NZMATHS new zealand maths

**Newsletter No. 69** **November 2007**

A colleague once asked me why I thought some teachers were reluctant to explore alternative approaches in the classroom, problem solving in particular. I think there are two main reasons: First, the fear that our pupils won't do themselves justice. This usually turns out to be wrong - given time and encouragement pupils usually exceed our expectations. Secondly, some us fear that we won't do ourselves justice. We might even be asked a question we cannot answer, or worse still have a pupil answer one we can't! Here, of course, we have a problem of definition. What is a teacher? Does a teacher have to be able to answer every question? Or rather, should it be that we know how to set our pupils off in the right direction, or any direction for that matter. Are we in fact worried about losing face? The difficulty with problem solving is that there is a high probability that we ***will*** be asked a question we can't answer. Not being able to answer a question doesn't mean that we can't respond, it offers a range of excellent learning opportunities. "O.K. Mary, this is a new problem for me too. What are your ideas?" Situations like this provide incredible motivation for students.

> *Mathematical presentations, whether in books or in the classroom, are often perceived as authoritarian by the student and this may arouse resentment.*
>
> **Reuben Hersch**

Having 'said' all that and a host of other things throughout the year and it being our last issue for 2007, it's time to wish everyone all good things for the summer. We hope you enjoy a relaxing break and return refreshed in the New Year ….

69 is the only number whose square and cube between them use all the digits 0 to 9 once only: $69^2 = 4761$ and $69^3 = 328509$.

Neville Chamberlain (any relation to Mary?) was aged 69 when he returned to England having just signed the Munich Agreement with Adolph Hitler. Ronald Reagan became the 40th President of the United States at that age. 69 is the atomic number for thulium. Louis Armstrong, Queen Elizabeth I, Buster Keaton and Emmeline Pankhurst all died aged 69.

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## Booke Review

**A Tangled Tale by Lewis Carroll**

This book was virtually unobtainable until Dover Publications republished it in 1958. It was originally published in 1885. In it Carroll presents a series of puzzles in ten chapters, which he calls 'knots', of a delightful story, typically Carrollean. The tale was originally printed as a monthly magazine serial before being reproduced in book form. The idea of setting problems in the context of a story was highly unusual at the time and gave the idea to later writers like H.E. Dudeney in the early 1900s (see last month's Booke Review) and Martin Gardner and Dennis Shasha in more recent times.

Carroll likened his problems in *A Tangled Tale* to medicine disguised by sweetness. He wrote in the preface: '(My) intention was to embody in each Knot (like the medicine so dexterously, but ineffectually, concealed in the jam of our early childhood) one or more mathematical questions - in Arithmetic, Algebra or Geometry, as the case might be - for the amusement, and possible edification, of the fair readers of the magazine.

The solutions given in the book are comprehensive with comments on those submitted by readers as well as additional information where appropriate. One thing you have to be very careful with when attempting the problems of this idiosyncratic puzzlist is the wordplay that is liable to occur in any problem - or in any solution, for that matter. As those who have read his childrens' books will know, plays on words were what delighted Carroll every bit as much as mathematics. Here's one of the problems:

> Place 24 pigs in four sties so that, as you go round and round, you may always find the number in each sty nearer to ten than the number in the last.

In the solutions Carroll comments that only two people solved the problem. In case you'd like to have a go at it the answer has been included in Afterthoughts below.

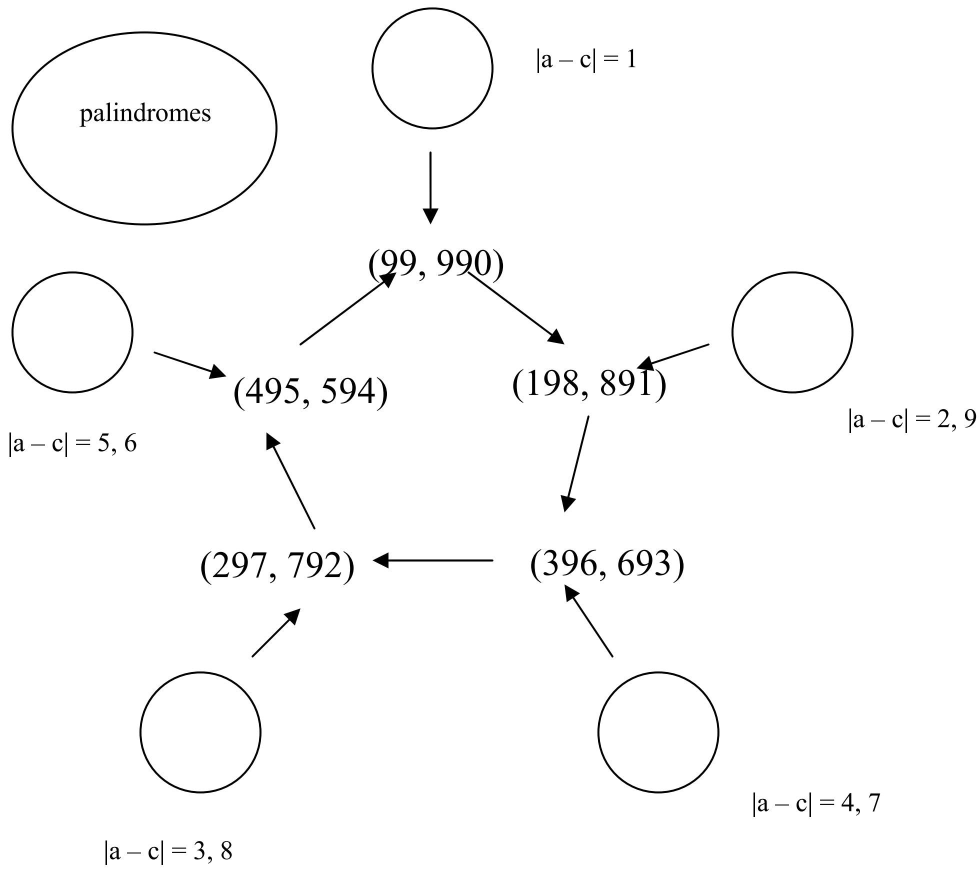
Here's another of his problems with the answer also in Afterthoughts below:

> A and B began the year with only £1,000 each. They borrowed nothing, they stole nothing but on the next New-Year's Day they had £600,000 between them. How did they do it?

Vive La Diffférence! III

I have to first of all confess that I'm worried by the heading. In fact, I've been worried by the heading since the beginning. All along I've suspected that it should have been "Vive La Difference III!" You can see how bad my life is going if I have to worry abut that sort of thing but there you are. But I like to get punctuation right.

When I left you last month you were looking hard at the following picture.




Any 3-digit number, and we include 099 as a 3-digit number that is not a palindrome or one of the numbers on the central 5-cycle, goes immediately to that cycle on reversing its digits and subtracting the smaller of the resulting numbers from the larger. So 785 – 597 = 198. Actually the picture tells you which number on the cycle the given number will hit. If we think of 785 as abc, then a = 7 and c = 5 and 7 – 5 = 2. With a difference of 2 we have to hit either 198 or 891.

My treat for you this month is to do the 4-digit case so that we can save the 5-digit case for February. From what we have seen before we would expect the palindromes to sit out by themselves and everything else to either be on a 5-cycle or one step off of it. So let's take an 'everything else'. 4371 will do. Actually it won't. Maybe my arithmetic went wrong somewhere because I get a whole lot of numbers before I get onto a 5-cycle. It just doesn't go straight to a 5-cycle. But at least a 5-cycle seems to exist. You try it.

So maybe I'll look at 6611. That's even worse. 6611 – 1166 = 5445 and that's a palindrome and it goes to zero at the next step. Nothing like that ever happened with the 2- or 3-digit numbers. What's gone wrong with 4-digits? And does that mean that we can't describe behaviour of 4-digit numbers? If that's the case I'm not going to be able to even *think* about 5-digit numbers. So does my series of articles end here? Can anyone help me? You've got a whole summer and a bit to find out what's going on. Why not get your students to help out after they've done their exams and are looking for something to do? Email derek@nzmaths.co.nz with your theorems, your conjectures, or even the odd calculation or two. Maybe we can work this one out together. There will certainly be no VLD IV if I don't get any input from someone else.

## Not This Month's Problem

Because there is such a long break between now and the next issue of the newsletter, we have decided not to have any contestable problems this issue. By the time you all get back next year you will have all forgotten (a) what answer you sent in; and (b) what the question was that you sent that answer in for.

But we're also thinking about not having a prize problem in February either nor the month after that. In fact we are thinking seriously about reviewing the whole newsletter. And that is where you, the reader, come in. What do you like about the newsletters? What don't you like? What would you like us to put here that we don't usually include?

Let's pull the whole thing to bits. We always start off with an 'editorial'. Is that a reasonable thing to do? Do you find that interesting? Have we got stuck in too small a rut? Could we be more cheerful? More serious?

Probably we'll keep the "What's new …" section as it is our opportunity to (a) brag; and (b) inform.

How often do you find out something for your diary that you didn't know already?

We've done a number of book reviews and web site reviews. Are they of interest? Would you like something that was more pedagogical than mathematical? In fact do we overdo the maths – the problem solving stuff? What with Frogs and so on we seem to use up quite a bit of space on problem solving. How much of the problem solving are you able to use in class? Is problem solving OK but are the problems we use too hard?

Do we have enough pedagogical material? Should there be more material related to the teaching of Numeracy?

Then there are the monthly problems. How often do you try them? How often do you get them out and don't bother to send in your solutions? How often do you tell your students about the Junior Problems? How often do they get them out and don't bother to send in their solutions?

Please let us know by emailing Derek at derek@nzmaths.co.nz or by completing the survey linked from the nzmaths home page www.nzmaths.co.nz. There is a book voucher available to one randomly chosen submission of the survey.

## Solution to October's problem

Suppose the total number of men in the company is m and the proportion refusing the bonus p. Then the proportion of men accepting the bonus is 1 - p and their number m(1 - p). The total amount paid out in bonuses will then be,

$$1000m(1 - p) + 835(350 - m)$$

Since the total amount in bonuses paid out can be calculated, the above formula must be independent of m, i.e., the terms containing m must cancel out. Hence,

$$1000(1 - p) - 835 = 0$$

from which p = 0.165.

Now, both m and 0.165m must be whole numbers less than 350 which means that m must be 200. Therefore the number of women is 150 and the total bonus paid out to them 835 x 150 = $125,250.

## Solution to October's Junior Problem

This was last month's problem. All of the different letters in the following correct multiplication represent different digits. Which letter is which digit?

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   TWO
x  TWO
THRE E
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Henry Yuen gave us the following solution almost before the ink dried on our web page.

T = 1, W=3, O=8, H=9, R=0, E=4

You can easily check that that's right. But how to get it and why is there only one solution?

Start off by noticing that 400 x 400 = 160000. That's a 6-digit number while the answer we expect to get is only 5 digits. So T = 1, 2, or 3. But 2WO x 2WO can't be equal to a number in the 20,000s. The same sort of thing works for T = 3. Hence T = 1.

Similarly, W can't be bigger than 4 (or else the T in THREE turns into a 2 or higher). So W = 2, 3 or 4 (it can't be 1 since T is). If W = 4, then O = 0 or 1. But in that case E = 0 or 1 so E = O. This is not possible. So W = 3 or 2.

In fact W is never 0, 1, 5 or 6 since they all force E to equal O. Perhaps the simplest way forward from here is to note that we have now reduced TWO to ten possible values: 123, 124, 127, 128, 129, 132, 134, 137, 138, 139. The only one that gives the two Es is 138.

## 2007 End Of Year Quiz

1. How do you spell isosceles?
(a) isosilly (b) issoseles (c) isoscales (d) isoseles (e) none of those.

2. Which of the following mathematicians did NOT come to a premature end?
(a) Hypatia (b) Isaac Newton (c) Evariste Galois (d) Alan Turing
(e) Archimedes

3. Which of the following works for the enemy?
(a) $e^{ix}$ (b) 2xy (c) $i^2\pi$ (d) tan x (e) $(x + y)^2$

4. What is the discriminant of a quadratic equation?
(a) An insect with taste which lives in the four rooms at the top of the house.
(b) The coefficient of $x^2$.
(c) The difference between the square of the coefficient of x and four times the product of the coefficient of $x^2$ and the constant term.
(d) The sum and product of the roots of the equation.
(e) The product of four times the coefficient of $x^2$ and the constant term subtracted from the square of the coefficient of x.

5. What is an algorithm?
(a) The music preferred by an American politician and environmentalist.
(b) A finite series of well-defined rules to solve a problem.
(c) An algebraic formula to solve cubic equations.
(d) An algebraic formula to solve quartic equations.
(e) A Euclidean method involving cyclic quadrilaterals.

6. My grandfather James loved to tell us that he was x years old in the year $x^2$. In what year was he born?
a. 1944 b. 1992 c. 1892 d. 1936 e. 1935

7. What is a topological map?
(a) One that's in the news.
(b) The landforms or surface configuration of an area.
(c) A survey typical of an area.
(d) One with its properties unaffected by ignoring named points.

(e) One with its properties unaffected by stretching.

8. Which long-standing problem did John Harrison finally solve?
   (a) Who threw the overalls in Mrs. Murphy's chowder.
   (b) Fermat's last theorem.
   (c) The four colour map problem.
   (d) Accurately determining longitude.
   (e) Determining the Earth's diameter.

9. Which of the following are not map projections?
   (a) The cinematographic projection.
   (b) The chronomatic projection.
   (c) The stereographic projection.
   (d) The orthographic projection.
   (e) The polar zenithal projection.

10. If n is an integer which of the following is/are always true?
   (a) $n^2 > n$ (b) $n^2 + n + 13$ is a prime number (c) $n + 3 > n - 2$
   (d) $2n^2 + 1 > n^2$ (e) $2n^3 + 1 > n^3$

11. At least one of the following is a perfect square. Which?
   (a) 24,876,222 (b) 80,209,937 (c) 14,107,536 (d) 26,225,213
   (e) 15,515,728

12. What is/are Euclid's Elements?
   (a) The name given to Euclid's pre-school students.
   (b) A early Greek version of the periodic table.
   (c) A geometry book written by Euclid.
   (d) A synopsis of the mathematical knowledge of his time written by Euclid.
   (e) A Greek guide to the weather.

13. Which of these relatives is the odd one out?
   (a) Cousin (b) Uncle (c) Mother (d) niece (e) Grandfather

14. What is a cardinal number?
   (a) One that is approved by the Vatican.
   (b) A number used to indicate the position of an item in a sequence such as first, second and so on.
   (c) A number that can be expressed as a decimal.
   (d) One used to quantify the size of, i.e. the number of elements in, a set.
   (e) None of the above.

15. What is a transitive relation?
   (a) One that has no fixed abode.
   (b) A relation that is symmetric, i.e., a relation R such that aRb implies bRa. For example; 'cousin of', if A is the cousin of B, then B is the cousin of A.
   (c) A relation that is not symmetric, like 'taller than'.
   (d) A relation R such that if aRb and bRc then aRc. For example; taller than. If a is taller than b and b is taller than c then a is taller than c.
   (e) One for which every member holds the relation to itself. For example; has the same name as.

16. For what is the mathematician Georg Cantor most well-known?
   (a) Horse riding.
   (b) Extending the idea of sets to compare infinite series of numbers.
   (c) Defining an algebra of sets that could be used to design simple electronic circuits.
   (d) Designing an electronic circuit that could be used to solve set problems.
   (e) Solving Zeno's paradox about the tortoise and the hare.

17. The following mathematicians are in alphabetical order. Can you put them in chronological order in terms of their birth dates? Descartes, Euclid, Fermat, Newton, Pythagoras.

18. What is a difference engine?
   (a) A calculating machine designed but not built by Charles Babbage.
   (b) A calculating machine designed and built by Charles Babbage.
   (c) A machine designed to solve subtraction problems
   (d) A machine designed to tell two items apart.
   (e) None of the above.

19. How many lines of rotational symmetry of order four does a cube have?
   (a) 7 (b) 8 (c) 10 (d) 12 (e) None of those

20. At least one of the following is a prime number, which?
   (a) 7919 (b) 9828 (c) 803,259 (d) 48,765 (e) 11,011,101

## Afterthoughts

Answer to Carroll's pig problem: Place 8 pigs in the first sty, 10 in the second, nothing in the third and 6 in the fourth. 10 is nearer 10 than 8, nothing is nearer 10 than 10, 6 is nearer 10 than nothing and 8 is nearer 10 than 6!

Answer to the second Carrollean problem: They went that day to the local bank. A stood in front of it while B went round and stood behind it - so they had £600,000 between them. No groans please!

Answers to 2007 End Of Year Quiz

1. Ha! (e) of course
2. (b) Isaac Newton. Hypatia was killed by a Christian mob in 415 CE. (There is a new book out on Hypatia – see http://www.maa.org/reviews/hypatiadeakin.html) Galois was killed as the result of a duel aged 21. Alan Turing committed suicide. Archimedes was killed by a Roman soldier. And you may argue that Newton died too soon.
3. (c) $i^2\pi = ii\pi$ = eyes $\pi$ = I spy !!
4. (e); careful (c) is only sometimes correct!
5. (b)
6. (c) The only possibility is for James to be 44 with $44^2 = 1936$. Hence the year he was born is 1936 - 44 = 1892.
7. (e)
8. (d) – he made a pretty damn good watch. And then the powers that be were very slow in giving him the prize that existed for that fete.
9. (a) and (b)
10. (c) and (d)
11. (c). We know at least one of the numbers is square, the others aren't since perfect squares can't end in any of the digits 2, 3, 7, 8.
12. (d) The Elements contained propositions from number theory, proportion and similarity as well as plane and solid geometry.
13. This is your bonus question because we're sure you can find reasons for choosing any one of the five. For example the answer could be:

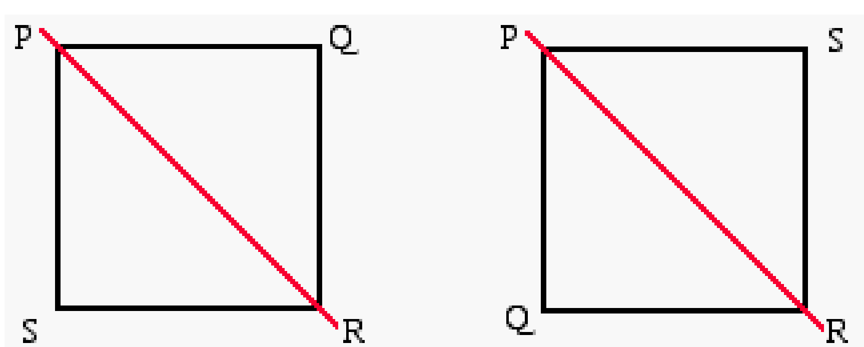
    (a) Because cousin is the only non-gender specific relative (gains you one point)

    (d) Because niece is the only relative with the first letter not a capital (worth $^1/_2$ point)

    Any of the others with a good reason gains $^1/_2$ point.
14. (d)

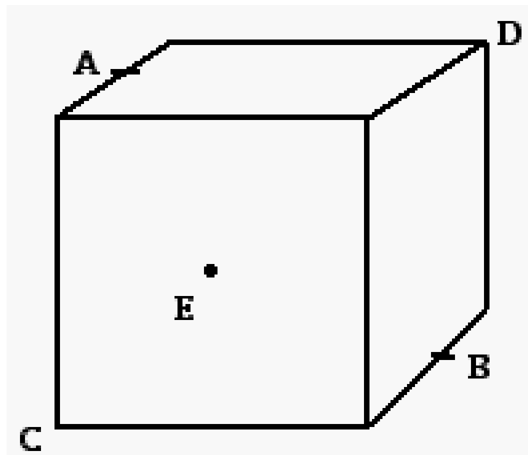
15 (d)

16. (b)
17. Pythagoras, Euclid, Descartes, Fermat, Newton
18. (a)
19. (e) Here's an example of what is meant by rotational symmetry. The order of rotational symmetry when this square is rotated about line PR is two (the two positions are shown) because these are the only two positions that look the same when the square is rotated about the line. If the square is rotated about a line through the centre and perpendicular to the plane of the square the order of rotational symmetry would be four.



A cube has 11 lines of rotational symmetry of order four:

- There are four about lines passing through diametrically opposite corners, e.g. CD (see diagram below).
- Four about lines passing through the mid points of diametrically opposite edges, e.g. AB.
- Three about lines passing through the centres of opposite faces, e,g, through E perpendicular to the face.




20. (b) is even, therefore divisible by 2 and not prime. The sum of the digits of (c) is 27, therefore it is divisible by 9, (d) is clearly a multiple of 5 and the sum of the digits of (e) is 6 therefore it is divisible by 3. Therefore (a) is the only possible prime number. If you work on it a bit you can see that it is prime.

**Scores:**

| | |
|---|---|
| 15 or more: | You won't need any Professional Development over the summer. |
| 10 - 14: | Take a maths book to the beach! |
| 5 - 9: | Take a maths book to the beach ***and*** go on some Professional Development. |
| < 5: | Do as many of the units on nzmaths as you can between now and next term! |

**Bonus Points:**

| | |
|---|---|
| +10 | If you knew where to find the hard ones on the web. |
| -10 | If one of your children told you an answer. |