Newsletter No. 71 **March 2008**

We mentioned last month that the International Congress on Mathematical Education is coming up this year and that it is one of the few conferences that covers all maths education from early childhood up. Not only that but the major activity for the Congress sessions is discussion rather than simply the presentation of papers so that even if you are not involved with the organising of sessions you will find yourself involved. There is ample opportunity to try out ideas in workshops at education levels that suit you and a range of action groups that look at recommendations and concerns suggested by teachers. The vast majority of participants at secondary level are classroom teachers and with all the opportunities to mix informally with others in your own field, conferences like ICME provide inspiration and the chance to share ideas. Living in New Zealand can make it difficult to get to exotic locations for such conferences but you might be surprised at the level of local funding you can obtain. Ask your principal if the school is willing to subsidise costs. Your local Maths Association or Community Trust might contribute. You could also write an item or three for this newsletter and obtain extra funding that way. Remember the $50 voucher that had previously been given for solutions to our monthly problem is now available for articles that are published in the newsletter. The article should be less than 2 pages long and can be on anything to do with maths or maths education. Send any submissions to derek@nzmaths.co.nz.

If by keeping the old warm one can provide understanding of the new,
one is fit to be a teacher.

Confucius

This is our 71st issue and 71 is the largest known solution to Brocard's problem. That is, $71^2 = 7! + 1$. In a pair of articles published in 1876 and 1885, H. Brocard asked for integer values of m for which $m^2 = n! + 1$. Three solutions were found. Can you find the two others? (Answers below in *Afterthoughts*). The Indian mathematician Ramanujan independently considered the same problem in 1913.

Like 11, 31, 41 and 61, 71 is prime. Are there lots of primes ending in 1? James Davies discovered, I'm not sure how, that $71^3 = 357911$ which has its digits the odd numbers 3 to 11 in sequence.

Sarah Bernhardt lost a leg at age 71. She was mucking about jumping off a parapet while making a movie - silly girl! Douglas McArthur was sacked at age 71 - he'd been getting too big for his army boots and had offended the top brass. Louis Armstrong, Enid Blyton, Daniel Defoe, Cole Porter, Neville Chamberlain and Tennessee Williams all died at that age.

INDEX

Diary Dates

World Mathematics Day.
I'm sorry that this may be a little late for some of you but have you registered for World Maths Day? It's on the 5th March this year. The aim is to do as many mental arithmetic questions as possible in 24 hours. Not just your school but the world! Apparently last year they had more than 287,000 from 98 countries solve 39,904,275 questions during the day. This year they would like to break the 50 million mark. Sound like a bit of fun? Check it out on http://www.worldmathsday.com/.

Booke Review

Disquisitiones Arithmeticae by Carl Friedrich Gauss

Gauss was born in Brunswick on 23 April 1777. His father was a bricklayer and Gauss enjoyed a liberal education, against the will of his parents who wanted him to work in the family business, thanks to his talents being noticed by the then reigning duke.

Most of the material for this book had been completed by the time Gauss was 22 although it was not published until 1801 when he was aged 24. A large part of it had been submitted to the French Academy in the preceding year but was rejected. Gauss was, not unexpectedly, upset and was afterwards hesitant about publishing his investigations. The book introduced fundamental theorems in many aspects of analysis including the theory of numbers as a separate, systematic branch of the subject for the first time. It was written in Latin and translated into French. Here are parts of the first section, concerning the congruence of numbers:

> If a number a divides the difference of the numbers b and c, b and c are said to be congruent with respect to a. We call a the *modulus*. Each of the numbers b and c is called a *residue* of the other.

> These notions apply to all integral numbers both positive and negative but not fractions. Obviously, the modulus is always taken absolutely - that is, without any sign.

The book continues with an analysis of residues and theorems on congruences of first and second orders. In it Gauss also discussed the solution of binomial equations of the form $x^n = 1$ and investigated the theory of determinants.

Vive La Différence! Le Fin.

Well I haven't had any ideas sent in this month so I have to go it alone once more. But it will be for the last time as I'm running out of energy, if not ideas.

You'll remember that we are looking at taking a number, like 4326, for example, reversing the digits, subtracting and seeing what happens. Well this is what happens here. We go through the pairs of numbers (4326, 6243), (1908, 8091), (3816, 6183), (2367, 7632), (5265, 5625), (370, 730), (360, 630), (270, 720), (450, 540), (90, 900), (180, 810), (360, 630).

You notice that we have the 5-cycle ending that we have come to know and love. Why does that happen? First remember that with 2- and 3-digit numbers we only had a 5-cycle or we ended on (0, 0). What else could happen with 4-digit numbers?

Let's take an overview for a minute. After we have taken a difference what could we have found? Either we got zero or we didn't. If we got zero we would have had to have started with a palindrome. If we didn't we could take another difference. That is either zero or it's not. Clearly things have to either stop or keep going.

But keeping going is a problem. There has to come some time when the difference gets us back to a number that we've had before. After all, there are only a finite number of numbers available. We can't go on getting new differences for ever. So every number that we start with has to come to a zero or has to cycle round.

There are two questions here. What numbers hit zero (possibly eventually)? How big can the cycles be? I'm going to concentrate on the cycles from now on. So far we have only found 5-cycles and these cycles are all numbers with a common factor. Is it true that when you get to a cycle all the numbers in that cycle have a common factor? If it is it will be interesting to find out what that common factor is. What are the possible factors of the difference? How many numbers exist with that number as factors? Does this cause cycles to be 5-cycles?

The MAA Web Site

Of course surfing the internet is an endless source of fun. There is seemingly no end of interesting stuff out there. I thought that you might not know about the MAA. From its site, http://www.maa.org, we see that "The Mathematical Association of America has been serving the mathematics community since 1915. We are a great source for expository articles and reviews, professional networking, a variety of workshops, minicourses and meetings, and information on the broad spectrum of mathematics and trends in teaching. We are almost

27,000 members including university, college and high school teachers; graduate and undergraduate students; pure and applied mathematicians; computer scientists; statisticians and many others in academia, government, business and industry." There are a range of things on the site that you might find interesting and useful. Don't be put off though by some of the maths that you may find a little hard going at times.

However, there is one section where you can find some particularly useful ideas that you might be able to use in the classroom. Go to Math in the News Archives, http://www.maa.org/mathdl/minarchive.html. These are snippets from the US press so you know that they can't all need a high level of maths ability. Here are a couple of things that have been cited there over the last couple of months.

Do you remember the tiger that escaped by jumping out of a zoo enclosure? Well two mathematicians proved that the enclosure wasn't tiger-proof.

Another animal story showed that monkeys can add as well as humans can. The monkeys were given two sets of dots. They then had to choose the sum of that pair from two other lots of dots. "Our data demonstrate that nonverbal arithmetic is not unique to humans but is instead part of an evolutionarily primitive system for mathematical thinking shared by monkeys," the researchers said.

In February a study of reasons why students took maths courses was cited. Apparently it was found that "Girls have caught up with boys in math course taking in high school but the reasons for taking math still differ by gender. For all adolescents, math course taking was associated with the achievement of their close friends and, to a lesser extent, their course mates. These associations tended to be stronger toward the end of high school and weaker among adolescents with a prior record of failure in school. Each of these patterns was somewhat more consistent among girls."

Comments on February's Endeavour

You were asked to find the length of the longest string of consecutive whole numbers between 1 and 100 that are neither a perfect square nor the sum of two such squares.

The longest string is six, from 91 to 96 …. but read on ….

This Month's Endeavour

In our last issue we looked at a problem concerning whole numbers between 1 and 100 that could not be expressed as the sum of two or fewer square numbers. Here are three related problems:

(a) How many whole numbers between 1 and 100 cannot be expressed as the sum of three or fewer square numbers?
(b) What are the two smallest consecutive whole numbers that cannot be expressed as the sum of three or fewer square numbers?

(c) How many whole numbers less than one million cannot be expressed as the sum of four or fewer square numbers? Don't spend too long on this problem!

This perhaps leads to the question of whether all whole numbers can be expressed as the sum of exactly five square numbers. Clearly not since it is impossible, among others, to express 15 in this way. Well then, is there a largest whole number that cannot be expressed as the sum of exactly five square numbers? The answer is yes and it is certainly less than 170 as the following demonstration from Ross Honberger's *Mathematical Gems III* (The Mathematical Association of America, 1985) clearly shows:

If $n > 169$ then LaGrange's result tells us that $n - 169$ can be expressed as the sum of four or less square numbers,

i.e. $$n - 169 = a^2 + b^2 + c^2 + d^2$$

where no more than three of a, b, c, d are zero.

So:

$n = a^2 + b^2 + c^2 + d^2 + 13^2$ (if none are zero)
$n = a^2 + b^2 + c^2 + 12^2 + 5^2$ (if one is zero)
$n = a^2 + b^2 + 12^2 + 4^2 + 3^2$ (if two are zero)
$n = a^2 + 10^2 + 8^2 + 2^2 + 1^2$ (if three are zero)

PLEASE NOTE: We are no longer giving a book voucher to one of the correct entries to this problem. Do still please send your solutions to derek@nzmaths.co.nz as the best will be published in next month's newsletter. The $50 voucher that has previously been given for this problem is now available to anyone who writes an article that is published in the newsletter. The article should be less than 2 pages long and can be on anything to do with maths or maths education. Send any submissions to derek@nzmaths.co.nz.

Solutions to Last Month's Junior Problem

First of all what was last month's problem? Recall that I had seen the letters DDSD on a manhole cover on the street and guessed that they stood for 'Dunedin and District Sewerage Department'. So I asked you to find some mathematical 'thing' that DDSD might stand for. It didn't matter whether it made too much sense or not. I was just looking for the funniest or most creative answer.

Well I got a whole stack of answers from Room 3, One Tree Point School, Ruakaka, Whangarei. They admit that they inadvertently did DSDD instead of DDSD but, hey, who cares. Their imaginations deserve the $20 book voucher.

Jared Don't Stop Doing Division

Jake Double Subtraction Division Diagrams

Dividing Sums Double Digits

Lekeesha	Does Subtraction Divide Digits
	Division Subtraction Diagrams Digits
Corey	Double Sided Double Density
	Divided Subtract Double Density
Jorgia	Digital Subject Double Digits
Justin	Double the amount then Subtract with the figure given to mean Divide this by two to find out Data needed
Xzavier	Do Some Division Daily
Lucy	Division Solving Double Digits
Hannah	Double Six Displays Dozen
	Doing Statistics Discovers Data
Taylor	Duplex Subtraction Direct Dissention
	Division Subtraction Double Digits

Shayla	Divided Subtraction Double Division
Nicole	Double Subtraction Double Division
	Do Subtraction Double Division
Jordan H	Do Subtraction Don't Delay
	Decimal System Dewey Decimal
Scania	Does Subtraction Divide Decimals
	Don't Subtract Divide Daily
Taylor T	Division Subtraction Diagonal Divide
Isaac	Divide Subtract Double Duplicate
	Divide Subtract Denominator Decimal
Rikiana	Double Study Division Decimals
Niamh	Divide Some Double Decimals
	Do Some Decimals Daily
Paris	Divide Subtract Deduct Decomposition

Dylan	Double Sided Division Device
Britain	Divide Subtract Double Digits
Christopher	Double Sided Double Density

This Month's Junior Problem

All of you who know Brian Bolt's books will be glad to hear that they are all being reprinted by Cambridge University Press. To celebrate the occasion I'm taking a problem from his Mathematical Activities that was originally published in 1982.

Explain this pattern and show that it is always true

$$3^2 – 2^2 = 5 = 3 + 2$$
$$4^2 – 3^2 = 7 = 4 + 3$$
$$5^2 – 4^2 = 9 = 5 + 4$$

The usual $20 book voucher is available for the best solution to this.

Afterthoughts

Two other known solutions to Brocard's problem are $11^2 = 5! + 1$ and $5^2 = 4! + 1$. It is extremely unlikely that there are other solutions but as yet this has not been proven.

Back to the difference problem, what are the possible factors of abcd – dcba = 999(a – d) + 90(b – c)? 9, 90, and 999 all look possible. What cycles do you get from these? Try a few numbers that have 9, 90 and 999 as factors. But 999e + 90f can have a factor of 99 (when e = f). What cycles do we get from 99? And no, I'm sorry but they are not always 5-cycles! Try 6534.