Newsletter No. 73 **May 2008**

A long, long time ago, in the predawn of mathematics teaching … well, my mathematics teaching anyway, I met my first Head of Department. He was a feisty, abrupt and innovative teacher who prided himself on being the first in his area to set up a maths lab in his classroom. Motivated greatly by that iconic teachers' reference, *Mathematical Models* written by H. Martyn Cundy and A.P. Rollett (Oxford, 1951 - see Booke Review below), his room was aswarm with meticulously made polyhedra including the thirteen Archimedean solids and their duals. He also had hyperboloids and hyperbolic paraboloids made from perspex and nylon filament, Möbius strips, Klein bottles and a number of mechanical models. Included in these was a bagatelle-like contraption used to demonstrate binomial probabilities, a coin-tossing machine, a series of linkages and a harmonograph for drawing curves, I still relish in the name, and other pieces of home-made equipment. According to Cundy and Rollett, the harmonograph was a popular diversion in Victorian drawing-rooms. It had suffered a decline and was rarely seen in the 1950s - or today I would have thought. In it a pen somehow harnessed the movements of two conical pendulums and the most delightful curves could be drawn. (see Afterthought.) As the pendulum motion subsided the continuous curve diminished in size so that a fingerprint-like pattern emerged - concentric but continuous curves, as it were. Alongside all these gadgets in his room were trays and trays of pricked-out plants - our HOD loved his garden you see - and a kettle always on the hob - he also relished a cuppa between classes. He continued teaching at the school long after I left, regularly falling foul of ever-changing Principals and Department personnel. Classrooms weren't supposed to provide tea or nurture plants, you see, at least, not maths classrooms. Eventually Department pressure became too great and he was closed down, so to speak. He didn't mind, he could live quite nicely on his pension and the little market-garden he'd set up during his teaching years.

Characters like that are probably long-gone from teaching, although we'd like to hear of any you've come across. Classrooms full of mathematical models and equipment are also unlikely to be seen these days, at least, at secondary school level - we won't conjecture why.

> *For the purpose of teaching, it is essential to master the primitive methods of practical mathematicians before attempting to introduce the strict methods of the pure mathematician.*
>
> **W.W. Sawyer**

Not only is 73 a prime number but all integers can be represented as the sum of at most 73 sixth powers. Not a lot of people know that!

Konrad Adenauer became Chancellor of Germany at age 73. The mathematician Arthur Cayley died at that age, as did Casanova, Noël Coward, Charles Darwin, Alexander Fleming and El Greco. (I'm a bit worried about what we're going to put in here when we get to 173.)

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What's new on nzmaths.co.nz

We have now updated the Achievement Objectives for every unit on the site so that they relate to The New Zealand Curriculum (2007) rather than Mathematics in the New Zealand Curriculum (1992).

Geometry

We seem to have put quite a lot of geometry in this issue. And it's beautiful though quite complicated stuff. Trying to describe algebraically the things that the harmonograph produces is pretty difficult. However, anyone can appreciate the prettiness of the shapes, hopefully even if that anyone is a young student. And we have shown in an Afterthought where to go on the web to produce your own versions.

We also appeal to the web for three-dimensional models of various shapes. There are nets available that you can copy for your students. You don't have to understand how these are achieved to enjoy them. What's more they look good coloured and hung around the class.

We'd like to include some more geometry. If you have any ideas from your own classroom please let us know about them. We pay in book vouchers for any article published. (Of course we pay for articles on things other than geometry too.)

Compendium

For the last few years the Ministry of Education has commissioned research on the Numeracy Project. This research has appeared in Compendia. Last year two appeared. One was called "Findings from the New Zealand Numeracy Development Projects 2006" and the other "Evaluations of the 2006 Secondary Numeracy Project." In a few months the results from last year's research will be published. When it is we'll announce it here but it will arrive in your school automatically.

We thought that it might be a good time to remind you of some of the findings from last year.

The primary book, which we will concentrate on here, was divided into three parts, four if you include a longish appendix on patterns of performance. This appendix was compiled from the data that teachers sent in after their initial and final tests of students. Incidentally, this data is a large and useful collection that surpasses almost anything else in mathematics education world-wide. I don't know of any country that has such an extensive collection of data collected at the chalk face. From this appendix you can find out such things as the composition of the Year 5-9 cohort in 2002-2006, the percentages of Year 2-9 students at framework stages on each domain; percentage of students who progressed to a higher stage relative to initial stage by ethnicity and so on. A number of these tables will enable you to compare your own students with the nation as a whole.

But what were the three sections I started to talk about earlier? Well there was Student Achievement, Leading Teachers and Sustaining Numeracy in Schools and Professional Practice. In the first of these, three things stand out. First, the performance of students who had been in schools that had been involved in the project for a while generally outstripped that of comparison groups. Second, there were still differences in performance between ethnic groups. Finally, students who persist with counting as a strategy have weaknesses in other areas.

In the second category it was interesting to find that lead teachers often had different priorities in their view of important aspects from teachers and principals. It was also interesting to note that teachers are starting to show use of the principles of the Numeracy Project in other strands than number, there is more carefully planned instruction, and the role of the NDP resource books seems to be changing from one of reliance to one of guidance.

In the Professional Practice section the three papers all had a different flavour but I'll take one of these because it was concerned abut fractions and this was a feature of other papers in the Compendium. It would seem from the data collected in New Zealand as well as that from overseas that the area of proportional reasoning/fractions is difficult for most members of society and not just for children in school. One of the papers in the book contained a survey of the literature on fractions and raised the following points among others.

- Fractions are an important area of mathematics;
- Learning about fractions is difficult for most students;
- Teaching understanding of fractions can aid students' learning of algebra; and
- "adding across" denominators as well as numerators is a common error.

This last point also appeared elsewhere and it might be worth closing with a discussion of "adding across". What does that actually mean? It appears that many students add $\frac{3}{4}+\frac{1}{2}$ by adding the numerators together to get 4 and the denominators to get 6 and then giving the answer as $\frac{4}{6}$. A quick check of that shows that it isn't correct. But it's perhaps even easier to see by adding $\frac{1}{2}+\frac{1}{2}$. Surely the answer to this is 1 but by adding across you get $\frac{2}{4}$ which is equivalent to $\frac{1}{2}$.

Unfortunately it is much harder to add fractions correctly. This requires bringing the fractions to the same division of unity. So in $\frac{3}{4}+\frac{1}{2}$ it's necessary to first think of the $\frac{1}{2}$ as being $\frac{2}{4}$. Now we have to add 3 lots of 4 to 2 lots of 4. Since we are dealing with the same size of fractions (fourths or quarters) we get 5 lots of 4. This is what we always do when adding like objects. So $\frac{3}{4}+\frac{1}{2}=\frac{3}{4}+\frac{2}{4}=\frac{5}{4}$.

It will be interesting to see what comes out of this year's reports.

Booke Review

Mathematical Models by H. Martyn Cundy and A.P. Rollett (see Afterthought)

The preface to the first edition of this book (Oxford, 1951) begins 'I have often been surprised that Mathematics, the quintessence of Truth, should have found admirers so few and languid.' The cause, the authors claim, is that ' Reason is feasted, Imagination starved.' The purpose of the book, the authors say, is to assist reason by the stimulus of imagination.

When it was first published this book had little impact on teachers - the world was not ready for practical applications of mathematics in the classroom. 'Modern Math' had not yet become the vogue. Within half a decade though teachers became aware of the immense motivational value of model-making as well as the overspill of good mathematics it engendered. *Mathematical Models* soon became a standard handbook for the classroom and remained so for next thirty years.

The first section describes how by using techniques of dissection, curve-stitching, paper-folding, knotting and tessellation much of the groundwork of plane geometry and trigonometry can be investigated.

The section on polyhedra is probably the one most familiar to teachers, with templates and theory for just about every polyhedra it is possible to construct. What enthusiastic mathematics teacher hasn't had a go at making a great stellated triacontahedron? (What teacher knows what that is?) For a hint as to where you could start looking for ideas of how to make polyhedral see the Afterthoughts.

The remaining chapters describe how to make wire and wooden models, quadric surfaces, Möbius strips and Klein bottles. They consider packing problems and models in mechanics and statistics to show such phenomena as cycloid curves, the binomial distribution and the evaluation of π. There is a chapter on models in hydrostatics and electrical models to solve equations with the end-piece devoted to making linkage machines like the pantograph for drawing curves.

It is a monumental and comprehensive work that should still be on every mathematics teacher's bookshelf.

Solution to April's Endeavour

'Search and find' readily locates the smallest number that is equal to the sum of two squares in two different ways. It is $50 = 1^2 + 7^2 = 5^2 + 5^2$.

Now find all other integers below 100 that can be expressed as the sum of two squares in two different ways. Answers in Afterthoughts below.

This Month's Endeavour

What is the smallest positive number that is the sum of two cubes in two different ways? And what has this to do with Hardy and Ramanujan (whoever they were, see Afterthoughts).

Solution to Last month’s Junior Problem

I’m afraid that I had no solution to last month’s problem so let me remind you what it was and then show how it might have been done.

Can you continue this pattern, perhaps say what the general pattern is, and give a diagrammatic justification (or any justification for that matter)?

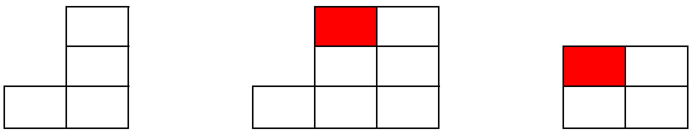
$$1 = 1^2$$
$$1 + 3 = 2^2$$
$$1 + 3 + 5 = 3^2$$
$$1 + 3 + 5 + 7 = 4^2$$
$$1 + 3 + 5 + 7 + 9 = 5^2.$$

It looks as if

$$1 + 3 + 5 + 7 + 9 + 11 = 6^2 \text{ and}$$
$$1 + 3 + 5 + 7 + 9 + 11 + 13 = 7^2.$$

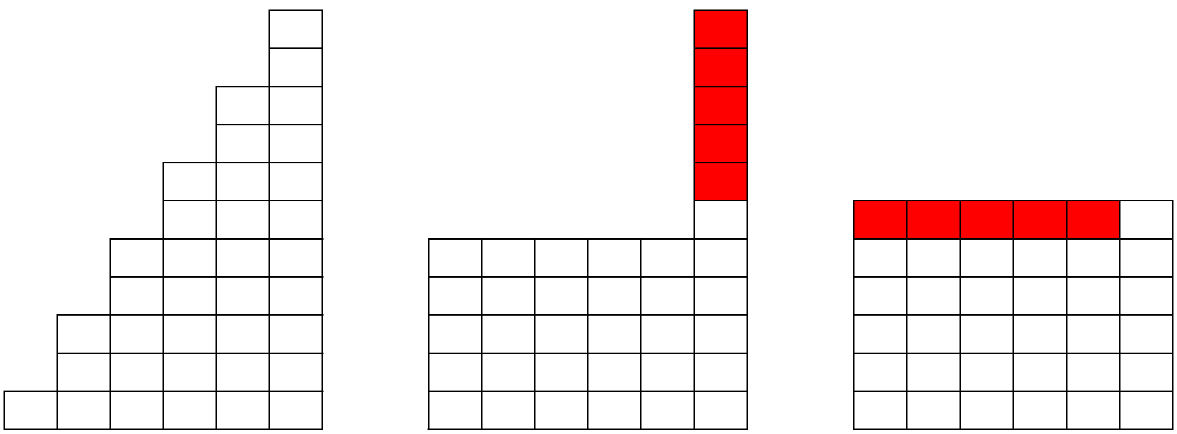
So how to show that these are correct?

One way to do this is with blocks. Let's start at the lower end and then do a higher one. First 1 + 3 = 2^2.



The left situation shows a 1-block added to a 3-block. In the middle we colour the top part of the 3-block. If we move that into the position of the coloured part of the final diagram we see we have formed two 2-blocks.

Now let's try 1 + 3 + 5 + 7 + 9 + 11 = 6^2. If we are moving slowly up the list, we know by this time that 1 + 3 + 5 + 7 + 9 = 5^2.



Here we start off with the blocks of the right heights. Then we know that the first five columns can be converted to 5^2 or a 5 x 5 set. The last column is 6 + 6 high and we colour the top 5 pieces. You can see how we move these 5 pieces on top of the 5 x 5 section to get a 6 x 6 set. Hence the original sum adds to 6^2.

Another way to do this is to start with a sum of odd numbers and call it S. So take the following sum for example.

S = 1 + 3 + 5 + 7 + 9 + 11 + 13.

We actually don't affect the sum by turning it round the other way. So

S = 13 + 11 = 9 + 7 + 5 + 3 + 1.

Now add these two term by term. Since 1 + 13, 3 + 11, 5 + 9, 7 + 7, 9 + 5, 11 + 3 and 13 + 1 all equal 14 we get

2S = 14 + 14 + 14 + 14 + 14 + 14 + 14 = 7 x 14

Dividing both sides by 2 we get

S = 7 x 7 = 7^2.

If you care to introduce a little algebra you'll be able to prove that the sum of the first n odd numbers is n^2.

This Month's Junior Problem

We've seen how to add up the first lot of odd numbers so what about adding the first lot of even numbers?

2 = 1 x 2
2 + 4 = 2 x 3
2 + 4 + 6 = 3 x 4
2 + 4 + 6 + 8 = 4 x 5

Does this continue? Can you justify it in any way? For a book voucher of $20 can you start it off?

Afterthoughts

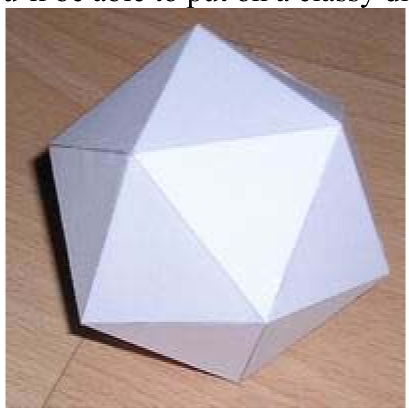
(1) Harmonograph

There is a harmonograph in the mathematics and Statistics Department at the University of Otago. Unfortunately it lives a precarious life and is prone to losing it's pen (through someone borrowing it) and never having fresh paper (for the same reason). However, it does perform interesting gyrations that amuse children of all ages. If you'd like to see the sort of pictures that a harmonograph produces you can do that by going to the site http://www.sequences.org.uk/harmonograph/ and using the interactive 'machine' there. You may have to install some software to make it run though. You can find out more about harmonographs in general, and can see some of the very pretty pictures that they produce, by going to http://en.wikipedia.org/wiki/Harmonograph.

The simpler form of this curve is the Lissajou's curves that can be found on http://www-groups.dcs.st-and.ac.uk/~history/Biographies/Lissajous.html along with a biography of the man. Some of you probably already know that the Australian Broadcasting Corporation has a Lissajou curve as it's logo. You can access a web site that will allow you to draw your own Lissajou curve too.

(2) Polyhedra

The pictures below come from http://www.korthalsaltes.com/. If you would like to produce some of these three-dimensional figures this site also has the nets of some pretty fantastic shapes. You can down-load some of them as pdf files that can be printed out for your students to construct. They give them a feel for three-dimensions and give you some interesting shapes (even more interesting than the icosahedron that we've shown) to hang around the classroom. With different coloured faces you'll be able to put on a classy display.



(3) Sums of squares

$65 = 1^2 + 8^2 = 4^2 + 7^2$ and $85 = 2^2 + 9^2 = 6^2 + 7^2$ are the only other two integers below 100 that can be expressed as the sum of two squares in two ways.

(4) Hardy and Ramanujan

Hardy might be considered as a boring old English academic (though that would be unfare) but Ramanujan was a bright Indian star that shone brightly for a while and then disappeared leaving behind some very interesting and deep ideas. You can find both of them on the 'MacTutor' site (see http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Hardy.html and http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Ramanujan.html, respectively.) To see what this has to do with a taxi you'll have to wait till next month's newsletter. But you might not be surprised to hear that many a mathematician has this month's number as their pin number.