Newsletter No. 64 **June 2007**

I've recently read a book entitled 'Modern Mathematicians' by Harry Henderson. It's one in a series of profiles on the lives and achievements of people from a wide range of cultures and fields of endeavour. For example, the series includes one on Latin American writers and another on contemporary African Political leaders. This one on mathematician's profiles 12 men and women whose contributions to mathematics have revolutionised modern thought and technology.

What particularly interested me was the use of the word modern in the title. In the book the mathematicians include Charles Babbage (1792-1871), George Boole (1815-1864) and Sofia Kovalevskaia (1850-1891). The most recent is John Conway born in 1937. Yet it was only last evening I heard a television newsreader say, " Way back in 1998" and we all know that, as far as the young are concerned, anyone born before 1980 is considered an 'oldie'.

Of course, no one disagrees with the idea that these mathematicians have revolutionised modern thought but then again so have Euclid, in a sense, and Isaac Newton. Maybe it's because a few years have elapsed since their contributions. We've had time to realise their importance and they are familiar. We can't say the same about the work of most contemporary mathematicians - it's too recent, it's mostly unfamiliar and we haven't had time to gauge its importance. There are parallels in the field of jazz. Jazz's history spans perhaps 100 years, yet modern jazz is said to have begun in the late 1940s - sixty years ago! I think perhaps in both cases we are using the word modern in an uncommon sense - not meaning recent but something much more subtle, reflecting the way people think about the subject.

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64 is, of course, a square, cube and sixth power. It is the smallest number with six prime factors, albeit them all being the same, 2. It is also the sum of the first four centred hexagonal numbers; thus 1 + 7 + 19 + 37 = 64.

The Scottish mathematician John Napier was 64 when he published, in 1614, his invention he called 'logarithms' after spending 20 years calculating the necessary tables. He was also responsible for the introduction of the decimal point.

John Galsworthy was awarded the Nobel Prize for Literature at age 64 in 1932 and Jean Argand, Pierre de Fermat, Karl Marx, Augustus De Morgan and Walter Raleigh all died at that age.

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What's new on nzmaths.co.nz

Several new links have been added in the links section of the site. We have also 'archived' those links with star ratings of less than three. You can still access these links by clicking from the bottom of each page of links.

Booke Review

Mishnat ha-Middot by R. Nehemiah, c. 150 C.E.

R. Nehemiah was a Hebrew rabbi, scholar and teacher and the author of the earliest Hebrew treatise on geometry known to us. He was one of the spiritual and intellectual leaders of the second century and as such his treatise became the basis of extensive future rabbinical literature.

Although some parts of the work are missing it is clear that it primarily dealt with elements of plane and solid geometry. The first five chapters were concerned with mathematical results generally known at the time while the remaining chapters looked at applications involving the calendar and measures of the Ark and the Tabernacle. The book is written in the form of maxims together with explanations and supplementary notes - a style still acceptable in school texts 50 years ago but no longer so. To give you an idea of this style here is part of the opening chapter:

> §1. There are four ways to grasp the area, namely; the quadrilateral, the trilateral, the circle and the bow-figure. The rule is as follows; the second figure is half the first one, and the fourth is half the third one. All the rest are closely connected with them, respectively as the hook with the ring.

The chapter continues with descriptions and terminology of concepts needed for deeper study including ideas of chord, angle, isosceles, bisection and the basic compass/ruler constructions.

Chapter two describes how to calculate areas of rectangles, triangles and circles and the volumes of solids built upright on these shapes. The units of measurement used are cubits. Later chapters involve the use of Pythagoras' theorem, Heron's formulae and the areas of sectors.

Numeracy in the Junior Classroom – Some Organisational Ideas by Jo Patrick

Having spent the last 7 years focusing on maths as a maths adviser and Numeracy facilitator I've recently returned to a school as a teaching AP with a Year 1 and 2 class.
No matter how passionate we are about maths, it is one of a number of curriculum areas we teach and we have only a limited amount of time to teach it. Keeping to a routine is crucial so that the children know what to do and teacher explanation time is kept to a minimum. Time is always a challenge in teaching. Here are some organisational ideas I've found useful:

Daily Starter Routine: – a little each day goes a really long way.
The daily warm-up time is a valuable time to teach knowledge in small, regular chunks. Young children tend to have shorter attention spans. Several quick, regular activities and songs are easier for them to say focused on than one long activity. Knowledge seems to be built up when a lot of repetition is involved. My class ranges in stage level from Counting From One on Materials to Advanced Counting, so counting is a daily occurrence. Up to 100 at least and / or back again. Tens frames also are displayed on the Maths Wall we use for Calendar Maths activities. We look at the tens frame patterns every day – saying how many dots there are, showing the numbers on our fingers, saying how many spaces. It takes less than 20 seconds. Every day. Last term we would have spent 15 minutes overall doing this in 20- second blocks. Almost every child can now say how many dots there are without counting (really important for developing part-whole strategies through the use of tens frames as a material.) Yet I wouldn't think they'd have picked it up quite as well in one 15 minute block of time, once.

Task Boards - Add Children's Names:
Task Boards are widely used by Junior teachers in maths. We have 4 numeracy groups and on my taskboard I have the names of the children within the shape of their group name (I've stuck to triangles, circles, squares, hexagons as I'm not overly creative!) I keep the group list on my computer so that as children change groups I can reprint the one A4 sheet and blue-tak the names on to the taskboard easily. This helps eases the disruption of having changed a group.

Planning:
To speed up planning writing I have an odd week and an even week template with the rotation of: Teacher, Practice, Group Box, Choice. Every group does 2 of these per day. So I see each group 5 times over 2 weeks – and don't have to change the rotation in my planning, just what the activities are in the boxes.

Independent Activities:
Independent Practice activities are set out on tables with a group card next to them – a piece of cardboard folded into a triangular shape, with the group shape on one side and the children's names on the other. They can quickly find their activity. Two groups a day do Practice activities so only 2 tables are used up, leaving the other 2 free for choice or Group Box activities.

Name Cards:
Name cards are invaluable for independent activities involving equipment. The children can put their name card next to an activity they have done or a pattern they have made and the

teacher can quickly roam towards sharing time to see what has been done. Photos can be taken to keep the activity for posterity – not so disappointing when it's pack-up time. It is also useful to have a space in the classroom where children can keep treasured constructions and equipment creations on show for a length of time. By viewing each other's work children can find inspiration for their own work.

Knowledge Learning Intention Wall Charts:
Sometimes Practice activities are a 'Partner Check' for a particular knowledge area. Children in the group pair up to check an aspect of each other's knowledge e.g. counting backwards from 100. (Parent helpers are very useful for this activity, too!) I display the Learning Intention on a sheet of paper on the wall, with the children's names penciled in that need to be practising that skill. When their partner or the parent helper or myself or the child thinks they have mastered that skill they write over their name with felt. It is useful for me to keep tabs on progress. In sharing time a child can be selected to show the class what they can do (and useful as a teacher check.)

Kho-Kho

In the last newsletter, Christopher Gall's article mentioned Kho-Kho. Now I had absolutely no idea what that was but, as with pretty well anything these days, it's easy enough to look it up on the web. There you'll find that Kho-Kho is an Indian team sport with twelve players on each side. The aim of the game is to avoid being touched by members of the opposition for as long as you can.

So, as I understand, Kho-Kho begins with one team sitting or kneeling in a row, with its members alternately facing in opposite directions. This team has a chaser whose job is clearly to chase the opposition. The other team has dodgers who try not to be tagged. Now this side can send two or three members in the court. The chasers can only run in one direction and cannot run between the sitters (unlike the dodgers who can run wherever they like). Chasers have to run round the entire row of their team in order to reach the other side. But what they can do for efficiency is to pass the chasing job to one of their sitting colleagues whose back is facing them. When the chaser touches a sitter, usually one nearer to a dodger the chaser shouts "kho" to signify the change of guard.

If your team can tag all your opponents in a shorter time than they can tag you, then your side wins.

I hope that that's all clear. Perhaps we can encourage Kho-Kho on the playing fields of New Zealand. We might even have a Test Match with India before long.

Frogs III

When we left you last month, we had discovered that in getting three green frogs to change sides with three brown frogs:

(i) it was a bad idea to allow two frogs of either colour to be together except at the end;
(ii) that it looked as if when we had to have f^2 jumps, where f was the number of frogs of each colour; and
(iii) that it looked as if when we had to have $2f$ slides – moves to the next vacant lily pad.

So how can we see that (ii) is right; that no matter how we go about it there have to be f^2 jumps?

Let's look at the number of jumps when $f = 3$. The first poor old brown frog is hovering around somehow and along comes a green frog. How can they get past each other? Well, one of them has to jump over the other. We don't quite know who the jumper is but it doesn't matter. When there is a brown/green encounter there has to be a jump.

Now the same thing happens when the next green frog comes along. For the brown frog to get past the green one, or vice-versa, there has to be a jump. And the same thing happens again when the third green frog comes along. There's a brown/green encounter and there's another jump.

Since this first brown frog is now on the other side of all of the green frogs the brown frog takes no more part in jumping – at least not with jumping with green frogs.

So the brown frog needs to be involved in 3 jumps. But the second brown frog has to be involved with 3 jumps to get past all of the green frogs. And the third brown frog has to get past the green frogs as the result of 3 more jumps. So there has to be $3 + 3 + 3 = 3 \times 3 = 3^2$ jumps.

Now we get the same sort of thing with four frogs. The first brown frog is involved with 4 encounters with green frogs and so is involved with 4 jumps. But the same thing can be said for all of the brown frogs. Since 4 brown frogs have to be involved with 4 jumps there have to be $4 \times 4 = 4^2$ jumps when $f = 4$. Of course you can use precisely the same argument with any value of f. This means that there have to be at least f^2 jumps.

It's not too hard, then, to show that there have to be a minimum of f^2 jumps but how do we manage the slides?

What do we have to do to establish that there are at least $2f$ slides? I'm afraid that's a bit more complicated, so again let's assume for the moment that $f = 3$. First of all let's see how far the brown frogs have to move. I'll do this in terms of numbers of lily pads. Now if the first brown frog moves to the furthermost lily pad it moves over 4 lily pads – the empty one in the middle and the 3 on the green side. If the first brown frog moves to the furthermost lily pad it moves over 4 lily pads – the empty one in the middle and the 3 on the green side. If the second brown frog moves to the furthermost but one lily pad it moves over 4 lily pads – the one before the empty one, the empty one in the middle and then 2 on the green side. If the third brown frog moves to the furthermost but two lily pad it moves over 4 lily pads – the two before the empty one, the empty one in the middle and then 1 on the green side. This gives a total of 3 x 4 = 12 lily pads to be covered.

Naturally the same thing happens for the green frogs. They move in the opposite direction but they move over the same number of lily pads – 12. This means that

altogether the frogs have to cover 24 lily pads.

So let's think for a bit. How many lily pads can be covered with a jump? Surely 2. One of these has a frog on and the other was empty before the jump. Now we know that there are 3^2 jumps. So the jumps

allow the frogs to cover 9 x 2 = 18 lily pads.

There are still slides to think about. Each slide covers 1 lily pad. But slides must cover the number of lily pads left after jumping. This number is 24 – 18 = 6. But 6 lily pads are covered by 6 slides. So we need 6 slides and 6 = 2 x 3.

With a little bit of work we can go through the same routine with f frogs. There we get $2f(f + 1)$ lily pads having to be covered altogether and $2f^2$ lily pads having to be covered by jumps. The number of slides therefore has to be

$$2f(f+1) - 2f^2 = 2f.$$

So it would seem that there have to be at least f^2 jumps and at least $2f$ slides so there have to be at least

$$f^2 + 2f \text{ moves}$$

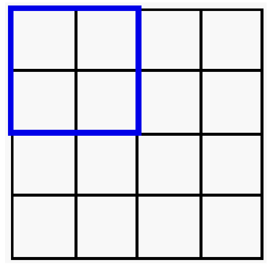
in total.

Now that's what we thought last month. So it looks as if we might have proved it. The only problem is that we can't be sure that we can always do it. Certainly we can do it for $f = 1, 2, 3, 4$, and maybe many many more. But how can we be sure that the method of moving frogs that we have shown for $f = 3$, works for $f = 3{,}000{,}000$? To settle the problem we would have to show that the method worked for every possible value of f. And I can't see a simple way to do that, can you?

But there's still more fun possible here. What if there were b brown frogs on one side and g green frogs on the other? What is the smallest number of moves that we would need then? Or suppose that the brown frogs have big legs; suppose that this doesn't stop them from sliding but that it does mean that when they jump they can only jump over two other frogs onto an empty lily pad? Or suppose …

Solution to May's problem

The problem was to determine the number of solutions to the game 4-Sudoku. Consider the top-left four squares (outlined in blue); there are 4! (four factorial, i.e. $4 \times 3 \times 2 \times 1 = 24$) ways in which the symbols 1 to 4 can be placed.



We thought that we had got this right using the following method. Consider the four squares at the top-right. In the top row there are two ways the remaining two symbols can be placed. This is also the case for the two symbols in the second row. So the symbols in the top-right four squares can be placed in $2 \times 2 = 4$ ways. The same is true when we consider the four bottom-left squares.

When all these symbols have been placed there are no choices left for the bottom-right four squares, i.e. the remaining symbols can only be placed in one way.

Hence, there are $24 \times 4 \times 4 = 384$ ways in which the symbols can be arranged according to the rules of the game. The 4-Sudoku has 384 solutions.

That is all very plausible until you delve a little further as Derek Smith did. When he got to 384 he stopped and thought.

4 #s	3 #s	2 #s	1 #s
2 #s	1 #s	2 #s	1 #s
2 #s	1 #s	1 #s	1 #s
1 #s	1 #s	1 #s	1 #s

Number of possibilities:
Upper Left 2×2 (UL) box: $4! = 24$
Upper Right 2×2 (UR) box given each UL: $2\text{x}2 = 4$

Lower Left 2×2 (LL) box given each UL: $2\times2 = 4$
Lower Right 2×2 (LR) box given LL, UR and LL: $1\times1 = 1$

Therefore there are:
$4!\times2^2\times2^2 = 24\times4\times4 = 384$ possibilities.
It is possible that some of these possibilities won't produce an answer. In other words, the LR section may be 'unfillable':

```
34  | 21
12  | 34
--------
23  | ??
41  | ??
```

here boxes in LR cannot be filled by any remaining valid number. Similarly for

```
34  | 21                  34  | 12                34  | 12
12  | 34                  12  | 43                12  | 43
--------       and        --------       and      --------
41 | ??                   23 | ??                 41 | ??
23 | ??                   41 | ??                 23 | ??
```

and using a cyclical replacement of the numbers in the 16 (= 4×4) possibilities Sudokus that can be formed with the UL being:

```
34  | ??
12  | ??
--------
?? | ??
?? | ??
```

4 of them result in an unfillable LR. Therefore 12 possibilities are good of these 16. For the total number of valid possible 4×4 Sudoku solvable puzzles is 24×12 = **288**.

Hence, using only 1,2,3 and 4 to complete a $2^2 \times 2^2$ Sudoku, 288 of these give a unique solution.

Well done Derek! You get (and deserve) this month's book prize.

Incidentally this must be generalizable to the n x n case.

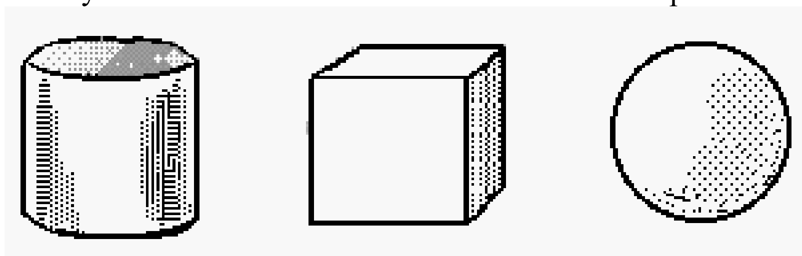
This Month's Problem

The ratio of surface-area to volume can be very important to organisms in respect of heat loss (or retention). Small animals have a relatively high surface-area to volume ratio, that is, their skin surface is large compared to their body's volume. Heat loss occurs through an animal's skin and so a high surface-area to volume ratio can cause them to chill quickly. High metabolic rates and fur are just two ways to help overcome this problem. In contrast, larger animals have a relatively small surface-area to volume ratio that can cause overheating problems. Such animals may need special techniques to reduce heat like sweating or tongue-lolling.

Here we have three organisms of different shape; a cylinder with height equal to its diameter, a cube and a sphere. Let us further suppose that the sphere fits snugly inside both the cylinder and the cube. Which of these three organisms do you think has the smallest surface-area to volume ratio? i.e. which is the most efficient heat retainer?

If the sphere is of radius r and it fits snugly inside the cylinder then the height of the cylinder is 2r and its radius is r. Also, if the sphere fits snugly inside the cube then the cube has each edge of length 2r. Lets us work out the surface-area and volume in each case:

	Cylinder	Cube	Sphere



Surface area:	$2\pi r^2 + 2\pi r.2r$ $= 6\pi r^2$	$6\times(2r)^2$ $= 24\pi r^2$	$4\pi r^2$
Volume:	$\pi r^2\times 2r$ $= 2\pi r^3$	$(2r)^3$ $= 8 r^3$	${}^4/_3\pi r^3$

So that in each case the surface-area to volume ratio is the same, i.e. 3/r . This implies, of course, that each organism is equally efficient in retaining heat which is clearly not the case.

The prize this month goes to the best explanation of this apparent contradiction. We will give a book voucher to one of the correct entries to the problem. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so we can send the voucher if you are the winner. Sorry but Ministry of Education employees and facilitators working on Ministry of Education contracts are not eligible to win this prize.

Solution to May's Junior Problem

Last month wee noted that 9 x 1089 = 9801. Here, multiplying by 9 we manage to reverse the digits of 1089. Are there any 4-digit numbers abcd so that multiplying by 4 reverses their digits? In other words 4 x abcd = dcba?

It turns out that … Why don't we do it from scratch? You can do this by playing with a calculator or by thinking. Of course thinking is harder.

For a start there are only two possibilities for a. If a = 3, then 4 x 3bcd is bigger than 1000 and so can't give the 4-digit answer dcba. The same problem arises if a > 3. So a = 1 or 2.

Suppose a = 1. To get the answer dcba, 4 x d = 1 or 11 or 21, and so on. But 4 x d is even, so we can't get an odd number ending with a units digit of 1.

Then a must be 2. To get 4 x d to end in 2, d can only be 3 or 8. Clearly 4 x a is no more than d, but if a = 3, 4 x a = 4 x 2 = 8 which is more than d = 3. So then d = 8. Similar reasoning gives b = 1 and c = 7. Indeed 4 x 2178 = 8712 and the way we've put things together this is the only 4-digit number that can be turned around by multiplying by 4.

Are there any 5-digit numbers? After all 9 x 10989 = 98901.

This Month's Junior Problem

This section contains a monthly problem competition for students up to Year 8 with a $20 book voucher available for the winner. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so that we can send you the voucher if you are the winner.

Take any piece of paper and cut it into two. Obviously you'll get two pieces of paper. Then take any of these two pieces and cut them into two. That will give you three pieces of paper. Then take any of these and cut it into two. Four pieces results. Is it clear that if you keep on going this way for ever you can get any number of pieces of paper you like?

But what happens if you take a piece of paper and cut it into either 3 or 6 pieces of paper? And then you take that many pieces and you cut one of them into 3 or 6 pieces. And then you take that many pieces and you cut one of them into 3 or 6 pieces. And then you take that many pieces and you cut one of them into 3 or 6 pieces. …

What number of pieces can't you get and why?