Newsletter No. 67 September 2007

The monthly problems on sequences we've included in our recent newsletters reminded me of Neil Sloane's book *Handbook of Integer Sequences* (Academic Press) published in 1974. I remember when I first came across it how impressed I was. I think it appealed to my fascination with lists. Neil had been collecting integer sequences from the mid- nineteen-sixties from every possible source. I think it's one of those things that once you've started you just can't give up. He had listed well over two thousand of them arranged lexicographically.

An integer sequence is simply a series of integers which may be specified *explicitly* by giving a formula for its *n*th term, or *implicitly* by giving a relationship between its terms. For example, the sequence 0, 1, 1, 2, 3, 5, 8, 13 ... (the Fibonacci sequence) is formed by starting with 0 and 1 and then adding any two consecutive terms to obtain the next one: an implicit description. The sequence 0, 3, 8, 15, ... is formed according to the formula $n^2 - 1$ for the *n*th term: an explicit definition.

The book created a great deal of interest and people rushed to add new sequences to the list. To date there are over a million but of course this is far too many to publish in hard-copy, the database would now fill over 300 volumes of the original book. Another book *called The Encylopedia of Integer Sequences*, was co-authored by Simon Plouffe, it contains nearly five-and-a-half thousand of the more important sequences. There is also an electronic *Journal of Integer Sequences* devoted to papers dealing with integer sequences and related topics.

But if you want to see the complete 300 volumes then you'll need to go on line to the Encyclopedia of Integer Sequences (http://www.research.att.com/~njas/sequences/). There you can look at sequences to your heart's content. But maybe the web site is more useful if you want to see how a sequence might be continued. For instance, you can just type in the sequences that we included in the July and August problems and out will come numbers of sequences (see more on the August problem below). And if you have some sequences that are not on the integer web site, then I'm sure that Neil would be interested in adding them.

I wonder if Neil knew what he was starting when he put his first book together. One thing leads to another, I guess.

67 is a prime number and the sum of five consecutive primes (you might like to find them, answer below).

The famous artist Grandma Moses began painting at age 67 and 67 is the atomic number of holmium. Ingrid Bergman, Augustin Cauchy, Paul Cezanne, Harry James, Leonardo da Vinci, a couple of King Georges (I and IV), Michel Rolle, Jacob Steiner, George Washington, Woodrow Wilson and cricketer W.G. Grace all died aged 67.

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What's new on nzmaths.co.nz

A collection of new activities have been added to the SNP section of the site, and more activities will be added in the next few weeks.
Māori vocabulary has been added to all of the activities in the Families section of the site.

Diary Dates

CensusAtSchool is currently underway – it runs from 13 August to 30 September 2007. Further information is available from http://www.censusatschool.org.nz .

Booke Review

Amusements in Mathematics by H.E. Dudeney

This collection of problems from Dudeney's columns in magazines such as *The Strand*, *The Queen* and *The Weekly Dispatch* was first published in 1917. It is still available via Dover Publications. Virtually every sort of mathematical and logical poser is included in the book – problems concerning number theory, tracing routes, moving counters, speed problems, measuring, weighing and packing problems, dissections, chessboard problems, mazes and magic squares. They're all there and a lot more besides. Each one is presented with Dudeney's unique urbane wit and sense of paradox and, you'll be pleased to know, each comes with its clearly explained solution and sometimes with discussion on where others tried to attack it and failed.

Most of the problems in the book are original creations and that's what makes *Amusements in Mathematics* such a landmark publication. Not only was Dudeney a very fine original problem compiler but also a savant and calculating prodigy. His memory held a host of number facts like all the cubes up to six figures, hundreds of primes and a range of number patterns. His problems were so popular in Britain that huge numbers of the book were sold. What made them particularly attractive was the way he put each one in a setting, telling a story, whether it was about giving change for purchases, making a patchwork for Mrs. Perkins or buying a Christmas present for Mr. Simpson. The book also includes notes on British coins and stamps and the game of cricket to help American readers understand some of the problems. The former makes very interesting historical reading and the latter makes it clear that even Dudeney did not fully understand the rules of cricket!

Amusements in Mathematics must have been an incredible source of material for teachers of mathematics and will no doubt continue to be so.

Frogs RIP: Vive La Différence!

I've had no correspondence about Frogs this month so I'm laying them to rest. I'll willingly raise them from the mud though, if I get any encouragement.

So let's take a look at some differences. Take any 2-digit number. I'll take 67 because that is the number of this issue of the newsletter. Reverse the digits to get 76. Take the smaller from the larger. 76 – 67 = 9. Think of 9 as 09. Reverse the digits to get 90. Subtract the smaller from the larger. 90 – 09 = 81. Do the same again. 81 – 18 = 63. And again, 63 – 36 = 27. Keep going. 72 – 27 = 45. 54 – 45 = 9. Oops! We've been here before.

So the question is, if you start with any 2-digit number and keep reversing and subtracting, will you always keep coming back to 9?

Play around with this for a while and see what you come up with.

Well clearly the answer to the question is "No". 88 – 88 = 0. So there are nine 2-digit numbers that are palindromes and give you zero. Set them all apart and ask again "if you start with any 2-digit number and keep reversing and subtracting, will you always keep coming back to 9?"

Let's put that question aside for a minute. Have you noticed anything strange about the differences that you keep getting? Well I suppose that it isn't strange exactly. But every difference is a multiple of 9. Is that always so?

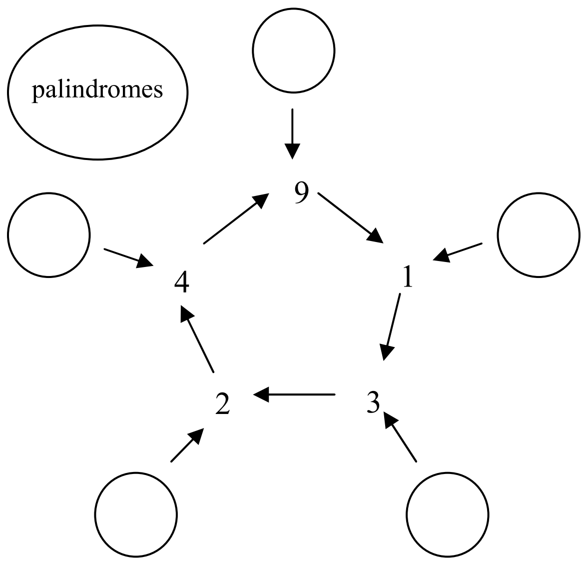
I'm sitting in a darkened room. There is strange music. Hang on I'm getting a proof. It's coming to me. Yes. You write the 2-digit number as ab. OK. So now look at ab – ba. Is it alright to assume that a > b? Yes, yes, OK. And ab – ba = a(10 – 1) – b(1 – 10). Keep going. Don't stop. And that is 9(a – b)? I don't believe it. Well, maybe I do. Hmmm.

In the light of day, what's so clever about that? Forgetting about the palindromes, every other number that I start with is going to give me a multiple of 9. And there aren't too many of them. Here they are, all five of them:

9, 18, 27, 36, 45.

You can see that you don't have to worry about 90 or 82 or 72 or 63 or 54. And we've seen that any of these numbers gives us 9. So there you go.

In fact we can map out all of the 2 –digit numbers like this. First we have palindromes in a group by themselves. Next we have the multiples of 9 sitting around a pentagon. Then every other number is directly attached to the pentagon.



Now I hope that you know what ‘Vive La Différence! II’ is going to be about.

Solution to August’s problem

The problem last month was an open-ended one. As in July, you were asked to find the next number in a sequence but this time the prize was to go not for a ‘correct’ solution but for the set of different solutions, with justification, that you offer. Subtlety, imagination and originality will be taken into account and not just the number of solutions. Here is the sequence, find the seventh term.

3, 1, 4, 1, 5, 9, ?

To make the idea clearer, here are a couple of possible solutions: 2 (the next digit in the decimal representation of π), 6 (even terms are random, odd terms are the consecutive whole numbers beginning with the number 3).

We had no solutions from anyone in New Zealand but we did have some from Germany. The answers below are from Christiane Schicke who lives in Braunschweig.

a) 3,1,4,1,5,9,1,9,3,9,5,9, …. First Christiane looked for someone who had a phone number starting with the magic six numbers. There wasn’t anyone in the book but there was a bus that went from the station in Rothenburger Strasse to the main hospital. It leaves on Sunday at 16.31, 16.41, 16.59 and 17.19. The rest is history! Here is the bus time table.

→ Gültig ab 10.12.06

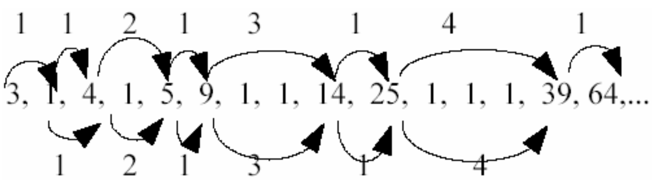
Samstag								
Uhr		5	6	7	8-15	16	17-22	23
Rothenburger Str. U R	ab	19 39 59	19 39 59	19 39 51	01 11 21 31 41 51	01 11 21 31 41 59	19 39 59	19 39
Hessestr.		21 41 01	21 41 01	21 41 53	03 13 23 33 43 53	03 13 23 33 43 01	21 41 01	21 41
Austr.		22 42 02	22 42 02	22 42 54	04 14 24 34 44 54	04 14 24 34 44 02	22 42 02	22 42
Gostenhof Ost U		24 44 04	24 44 04	24 44 56	06 16 26 36 46 56	06 16 26 36 46 04	24 44 04	24 44
Gostenhof West U		25 45 05	25 45 05	25 45 57	07 17 27 37 47 57	07 17 27 37 47 05	25 45 05	25 45
Deutschherrnstr.		26 46 06	26 46 06	26 46 59	09 19 29 39 49 59	09 19 29 39 49 06	26 46 06	26 46
Großweidenmühlstr.		27 47 07	27 47 07	27 47 00	10 20 30 40 50 00	10 20 30 40 50 07	27 47 07	27 47
Brückenstr.		29 49 09	29 49 09	29 49 02	12 22 32 42 52 02	12 22 32 42 52 09	29 49 09	29 49
Klinikum Nord		30 50 10	30 50 10	30 50 04	14 24 34 44 54 04	14 24 34 44 54 10	30 50 10	30 50
Lange Zeile		32 52 12	32 52 12	32 52 06	16 26 36 46 56 06	16 26 36 46 56 12	32 52 12	32 52
Friedrich-Ebert-Platz	an	34 54 14	34 54 14	34 54 08	18 28 38 48 58 08	18 28 38 48 58 14	34 54 14	34 54

Sonn-/Feiertag					
Uhr	5	6-7	8	9-22	23
Rothenburger Str. U R	13 33 53	13 33 53	19 39 59	19 39 59	19 39
Hessestr.	15 35 55	15 35 55	21 41 01	21 41 01	21 41
Austr.	16 36 56	16 36 56	22 42 02	22 42 02	22 42
Gostenhof Ost U	18 38 58	18 38 58	24 44 04	24 44 04	24 44
Gostenhof West U	19 39 59	19 39 59	25 45 05	25 45 05	25 45
Deutschherrnstr.	20 40 00	20 40 00	26 46 06	26 46 06	26 46
Großweidenmühlstr.	21 41 01	21 41 01	27 47 07	27 47 07	27 47
Brückenstr.	23 43 03	23 43 03	29 49 09	29 49 09	29 49
Klinikum Nord	24 44 04	24 44 04	30 50 10	30 50 10	30 50
Lange Zeile	26 46 06	26 46 06	32 52 12	32 52 12	32 52
Friedrich-Ebert-Platz	28 48 08	28 48 08	34 54 14	34 54 14	34 54

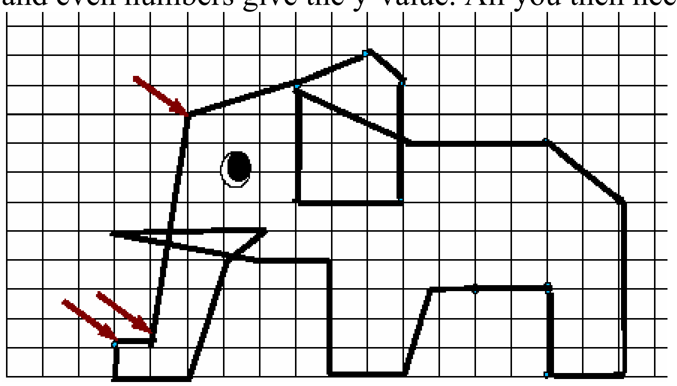
ZEICHENERKLÄRUNG: V01 = nur an Schultagen F04 = nicht während der Sommerferien

An den Adventssamstagen werden zwischen 16:00 Uhr und 20:00 Uhr Verstärkungsfahrten durchgeführt !

b) 3, 1, 4, 1, 5, 9, 1, 1, 14, 25, 1, 1, 1, 39, 64, …. Here you oscillate from a ‘top’ rule to a ‘bottom’ rule. Along the top you first add 3 to 1 (one apart) to get 4. On the bottom you add the 1 in the second position to the 4 (one apart) to get 5 but put this two positions along. Now go back to the top. Add 4 to 5 (two positions apart) to get 9. Now go back to the bottom and add the 5 to the 9 to get 14. As you go you have left some spaces so put 1 in each of these spaces. It turns out that first you need one 1, then two, then three and so on. The diagram below might help.



c) 3, 1, 4, 1, 5, 9, 3, 12, 3, 15, 27, 9, 36, 9, …. Since 9 is 3 x 3, multiply the 1 in the second position by 3. Keep multiplying by 3.
d) 3, 1, 4, 1, 5, 9, 6, 27, 7, 625, 8, 3125, …. The odd terms are just going up by one each time. The even terms come in pairs are n^{n-1} and n^n as in 1^0, 1^1, 3^2, 3^3, 5^4, 5^5, …
e) 3, 1, 4, 1, 5, 9, 8, 10, 8, 6, … Odd numbers give the x-value of a point in 2-dimensions and even numbers give the y-value. All you then need is an elephant!



f) 3, 1, 4, 1, 5, 9, 58, …. This comes from fitting a degree 5 polynomial to the first six numbers in the sequence. It’s likely that the next numbers may not be integers though.
g) 3, 1, 4, 1, 5, 9, 22, 41, … Now fit a quadratic through various subsequences of three terms. Things get difficult from here so we leave that to you to work out.
h) 3, 1, 4, 1, 5, 9, -1. 9, 14, -3, 13, 19, …. Take (3, 1, 4) and (1, 5, 9) as coordinates on a straight line in 3-dimensions. The other sets of three numbers are coordinates of points on that line. It’s up to you to choose some rule to give the ‘next’ point for the sequence.

But then I thought that I’d look at the on line Encyclopedia of Integer Sequences. It came up with 9 possibilities none of which is as original as the bus timetable. But here they are anyway. Can you really believe some of these?

1. Their first choice was the integers of π.
2. Then there was the decimal expansion of 104348/33215. My guess is that this fraction is someone’s approximation of π.

3. In the decimal expansion of π: greatest number contained as a string in the first n positions, which is also contained in the next n positions.
4. The decimal expansion of 355/113.
5. Decimal expansion of $4\sum_{1}^{500000}(-1)^{(k-1)(2k-1)}$.
6. Decimal expansion of $(\sqrt{20467}-\sqrt{19578}+\sqrt{10117}-\sqrt{9553})\div 2$.
7. $\sqrt[9]{10e^8}$.
8. Erroneous version of decimal expansion of π (see A000796 for the correct version).
9. Decimal expansion of $\sum_{k=-\infty}^{\infty}\frac{\log 2}{2^{-k/2}+2^{k/2}}$.

Where on earth did they get some of these from?

Anyway, I hope that you have been inspired by all of these answers. So I'm holding open the competition until next month for anyone who can come up with two or more new ways to get 3, 1, 4, 1, 5, 9, into an integer sequence.

This Month's Problem

Here are three little mathematical poems written by John McClellan in the style of the limerick.

A man in an ivory tower
Dreamed of π to the 200th power
(this is apt to occur
when you mix a liqueur
with clams at a very late hour).

If you want to be famous like Miro
Write a book on division by zero
or give a long lecture
on Goldbach's conjecture -
you'll end up a bit of a hero.

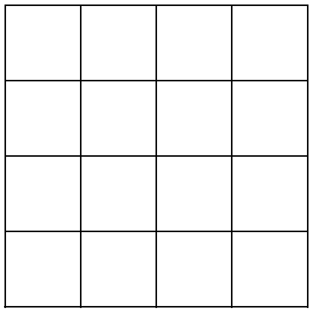
Our Mabel has saved up her dimes
To purchase a table of primes.
We never thought Mabel
Would need a prime table -
Perhaps it's a sign of the times.

Yes, you've guessed it, your task this month is to write a mathematical limerick. We will give a $50 book voucher for an original and amusing limerick. Please send them to derek@nzmaths.co.nz and remember to include a postal address so we can send the voucher if you are the winner.

Solution to August's Junior Problem

On a chess board a queen can move horizontally, vertically and diagonally. She also can take another piece that is on the horizontal, vertical or diagonal line that she is on (if there is no other piece in between).

The normal chessboard is an 8 by 8 board but we want to think only about a 4 by 4 board (see below).

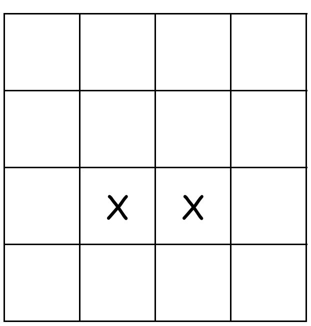


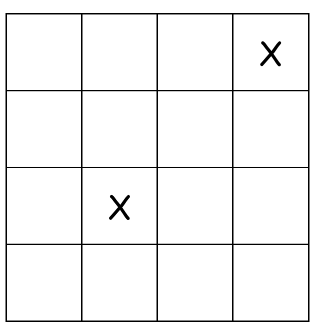
What is the smallest number of queens that you can put on this board so that they are attacking all of the squares that they are not on?

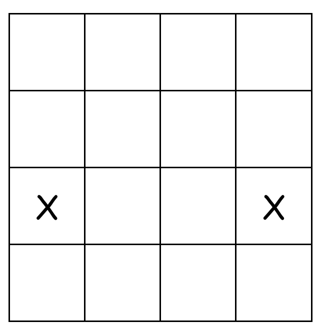
How many different solutions of this problem are there?

I didn't have any entries to this so let's see how to do it. First of all you can't cover all the squares of the 4 x 4 board with one queen so you need at least two. I've shown how you can do the covering with two queens. So what I have to do now is to show that there are essentially (with respect to rotations) only three ways that it can be done. (I would have accepted the three ways without explanation if that had been the only entry I had.)

Let's first suppose that both of the queens were in the middle 2 x 2 square. If they are on the same diagonal then there are two corner squares that are not covered. So the two queens have to be as I've shown on the first board.







Suppose that one of the queens is on the interior 2 x board. There are four positions for this queen but they are all symmetric so assume that the queen is in the bottom left corner of the 2 x 2 square. There is symmetry then about the diagonal through the interior queen. So we only have to try seven squares around the side of the board. Only one of these squares works.

So now assume that the queens are both on the side of the board. By symmetry, this gives two positions for the first queen. Putting one on a corner square there are two interior squares not covered. To cover these you have to put the other queen on a diagonal. But then there are still squares left uncovered.

So don't put the first queen on a corner square. By trial and error there is only one place that the second queen can be put.

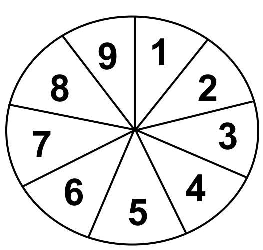
> Is anyone game to try the 5 x 5 board? I'll give the August prize to anyone who can make some progress with the 5 x 5 board.

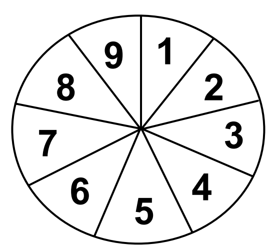
This Month's Junior Problem

This section contains a monthly problem competition for students up to Year 8 with a $20 book voucher available for the winner. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so that we can send you the voucher if you are the winner.

A fair game is one in which all players have the same chance of success. So a rugby game between Japan and the All Blacks is probably not fair but Snakes and Ladders probably is fair.

Ann and Bob's Game: Ann and Bob take it in turns to spin both of the spinners shown below. The numbers that come up are added. If the result is even, then Ann wins; if the result is odd, then Bob wins.





Is this a fair game? If it is, why is it fair? If it isn't, why isn't it and who is better off as a result?

Afterthoughts

67 = 7 + 11 + 13 + 17 + 19