



Newsletter No. 66

August 2007

In our last issue we included a cartoon showing the insect Hex (how did it get that name, we wonder?) climbing the hypoteneuse of a right-angled triangle - a touch of humour amidst the mathematics. Few people associate humour with mathematics. For many mathematics created only misery at school and was their least favourite subject. As Roger Highfield once wrote when he was Science Editor of the *Daily Telegraph*:

If you are introduced to a mathematician at a party there is a great groan. People boast about how bad they were at school, yet they would never boast how bad they were at English.

Some observers are prepared to take a wry approach in their references to the subject. Fran Lebowitz once wrote:

If you are really concerned about your child's education don't teach it to subtract, teach it to deduct.

And you'll recognise:

The word 'lesson' came back to Pooh as one he had heard before somewhere. "There's a thing called Twy-stymes", he said, "Christopher Robin tried to teach it to me once but it didn't."

On the other hand people can be mathematically amusing without always realising it. Take these two questions innocently asked of me in times past:

Which half do you want, the bigger half or the smaller half?

#### **Canteen Lady**

Which way's horizontal, up or down?

#### Student

We'll return to the subject of humour and mathematics but I must just add a little anecdote relating to the cartoon in *Afterthoughts* below. I used to keep a copy on my classroom door - only those for whom it produced a chuckle were allowed to enter to our lunchtime maths club (the others were deemed to have missed the point!).

This is our 66<sup>th</sup> issue and the sum of the divisors of 66, including 66 itself, is a square number (144). The sequence of numbers with this property starts: 3, 22, 66, 70, 81, ....

Although some very well-known people died aged 66 it is not all doom and gloom. John Betjeman became Poet Laureate at that age in 1972 and two people were awarded Nobel Prizes aged 66 - François Mauriac for Literature in 1952 and Niko Tinbergen for Medicine in 1973.

Those who died aged 66 include W.H. Auden, Hieronymus Bosch, Thomas Cranmer, C.S. Forester, Christian Huygens and Joe Louis.

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### **Diary Dates**

Maths Week is, as always, the second full week of August. This year that is the 13<sup>th</sup> to the 17<sup>th</sup> of August. The site is not quite ready yet, but you can bookmark it at: <u>http://www.mathsweek.org.nz</u>

### **Booke Review**

#### De Thiende by Simon Stevin

The extremely versatile mathematician, scientist and engineer, Simon Stevin, more than any other was responsible for the widespread adoption of the decimal system. He was born in Bruges, Flanders in 1548.

He wrote a number of books but *De Thiende* was by far his work of greatest importance. It contained a complete decimal system consistently applied to integers and fractions. In the book Stevin demonstrated the simplicity, feasibility and advantage of decimals to anyone who dealt with measure - astronomers, surveyors, bankers and so on. He called for the system to be applied to all weights and measures and coinage - in this he was only partially successful. For example, it was to take many years before some countries adopted the decimal system for coinage. De Thiende, written in Dutch, was originally published in 1585.

The book covers the four rules of arithmetic with an appendix covering applications in measuring land, cloth, liquids - areas or volumes where appropriate - and money. One drawback was that the notation used, based of course on place value, was nevertheless somewhat awkward, however the methods of handling the four rules were as we know them today.

Here's the first definition from De Thiende,

Dime is a kind of arithmetic, invented by the tenth progression, consisting in character of ciphers, whereby a certain number is described and by which also all accounts which happen in human affairs are dispatched by whole numbers, without fractions or broken numbers.

The awkwardness of the notation is best shown by example. What we write as 634.76 Simon Stevin wrote as 63@4①7@6③. The digit before the ①, the number in the units column, he called the prime. Those before the ③ were the higher place-valued digits (tens, hundreds and so on). That before the ② was, according to Stevin, 'the tenth part of unity of the prime we call the second'. He continued, 'and so of the other; each tenth part of the unity of the precedent sign, always in order one further.'

The first English translation of the book was published in 1608 and numerous reprints appeared from then on.

## Frogs V

I'm sorry I really wanted to stop Frogs at IV but they won't lie down (or whatever frogs should do). I've had contributions from two people this month that I think are worth passing on, so here goes.

Marion Steel came up with two things so she goes first. "In the July newsletter, you said that you weren't able to easily generalise the algorithm for b and g frogs. The way I think of it is that the brown frogs have to each move g places plus the empty space for a total of b(g+1). The green frogs each have to move b spaces plus the empty space for a total of g(b+1). Each frog must pass each frog on the other team exactly once, and one must jump over the other to pass so the number of jumps is bg. The number of moves is equal to the total of these spaces minus the number of jumps (since each jump is two spaces). Altogether, the number of moves is b(g+1) + g(b+1) - bg = bg + b + g. Is this what you were looking for?"

I have to say that that was exactly what I was looking for. But she went on: "I've been playing with the long-legged frog problem. If the frogs can jump over each other, and the brown frogs have long legs, then for team size (1, 1) 3 moves; (2,2) 5 moves (ac, da, bd, eb, ce – where each move is from and to a position); (3,3) 11 moves (bd, eb, de, gd, eg, ce, ac, da, cd, fc, df); and I have got (4, 4) 20 moves; (5,5) 28 moves; and (6,6) 40 moves. But I'm not sure those are a minimum. No pattern yet."

I know that I'm laying myself open for Frogs VI here but can anyone find the time to (i) check Marion's numbers to check that they are the minimum possible on that number of frogs; and (ii) suggest a pattern.

The next contribution comes from Andrew who many of you know from your contact with him in the Maths Technology (they run nzmaths) office. So here is

**Andrew's problem:** What if we put the lily pads in a circle? Have b boy frogs next to each other, then an unoccupied lily pad, then g girl frogs all next to each other, then another unoccupied lily pad. The usual moves of slides and jumps are still the only ones allowed. What is the minimum number of moves needed to change the positions of the frogs?

I want to stress first that there are two versions of this problem. It all depends on how you feel about empty lily pads. You might say that the same lily pads have to be unoccupied after the interchange as were unoccupied before. Or you might say that you don't care as long as all the boy frogs are together and all the girl frogs are together and there are two lily pads between them. Of course you might say that you can't see any difference in these two. If you do then you should perhaps think about what happens if we go back to the linear version. If b and g are different, the lily pad that is unoccupied at the start is not the same as the one that is unoccupied at the end. So we don't have to insist on an invariance of unoccupied lily pads.

I've had a little crack at this with b = g. Below I've put a table of values that (i) has my best results; and (ii) that compares this with the original problem. I would be super interested if you can improve my answers and if you could do something for  $b \neq g$ .

b = g	1	2	3	4	5	6	7	8	9
linear	3	8	15	24	35	48	63	80	99
circular	3	6	11	16	23	30	39	48	59

By the way, I've assumed that the unoccupied lily pads stay in the same place.

### **Bill's Number Plates**

A long time ago I wrote the Level 5 Problem Bill's Number Plates. So that you don't have to look it up I've listed it below.

Bill is making a small profit by designing and selling (through the appropriate channels of course) number plates. The only rule that he has to obey is that a maximum of six characters, numbers or letters, is permitted. (However, plus and minus signs, or any other mathematical symbols, are not characters that he can use.)

He decides to make up some number plates. For instance, he can make 10 by using '10' or by 'TEN'. In how many ways can he make up 12?

What number less than 20 can be represented in the most ways?

Out of the blue, I suddenly get an email from Angela Cook saying:

Our class has LOVED this task - THANK YOU!! [Room 1 Maths, *Dunedin North Intermediate*, Dunedin]

Here's one of the most interesting lists, led by Samantha Bucky. She'd be honoured should it be included on the *nzmaths* site.

THREE, THR33, NO3, N03, THWEE, NUMBA3, NOIII 1PLUS2, 2PLUS1, 0PLUS3, 3PLUS0, 0PLUS3, 3PLUS0, 1AND2, 2AND1, 3AND0, 0AND3, THR3E, THRE3, 3BY1, 1BY3, 3X1, 1X3, 9MNUS6, 8MNUS5, 6MNUS3, 7MNUS4, 5MNUS2, III, IIANDI, 9TAKE6, 8TAKE5, 7TAKE4, 6TAKE3, 4TAKE1, TAKE0, 1ADD2, 2ADD1,.... TORU, TROIS, DREI, TRE, TRES,

Thanks for the list Samantha. It has also been added as an example solution to the problem on the nzmaths site.

### Solution to July's problem

Last month the problem went like this.

I expect you remember those problems much beloved by compilers of so-called 'intelligence' tests. You are more likely to see them in the puzzle corners of magazines these days. They go something like: What is the next number in the sequence: 1, 2, 4, 8, 16, ...? The problems are almost always ambiguous but that doesn't stop us having a go at finding the most likely solution to the problem.

And that's your task this month! Here are the sequences, have a go. In each case can you supply the next missing number?

- 1. 77, 49, 36, 18, ...?
- 2. 2, 11, 23, 31, 41, 53, ...?
- 3. 3, 3, 5, 4, 4, 3, 5, 5, 4, 3, ...?
- 4. 2, 3, 5, 7, 11, 101, 131, 151, 181, 191, ...?
- 5. 1, 4, 9, 121, 484, 676, ...?
- 6. 6, 8, 10, 14, 15, 21, 22, 26, 27, 33, ...?

This month's winner is John Kramer of Whangarei. Congratulations John. Here are his solutions.

- 1. 8. (Given a start of 77, the product of the digits of the number defines the next in the sequence.)
- 2. 61. (The terms are the smallest primes in each block of ten 0 to 9, 10 to 19, 20 to 29, etc.)
- 3. 6. (The terms are the number of letters in spelling each of the words one, two, three and so on. The required term was the number of letters in the word eleven.)
- 4. 313. (The next palindromic prime number.)
- 5. 10201. (The next palindromic square number.)
- 6. 34. (The next number that has four divisors.)

We just want to underline the fact that we are only giving a rule that we can see. There is no reason why you can't get any number you like. In fact let's see how to get an infinite number. In the first sequence we gave four numbers. So let's take a general cubic f(n) = $an^3 + bn^2 + cn + d$ , where n is the number in the sequence of the number f(n). So f(1) = $77 = a(1^3) + b(1^2) + c(1) + d$ ;  $f(2) = 49 = a(2^3) + b(2^2) + c(2) + d$ ; f(3) = 36; f(4) = 18; if you have patience you could solve all of these equations to find a, b, c and d and so get f(n). You can bet your life that f(5) isn't 8.

Now if you are prepared to do this for a cubic you can do it for any polynomial. You'll get a different function each time; that will give a different f(5) each time; and so you can get an infinite number of genuine fifth terms of the sequence.

In the same vein you might like to play around with one of the questions from this year's National Bank Junior Mathematics Competition. You will have to work until the end to see why it is in the same vein.

If you want to find the rest of the questions or some more material on the Competition, then use <u>www.maths.otago.ac.nz</u> and click on the 'Schools section' of the pie chart. Then click on the competition button on the page that comes up next.

(By the way, you might like to hunt down the Problem Challenge if you don't know about it already. It's on the same web site in roughly the same position. There are a lot of useful problems there for primary school students.)

### **Question 2 (All Students)**

Sally stores mathematical expressions in her calculator. She uses special function buttons f1, f2 etc. to store expressions which she wants to use later. One day her little brother Robert is looking at her calculator and he asks Sally about the special function buttons. Sally shows him how they are used (although she doesn't use any fractions at all because Robert doesn't really understand fractions yet). Sally tells Robert that the expression she stored using f1 is  $4x^2$ . This means that if Robert enters a number into this expression, the number will be squared and multiplied by 4. (a) Write down the result when Robert enters the number 3 into the expression Sally stored using f1.

(b) When Robert presses INV f1 the calculator undoes the expression stored using f1. This means that if Robert uses INV f1 for the result he obtained in part (a) then the answer would be 3. Write down the result when Robert uses INV f1 for the number 64.

Sally doesn't tell Robert what expression she stored using  $f^2$ , but she does tell him that if he enters 2 and uses  $f^2$  the result will be 12, while if he enters 3 and uses  $f^2$  the result will be 36.

(c) Find a possible result when the number 4 is entered using the stored result  $f^2$ . Explain what expression has been stored using  $f^2$  to give your result.

(d) Is the result you found in (c) the only possible result? If it is, explain why. If it isn't, write down another possible result when the number 4 is entered using  $f^2$  and also give the expression you used to obtain your new result.

## This Month's Problem

Last month we asked you to give the next term in a variety of sequences. Such problems are not usually well defined but that doesn't stop us offering solutions. To illustrate a little more about what we mean by 'not well defined' consider the sequence 2, 4, 6, 8, ... If you were asked to give the next number you probably wouldn't hesitate in saying 10, on the assumption, applying Occam's Razor (see below), that the sequence was defined by  $n \rightarrow 2n$ . Of course, without additional information, it might equally well be defined by  $n \rightarrow (n - 1)(n - 2)(n - 3)(n - 4) + 2n$  (or a million other formulae, no, more than a million other formulae).

Your problem this month is an open-ended one. Again, you are asked to find the next number in a sequence but this time the prize goes not for a 'correct' solution but for the set of different solutions, with justification, that you offer. Subtlety, imagination and originality will be taken into account and not just the number of solutions. Here is the sequence, find the seventh term.

### 3, 1, 4, 1, 5, 9, ?

To make the idea more clear here are a couple of possible solutions: 2 (the next digit in the decimal representation of  $\pi$ ), 6 (even terms are random, odd terms are the consecutive whole numbers beginning with the number 3)

We will give a book voucher to one of the entries to the problem. Please send your ideas to <u>derek@nzmaths.co.nz</u> and remember to include a postal address so we can send you the voucher if you are the winner.

## Solution to July's Junior Problem

For the Junior problem we produced some dice.

In the picture we have three dice with letters on them. The letters are M, A, T, H, S, Y. What letter is opposite M?



Did you really need three dice to find the answer? Can you do it with only two?

Megan Hiew of Auckland came up with the correct answer which is 'H' and doesn't need all three dice.

In fact you can get the answer from just the first and second dice or the second and third dice but not from the first and third.

If you want to know what is actually going on, M and H are on opposite sides as are A and S, and T and Y.

## This Month's Junior Problem

This section contains a monthly problem competition for students up to Year 8 with a \$20 book voucher available for the winner. Please send your solutions to <u>derek@nzmaths.co.nz</u> and remember to include a postal address so that we can send you the voucher if you are the winner.

On a chess board a queen can move horizontally, vertically and diagonally. She also can take another piece that is on the horizontal, vertical or diagonal line that she is on (if there is no other piece in between).

The normal chessboard is an 8 by 8 board but we want to think only about a 4 by 4 board (see below).

What is the smallest number of queens that you can put on this board so that they are attacking all of the squares that they are not on?

How many different solutions of this problem are there?

# Afterthoughts 1

Occam's Razor loosely suggests that if two or more explanations describe a phenomenon you should go for the simplest in the absence of additional information.

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# Afterthoughts 2

I was only joking about not letting them come in and enjoy the maths club. All were welcome. Those who didn't chuckle were, in fact, deemed to be in greater need!

# Afterthoughts 3

For those who haven't heard them before:

There are three kinds of mathematicians: those who can count and those who can't.

There are 10 kinds of mathematicians: those who understand binary and those who don't.