

Newsletter No. 70

February 2008

Hi, welcome back to the nzmaths site newsletter. We hope you are all relaxed and ready to go again after the summer break.

It's at the beginning of each year that some teachers encourage their pupils' interest by setting them the task of writing as many consecutive whole numbers, starting from 1, using the digits of the year in order and a range of mathematical operations. For example, this year the digits 2, 0, 0 and 8 would be used and perhaps the operations +, -,  $\times$ ,  $\div$ , !,  $\sqrt{}$ , powers, brackets and concantenation (i.e. adjacently placed digits like 20 or 200). I wonder how far you can get under these conditions. I think you might find this year much more of a problem than previous years although things might begin fairly straightforwardly. For example;

$$1 = 2^{0} + 0 \times 8$$
  

$$2 = 2 + 0 + 0 \times 8$$
  

$$3 = 2 + 0! + 0 \times 8$$
  

$$4 = \sqrt{((2 + 0 + 0) \times 8)}$$

On my first attempt I got as far as 16 without a break, and then things became problematic. How far can you get without a break? If it's any help,  $25 = 200 \div 8$ .

Just as an aside you might like to look at the following little conundrum. Think of a number and count the letters in writing it. For example, one hundred and twenty five uses 23 letters. Repeat the process with the new number, i.e. twenty three. It has 11 letters. Keep repeating the process. What do you notice? What happens with different starting numbers? Can you generalise your results?

*Children's informal arithmetic is powerful, their understanding of written symbolism is weak.* 

#### A. Ginsberg

This is our 70<sup>th</sup> edition. The sum of all the divisors of 70 (including 70) is a square number. Is this common? 70 is the smallest *weird* number. Weird numbers are defined as those that are abundant without being the sum of any of their own divisors. Abundant numbers are those which are smaller than the sum of their divisors excluding themselves. 70 is abundant because 1 + 2 + 5 + 7 + 10 + 14 + 35 = 74 > 70. Weird numbers are rare. There are only seven below 10,000. By the way, what is the smallest abundant number (answer in Afterthoughts below)?

Mathematicians Lazare Carnot and Gottfried Leibnitz died aged 70 as did roundthe-world sailor Francis Chichester who emigrated to New Zealand from England when he was 18. Copernicus published his great work *The Revolution of Heavenly Bodies* when he was 70 and famous Cornish painter Alfred Wallis began to paint at that age.

To be seventy years young is sometimes far more cheerful and hopeful than to be forty years old.

#### Julia Ward Howe

But which would you rather be?

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#### What's new on nzmaths.co.nz

A large amount of new material has been added to the nzmaths site over the summer period.

In particular it is worth your while having a look at the new Online Numeracy PD material (<u>http://www.nzmaths.co.nz/Numeracy/onpd/Intro/01.aspx</u>). It replaces the old Online Numeracy Workshops, and unlike its predecessor, does not require a CD or a username and password.

There are also 10 new equipment animations, in both English and Māori, and a large collection of material in both the Families and Secondary Numeracy Project sections of the site.

#### Booke Review Pillow Problems by Lewis Carroll

In our last issue we reviewed Lewis Carroll's book *A Tangled Tale*. Published in 1895, *Pillow Problems* is by far the rarest of all his books. It contains 72 mathematical problems ranging from those that can be solved by arithmetic, algebra or plane geometry to those that require more advanced algebra, trigonometry, algebraic geometry, differential calculus and probability theory. Despite being written, as Carroll said, for *ordinary* mathematicians, it is not for the faint-hearted. Lewis Carroll, real name Charles L. Dodgson, after all, was a mathematics lecturer at Christ College, Oxford as well as a children's writer.

Carroll claimed to have solved nearly all of the problems in his head while lying awake at night. In his introduction he says, 'No. 37 and one or two others belong to the daylight, having been solved while taking a solitary walk.'

To give you an idea of the level of complication here's the first problem:

Find a general formula for two squares whose sum is two.

Unlike *A Tangled Tale*, there is no wordplay involved with these problems. They are seriously written and require solid mathematics for their solution.

The following problem, which defies common sense, has spawned a whole raft of follow-ups. They recur every few years as each generation discovers the problem's contrariness.

A bag contains one counter known to be either black or white. A white counter is put in, the bag shaken and a counter drawn out, which proves to be white. What now is the chance of drawing a white counter?

The unexpected answer to the problem is also given below but can you explain why?

Dover Publications have an edition of this book but perhaps it is of less interest to teachers than Carroll's *A Tangled Tale*, even though it is historically interesting and mind-stretching,

#### **Timothy Gowers**

Timothy Gowers is a mathematician and indeed a mathematician's mathematician. He has to be. He is Rouse Ball Professor of Mathematics at Cambridge University and was awarded the 1998 Fields Medal. Let this be enough to tell you that this man is no mean mathematician. If you want to find out more about him you can just Google 'Timothy Gowers' and you can find as much about him as you want.

The reason that I have mentioned him is because his YouTube presence was recommended to me by a friend. Now I have to say that I have heard of YouTube but had never bothered to find out what it is about. You have to realise that I am a technologically illiterate old man who should have retired a couple of years ago but vanity has kept me going. I don't even own a cell phone for crying out loud. Why would I want people to be able to phone me? Perhaps it would give me an excuse to get out of a conversation because my phone has just rung or to annoy people in a meeting by the last post announcing an incoming call. But I've managed very well speaking to people by mouth when I need to and by land based phone if they were some distance away. I don't have enough friends or business associates to tell everyone at an airport my most intimate details. And if I want to know how to get my DVD player working I just call on a passing 8 year old. But making YouTube work ain't hard after all and it was worth the trouble to get Timothy Gowers thoughts. I suggest that you try <u>http://www.youtube.com/watch?v=BsIJN4YMZZo</u> for the Importance of Mathematics Part 1 and you should be able to get to the other seven parts yourself from there.

While trying to prove to a non-mathematical audience in 2000 that it really is important, he touches on a lot of what is fundamental to mathematics. And since there are 8 parts to an hours lecture you can take it a bit at a time.

This is going to be recommended viewing for my university students this year but teachers should be able to get quite a lot out of it and it would be useful to suggest to the better students.

## Vive La Différence IV!

I know that you have all been working hard over the summer on nothing else but the four digit version of the problem I've been looking at over the last three months of 2007.

Let me refresh your memory, just in case you forgot to even think about it. Take a 4digit number, say 4371, reverse it's digits to get 1734, and subtract the smaller from the larger. This gives 2637. Now reverse the digits again and subtract the smaller from the larger. It turns out that 7362 - 2637 = 4725. Keep doing this and you should get

5274	9440	8991	6993	7992	5994	9990
- 4725	- 0449	<u>- 1998</u>	<u>- 3996</u>	<u>- 2997</u>	<u>- 4995</u>	<u>- 0999</u>
0449	8991	6993	2997	4995	0999	8991

Now, of course, it all comes back to you. With 2- and 3-digit numbers (see September and October 2007 Newsletters), palindromic numbers immediately go to zero (as they do, naturally with 4-digit numbers), some five numbers (and their reverses, go round in a cycle (as do the numbers starting from 8991 here), and the rest go immediately to one of these cycling numbers. While we get the same cycling with four digit numbers, the number 4371 doesn't immediately go to one of the cycling numbers. In fact it takes a few steps before it gets to the cycle.

So let me just go back over what most of you have discovered over the summer and I won't even mention again the palindromic numbers.

The cycle is pretty predictable. In the 2-digit case it was the 10 multiples of 9 that formed this. In the 3-digit case it was the 10 multiples of 99 that formed this. And in the 4-digit case it is the 10 multiples of 999 that form this. It's likely that we can generalise this. Wouldn't you think that for the n-digit case, n > 1, the 10 multiples of 9...9, with n - 1 lots of nines, form the 5-cycle. Probably to prove this all you have to do is to show that the reverse of a multiple of 9...9 is also a multiple of 9...9. Then you should pretty well be there.

The reason that the non-cycle numbers jump straight to cycle numbers in the 2- and 3digit cases is that the difference between a 2-digit number and its reverse is a multiple of 9 (and so on the cycle); and the difference between a 3-digit number and its reverse is a multiple of 99 (and so on the cycle). Unfortunately the difference between a 4-digit number and its reverse *isn't necessarily* a multiple of 999! The only guaranteed factor is 9 because abcd – dcba = 999(a - d) + 90(b - c). It would be nice if it were true that all numbers eventually came to a situation where b equalled c and we'd be on the cycle the next time round. Unfortunately this is not the case.

We already know about 6611. Now 6611 - 1166 = 5445. Yes b = c but a = d too and we have a palindromic number. So we now have to show that every number we start with is either a multiple of 999 (and so it's on the cycle); or it goes to a number where b = c and a and d aren't equal; or it goes to a number where a = d and b = c. The big problem, of course, is that b may never equal c and we have some other form of cycle.

Is that possible? Is the 5-digit case just the same but with multiples of 9999? Get your students to try this one out. If nothing else they will have done a lot of arithmetic in a good cause. I'll put anything that I get on this in next month's newsletter.

# Terence Tao

Here we have another Fields medallist, an Australian one at that. This medal caused some distress in New Zealand circles because up to that point we led the Australians 1 - 0 in terms of Field medallists. Anyway, suffice to say that he is a brilliant young mathematician. (With the emphasis on the 'young'. You might like to find out how young he actually is.)

Again, by Googling, this time using 'Terence Tao', you can find out all that you want about Terence. One place that has information is Wikipedia (<u>http://en.wikipedia.org/wiki/Terence\_Tao</u>), so this link will give you a start.

Somewhere to follow up is the article from the Herald Tribune at <u>http://www.iht.com/articles/2007/03/13/news/math.php</u>. You can get from there to a set of 8 questions that he was asked to solve when he was only 8. Here are the first four of them. You might like to see what your class can do with them (or your colleagues – just bring them up in the staffroom).

**Question 1:** Two circles have radii equal to 2 cm and 3 cm. The distance between their centres is 4 cm. Do they intersect?

Question 2: What angle does an hour hand turn through in 20 minutes?

**Question 3:** A can of kerosene weights 8 kg. Half of the kerosene is poured out of it, after which it weighs 4.5 kg. What is the weight of the empty can?

**Question 4:** What time is it now, if the time that has passed since noon constitutes a third of the time that remains until midnight?

Have a go at that, or better still find some young people to attack them. Afterthoughts is where we'll discuss them (but don't peek!).

## Conferences

There's an international conference that's coming up this year that, like the Olympics comes around only every four years. This is the International Congress on Mathematical Education. It's one of the few conferences that I know that covers all of maths education from woah to go. So you'll find something interesting there if you are a top researcher in maths education or a primary teacher or a teacher trainer or even a facilitator in the Numeracy Project.

To make it even more interesting it is in Monterrey, Mexico this year. I guess that makes it somewhat expensive to get to but maybe you can fit in a holiday in South America at either end of the conference.

The dates of the conference are July  $6^{th}$  to  $13^{th}$  and you can find out more about it on the web at http://icme11.org/.

## This Month's Problem

Here's a bit of a search and find for you, with or without the use of a computer spreadsheet. Just under half of the whole numbers from 1 to 100 are either a perfect square or the sum of two such squares. What is the length of the longest string of consecutive whole numbers between 1 and 100 for which this property is not true?

P.S. Don't throw away your working to this problem as there will be a follow-up next month!

**PLEASE NOTE:** We are no longer giving a book voucher to one of the correct entries to this problem. Do still please send your solutions to <u>derek@nzmaths.co.nz</u> as the best will be published in next month's newsletter. The \$50 voucher that has previously been given for this problem is now available to anyone who writes an article that is published in the newsletter. The article should be less than 2 pages long and can be on anything to do with maths or maths education. Send any submissions to <u>derek@nzmaths.co.nz</u>.

## This Month's Junior Problem

This section contains a monthly problem competition for students up to Year 8 with a \$20 book voucher available for the winner. Please send your solutions to <u>derek@nzmaths.co.nz</u> and remember to include a postal address so that we can send you the voucher if you are the winner.

I've tried to make things a little different for this first set of problems in 2008. It's come out of walking to work every day and walking with my head down looking at the footpath. I don't know why I do that. I hope I look up when I have to cross the road but I can't always remember if I did. Anyway on those metal things on the footpath, there are often initials. One that I can remember is DDSD. I assume that it stands for 'Dunedin and District Sewerage Department'. It seemed to me that here was a chance for you to exercise your imagination. Find some mathematical 'thing' that DDSD might stand for. It doesn't matter whether it makes too much sense or not. I'm really looking for the funniest or most creative answer. Feel free to put in as many entries as possible. With any luck I hope to be able to put several hundred of your answers in the March issue.

# Afterthoughts

The smallest abundant number is 12.

The answer to the Pillow Talk counter problem is 2/3. As we suggested above though, you might like to explain why.

**Question 1:** How far apart do the circles have to be if they don't intersect? Surely at least 5 cm. (You might like to check this out by drawing them.) So the two circles do meet.

**Question 2:** In twelve hours an hour hand goes all the way around the clock. So in one hour it goes a twelfth of the way. That  $360^{\circ}/12 = 30^{\circ}$ . But 20 minutes is a third of an hour. So the hour hand goes through  $30^{\circ}/3$  in that time. It turns through  $10^{\circ}$ .

**Question 3:** In pouring away half of the kerosene, how much weight is lost? So the weight of half the kerosene must be that. The weight of all of the kerosene is twice that. So the weight of kerosene in a full can is  $2 \times 3.5$  kg. The can must therefore weigh 8 - 7 = 1 kg.

**Question 4:** Let's try this two ways, ah, may be three ways! Let's start by guessing. Suppose that it's 1 pm now. Then there is 11 hours till midnight. That means that the time now is one eleventh of the time till midnight. This is smaller than 1/3 so the time must be past 1 pm. So try 2 pm. Now 2/10 = 1/5 and again this is smaller than 1/3. So try 3 pm. Ah, as Goldilocks once said, just right.

On the other hand, the twelve hours between noon and midnight is made up of a block from noon till now, and a block from now till midnight that's 3 times as long as that. The time form noon to midnight is 4 times as long as the time till now. So one block of time is 12/4 = 3 hours. It must be 3 pm.

And there is always algebra. Suppose it is x hours (x may not necessarily be a whole number) from noon to now. So from now to midnight it has to be 3x hours. So 4x hours is the same as 12 hours. Since 4x = 12, x = 3. It's 3 pm again!