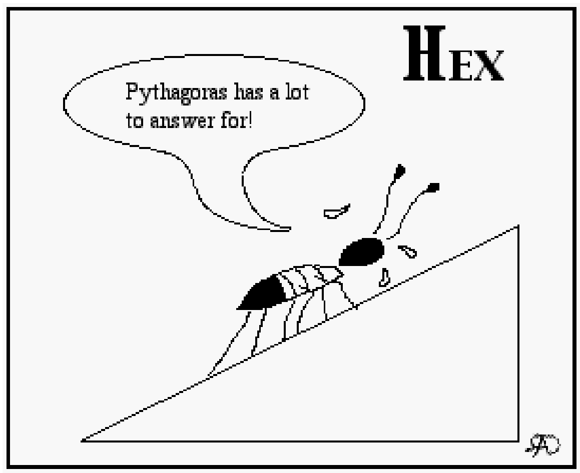
Newsletter No. 65 **July 2007**

65 is a multiple of 13 and triskaidekaphobia, fear of the number 13, costs America a billion dollars in absenteeism, rail and air cancellations and the like each year. I don't think we have figures for New Zealand but I guess the cost would be in the millions of dollars. It is commonly believed that aversion to the number 13 can be traced to the Biblical Last Supper where one of the 13 present betrayed Jesus. That maybe true although it only appears in literature from the Middle Ages. When the 13th falls on a Friday people get particularly apprehensive.

It's amazing that we are still superstitious about numbers, and we certainly are judging by the number of numerologists around the place who make a good living. Some of that could date back two-and-a-half thousand years to the time of Pythagoras who formed a religious brotherhood based around the 'magic' of numbers. Having found that musical harmony could be expressed in numerical formulae the Pythagoreans went on to look for numerical patterns in all branches of life. They applied their theories to music, astronomy and branches of science. For example, since 10 could be expressed as 1 + 2 + 3 + 4 and these four numbers were the building blocks, as they put it, of all numbers, 10 could be considered as perfect. As a result they believed there should be ten 'heavenly bodies' (the planets, sun and moon) - they only found nine but were convinced there was another up there somewhere.



65 is the second number to be the sum of two squares in two ways (see 'Afterthoughts' below) and is also the magic constant in a five by five basic magic square.

65 is of course the typical age of retirement when in most Western countries people are eligible for social security payments. However, it's not too late to begin a new career at that age. Winston Churchill was 65 when he first became Prime minister in 1940.

The following people, though, did end 'this mortal coil' aged 65; Matthew Arnold, Johann Sebastian Bach, Francis Bacon, Jean D'Alembert, Walt Disney, Nelson Eddy, Genghis Khan, Oscar Hammerstein and Queen Salote of Tonga.

INDEX

What's new on nzmaths.co.nz

A large collection of new activities have been added to the Number Facts section of the site (http://www.nzmaths.co.nz/Number/numberfacts.aspx).
There are also 6 new units of work in the Operating with Number section (http://www.nzmaths.co.nz/Number/Operatingwithno.aspx).

Diary Dates

Maths Week is, as always, the second full week of August. This year that is the 13th to the 17th of August. The site is not quite ready yet, but you can bookmark it at: http://www.mathsweek.org.nz

Booke Review

Prelude To Mathematics by W. W. Sawyer

This was a landmark book when it was first published in 1955. It came before the advent of 'modern mathematics' in the school curriculum, or 'modern math' as the Americans called it. It moved away from the traditional approaches of mathematics used by engineers and scientists to look at topics that were just plain interesting, exciting even. They included set theory, matrices, transformation geometry and group theory. The book paved the way for that insurgence of ideas that were to take over school mathematics curricula in the sixties.

Prelude to Mathematics opens with the sentence 'This book is about how to grow mathematics.' For a lot of young teachers this came as quite a shock. It described not only what was mathematics but what was a mathematician. It allowed teachers to ally themselves with a philosophy and offered an approach on how to go about teaching the subject. It was a radical approach at the time, one that encompassed an understanding of the historical background to the subject and the role of creativity. It described the beauty of mathematics - this at a time when maths was all long multiplication, working

Euclid's theorems and understanding decimal place. For those who had only met matrices, for example, in abstract university courses it introduced them in context and not just any context but one that could be built on in the secondary school classroom. It described geometries other than Euclid's and alternative algebras - wow!

Those that read the book began to cry out for change. "Remove the dryness from our subject, make it exciting" was the call. The modern mathematics movement was the result. Even that didn't get it all right but it was a move and one that encouraged educators to look at change, to challenge the *status quo*, keep exploring in a creative way.

Frogs IV

Maybe this should be the end of Frogs for a while. Let me try to finish off the questions that I raised last month and then put Frogs to bed. The first question was:

What if there were *b* brown frogs on one side and *g* green frogs on the other? What is the smallest number of moves that we would need then?

Let's think about what happens if there are 2 frogs on one side and 3 frogs on the other. Recall that we have 6 lily pads with the 2 green ones on the left lily pads, the 3 green ones on the right ones, and one empty lily pad in the middle. Also recall that frogs can move into an empty spot either by moving from an adjacent lily pad or by jumping over another frog onto the vacant lily pad. The aim is to get the green frogs to the right hand end and the brown frogs to the left hand end.

This game is best played first with children as the frogs. You might even have a 'frog master' who decides which frog should move where. Get the children to play the game a couple of times until they are sure that they have interchanged the frogs AND have done it in the fewest number of moves possible.

Since we want to find the answer for a general number of frogs it is a good idea to do a lot of examples and record the number of moves. At each stage query whether the number of moves is as small as it can be.

In doing this experimentation you might keep things as simple as possible. In this case I mean that you might keep the number of green frogs the same for a while while you vary the number of brown frogs. That way you get some idea of how the number of moves changes as you change the number of brown frogs.

At this point it's also a good idea to make a table to keep track of the results that you are getting. I'll start things off with 2 green frogs and 3 brown frogs to see what happens.

If we play around with the 3 brown frog case, the table starts to look like this.

Green frogs	1	2	3	4
Brown frogs	3	3	3	3
No. of moves	7	11	15	19

That looks like a simple pattern. How do things change if we stick with 2 brown frogs? What does the general situation look like?

One other thing worth thinking about is that if we get a formula with g and b in it, then it should become the formula for f frogs on both sides when we put $g = b$. (That's what happens with generalisations.)

It turns out that we can do the same thing with g and b frogs as we did with f frogs in Frogs III last month. We can work out the minimum number of moves that are possible. Unfortunately I still don't have a way of showing that this number, whatever it is, can be achieved for all values of b and g because I can't easily generalise the obvious algorithm that works for small numbers of frogs.

The second question that I asked last month was:

Suppose that the brown frogs have big legs; suppose that this doesn't stop them from sliding but that it does mean that when they jump they can only jump over two other frogs onto an empty lily pad?

If we go with that idea, what formula for the minimum number of moves can we find? The thing is that I haven't had the chance to completely analyse what happens here. I don't know what answer to expect. I have a couple of conjectures though. The first one is that if a frog isn't allowed to jump over one of its own colour, then the smallest number of moves is just the same as if no frog had long legs. The second one is that we get fewer moves than this when frogs can jump over one of their own kind. But how many fewer I'm not sure.

I'd be interested to hear from you though about the sorts of results that you get. Please send me any numbers that you produce by experimentation for the long-legged frog problem. Perhaps we can pool our ignorance and get a better couple of conjectures.

In the meantime, I think that you might find that $bg + b + g$ gives something you might have been looking for. If I get enough responses from you then maybe we can have a Frogs V after all.

So What is Mathematics?

I'm sorry that this month we seem to have gone very problemy and philosophical. To add to the philosophy of the Booke Review, for a while I want to think about what mathematics is.

It's easy to say what it isn't. **It isn't a collection of things that have to be learnt.** Perhaps it might be better to say that it isn't ***just*** a collection of things that have to be learnt.

Mathematics isn't just about being able to add, subtract, multiply, divide, learn the basic facts, use a bit of algebra, construct pie charts, and so on. This is not to say that these skills aren't important. There is not much that you can do in mathematics if you don't know how to do these and a variety of other things. This collection of 'things' that we learn in school is, generally speaking, worth knowing.

But research mathematicians don't spend their lives learning more skills. What would be the point? What research mathematicians do do is to solve unsolved problems. These problems might be old ones that they find in the literature, new ones that someone told them about, or really new ones that they have just made up themselves. They might be off-the-planet problems that only a pure mathematician might want to know the answer to or they may be extremely practical and useful ones that will help to put a rocket on the moon, lead to the invention of ipods, or help produce new sources of energy.

So how do they solve these problems? Well it's always useful to have a few examples, so one way to start might be to generate a whole lot of examples. (Does that sound familiar? If not, go look back at the Frogs series or at least Frogs IV.) From these they might be able to see some pattern so they make a conjecture – a guess as to what the answer might be.

Then they try to establish that conjecture; they try to justify what they have guessed. Here and throughout the process, they will use all of the known skills in their armoury. And if these skills aren't strong enough, then they may have to invent new ones of their own. (These, of course, go into the total set of mathematical skills and may eventually end up being taught in school or at university.)

If they can't justify their conjecture they look for a reason why the conjecture is false. This might lead to a new conjecture. If they can justify it, then they have a theorem and they try to extend or generalise this theorem.

The process that they have gone through may end up posing more questions than it answers and so there is a problem to solve for the next day and possibly for the next research mathematician.

But you know all of this. It's exactly what we have been doing with the Frogs series. There we started off with a few frogs on each side. That was our original problem. We played with that until we had that pretty well solved. Then we tried to see if we could find the number of moves needed with an arbitrary number, f, of frogs. This was our first generalisation; our first attempt to make a general statement rather than a statement about a specific number of frogs. We managed to get the right conjecture and then get a reasonable idea of how to solve that conjecture.

This led us to think about having a different number of frogs on each side. This was a further generalisation. Again we experimented and got some evidence before making a conjecture. The proof of this conjecture follows along much the same lines as that for f frogs on each side. And the proof used techniques that already existed: various methods of counting and some algebra.

But while we were doing this we thought up the long-legged possibility. This is an extension of our original problem because (i) it comes out of it; and (ii) if we solved the long-legged problem we wouldn't automatically have a solution of the normal-legged cases. I'm sure that you can think up a number of variations for yourself.

So how do you teach so that your students can see this other creative, problem solving part of the subject? You shouldn't have to think too hard about that because that is what the Numeracy Project is all about. First you find some nice situations or problems and let the children play and come up with ideas. Then, if they can't find solutions or answers for themselves, you guide them to discover them. Hopefully this whole process is then more interesting for you (if a bit harder than just telling them answers and letting them practice things), and more fun for them. Surprisingly at the same time they are finding out more of what mathematics is than they did the old way.

Solution to June's problem

You were asked to explain why the surface-area to volume ratios for three different solids appeared to be the same. They are not, of course, and this is because the ratios are sensitive to the units of measurement. The result 3/r can take any value depending upon that of the variable r, so to say the three results are the same is to say that the three results which can take any value are the same. Let us look at a two-dimensional analogue.

Consider a circle of radius r, then the circumference to area ratio is $2\pi r/\pi r^2 = 2/r$.
Consider a circle of radius 3r, then the circumference to area ratio is $6\pi r/9\pi r^2 = 2/(3r)$

These formulae are different!

The surface-area to volume ratio is dependent on the value of some measure, it is not constant.

This Month's Problem

I expect you remember those problems much beloved by compilers of so-called 'intelligence' tests. You are more likely to see them in the puzzle corners of magazines these days. They go something like: What is the next number in the sequence: 1, 2, 4, 8, 16, …? The problems are almost always ambiguous but that doesn't stop us having a go at finding the most likely solution to the problem.

And that's your task this month! Here are the sequences, have a go. In each case can you supply the missing number?

1. 77, 49, 36, 18, ?
2. 2, 11, 23, 31, 41, 53, ?
3. 3, 3, 5, 4, 4, 3, 5, 5, 4, 3, ?
4. 2, 3, 5, 7, 11, 101, 131, 151, 181, 191, ?
5. 1, 4, 9, 121, 484, 676, ?
6. 6, 8, 10, 14, 15, 21, 22, 26, 27, 33, ?

We will give a book voucher to one of the correct or part-correct entries to the problem. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so we can send the voucher if you are the winner.

Solution to June's Junior Problem

Problem: Last month we asked the following question.

Take any piece of paper and cut it into two. Obviously you'll get two pieces of paper. Then take any of these two pieces and cut them into two. That will give you three pieces of paper. Then take any of these and cut it into two. Four pieces results. Is it clear that if you keep on going this way for ever you can get any number of pieces of paper you like?

But what happens if you take a piece of paper and cut it into either 3 or 6 pieces of paper? And then you take that many pieces and you cut one of them into 3 or 6 pieces. And then you take that many pieces and you cut one of them into 3 or 6 pieces. And then you take that many pieces and you cut one of them into 3 or 6 pieces. …

What number of pieces can't you get and why?

Solution: It seems that you can get any number of pieces except 1, 2 and 4.

You can surely get 3 pieces but there is no way of getting any fewer than 3 pieces.

But how do you get 4 pieces? When you take a piece of paper you can either cut it into 3 or 6 pieces so the number of pieces goes up by 2 or 5 each time. To get 4 pieces you would have to start with 2 pieces (or -1 pieces!) and that's not possible.

However we can get 5 pieces by cutting the first piece into 3 and then cutting one of these pieces into 3. And we can go on from here in jumps of 2. So we can certainly get all of the odd numbers from 5 upwards.

On the other hand if you start off with 6 pieces, you can get 2 more pieces by cutting 1 piece up into 3. And you can get 6 + 2 pieces in the same way. So it looks like you can get any number of even pieces from 6 on.

As a result, overall we can get any number of pieces from 5 onwards. So only 1, 2 and 4 pieces can't be got in this way.

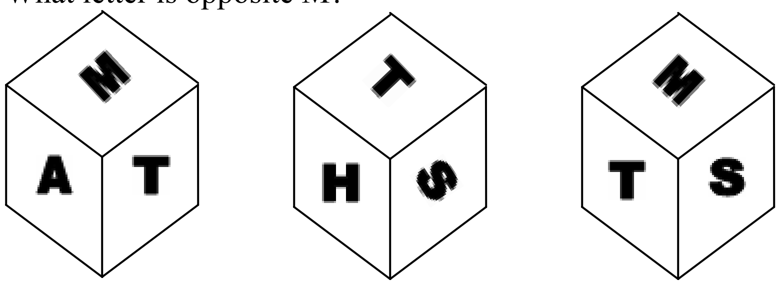
Extensions: What can you do now? What numbers of pieces can you get with 4 cuts and 6 cuts? Or 4 cuts and 5 cuts? What numbers of cuts mean that we can always get all possible numbers of cuts except for a finite number? What numbers of cuts mean that we can always get all possible numbers of cuts except for a infinite number?

Let me know what answers you get.

This Month's Junior Problem

This section contains a monthly problem competition for students up to Year 8 with a $20 book voucher available for the winner. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so that we can send you the voucher if you are the winner.

In the picture we have three dice with letters on them. The letters are M, A, T, H, S, Y. What letter is opposite M?



Did you really need three dice to find the answer? Can you do it with only two?

Afterthought

$65 = 8^2 + 1^2 = 7^2 + 4^2$.