 6^3 + 8^3 = 9^3 + (-1)^3$, but of course you have to allow negative numbers.

If the word ‘positive’ is omitted from the original problem so that you are asked to find, the smallest number that is the sum of two cubes in two different ways, there are smaller solutions. Can you find any? One is given in *Afterthoughts* below.

This Month’s Endeavour

Can you find the three smallest whole numbers for which the sum of the squares of the reciprocals of two of them is equal to the reciprocal of the third? That is, find the smallest x, y, z such that:

$$\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{z^2}$$

Solution to the month before last’s Junior Problem

I can’t get rid of the problem from April that asked for a give a diagrammatic justification (or any justification for that matter)?

$1 = 1^2$
$1 + 3 = 2^2$
$1 + 3 + 5 = 3^2$
$1 + 3 + 5 + 7 = 4^2$
$1 + 3 + 5 + 7 + 9 = 5^2$.

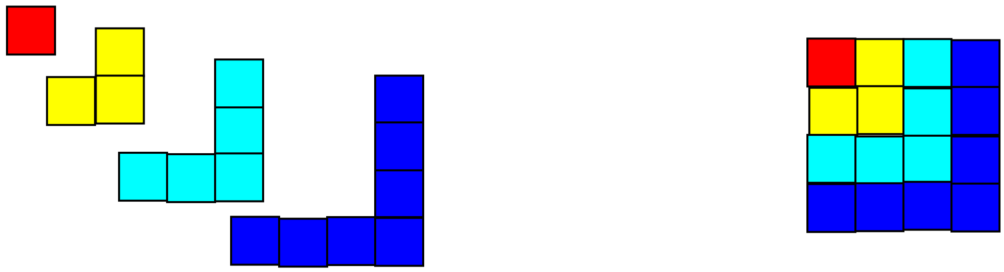
It looks as if

$1 + 3 + 5 + 7 + 9 + 11 = 6^2$ and
$1 + 3 + 5 + 7 + 9 + 11 + 13 = 7^2$.

I found an answer but it isn’t the best one around. Jim Hogan, a secondary maths advisor from Taupo, gave me the better one below. To rub salt into the wound, Jim reminded me that this method was included in the “Thinking inside the square” unit for the Secondary Numeracy Project and it’s actually on the nzmaths web site.

The teachers’ notes for the unit are available here:
http://www.nzmaths.co.nz/Numeracy/SNP/Activities/ThinkingInsideTheSquare.aspx

The diagram below comes from Exercise 4 of the downloadable PDF.



You can see that the odd numbers can be made to form L-shapes and that these can be easily put together to make a square. Clearly this gone on for ever.

We can put this down in algebraic terms by noting that

$1 = 2 \times 0 + 1$ $3 = 2 \times 1 + 1;$ $5 = 2 \times 2 + 1;$ $7 = 2 \times 3 + 1.$

Every odd number can be written in the form $2 \times n + 1 = 2n + 1$.

Now from what we have just seen

$$1 = 1^2 = (0 + 1)^2$$
$$1 + 3 = 2^2 = (1 + 1)^2$$
$$1 + 3 + 5 = 3^2 = (2 + 1)^2$$
$$1 + 3 + 5 + 7 = 4^2 = (3 + 1)^2$$
$$1 + 3 + 5 + 7 + 9 = 5^2 = (4 + 1)^2$$
$$1 + 3 + 5 + 7 + 9 + 11 = 6^2 = (5 + 1)^2 \text{ and}$$
$$1 + 3 + 5 + 7 + 9 + 11 + 13 = 7^2 = (6 + 1)^2.$$

So if we try to add the odd numbers from 1 up to $2n + 1$, whatever that is, we'll get $(n + 1)^2$. This is because the L-shaped piece that that makes up the number $2n + 1$ has $n + 1$ squares on each side.

Solution to last month's Junior Problem

Last month we went on to even numbers and asked this one.

We've seen how to add up the first lot of odd numbers so what about adding the first lot of even numbers?

$$2 = 1 \times 2$$
$$2 + 4 = 2 \times 3$$
$$2 + 4 + 6 = 3 \times 4$$
$$2 + 4 + 6 + 8 = 4 \times 5$$

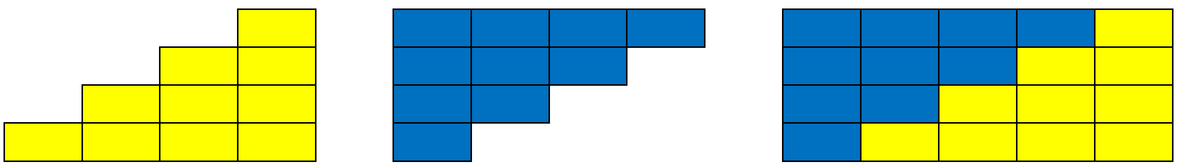
Does this continue? Can you justify it in any way? For a book voucher of \$20 can you start it off?

Well this certainly continues and works by the rule that we developed in the lsat newsletter. It turns out that if you add the first and the last number and multiply by the number of terms you get the answer you want.

But the nice way to see these sums is to use some more diagrams and the fact that

$$2 = 2\{1\} = 1 \times 2$$
$$2 + 4 = 2\{1 + 2\} = 2 \times 3$$
$$2 + 4 + 6 = 2\{1 + 2 + 3\} = 3 \times 4$$
$$2 + 4 + 6 + 8 = 2\{1 + 2 + 3 + 4\} = 4 \times 5$$

So what we do is to take two lots of, say 1 + 2 + 3 + 4, to first form a ‘triangular shape. Then we add these two shapes together to get a rectangle. Here’s how it works.



This Month’s Junior Problem

I don’t think that we have done codes before. So this month I’m going to use a simple code to send you a message. Just so that it’s not impossibly difficult I’ll tell you two things and give you a hint. First I have written out (but not where you can see it), all the letters of the alphabet in order and put the numbers from 0 to 25 underneath them. So A is 0, B is 1, and so on up to Z which is 25. Second, I’m using a shift cipher or Caesar cipher. (At worst now you can look this up on the web.) Then the hint is that there is one letter in the alphabet that gets used more than any other.

So here is the message.

G L Z K X E   U A N G B K   J K I U J K   J Z N O Y S

K Y Y G M K   K S G O R Z   N K C U X J   K K F K E V

K K F K E Z   U J K X K Q   G Z T F S G   Z N Y . I U . T F

If you can decode this you will know what to do next.

Afterthoughts

$-1729 = (-1)^3 + (-12)^3 = (-9)^3 + (-10)^3$. When the question is asked in this way there is, of course, no such thing as a smallest solution.