NZmar...
Newsletter No. 63
Educators in the field of mathematics suggest that decimals, with their logical base 10 place value construction, are straightforward and simple
 to use, at least, compared with what went before. I think that what they are really saying is, "We understand them so they can't be that difficult". Yet the late development of the decimal system in the history of mathematics would suggest a note of caution. If it's that easy how come it wasn't invented earlier? Early forms of numeration were based on simply making slashes in the sand (or whatever), using shorthand when the number of marks became larger and not easy to distinguish. The difference between five slashes and six is not readily discerned, at least, at a quick glance, so V was used to denote five and the slashes continued, VI, VII and so on for six and seven. Similar techniques were developed by other civilisations. Systems of measurement were based on everyday items like hand widths, stride lengths and so on. The movement towards decimalisation was contrived rather than natural in an attempt to (a) simplify arithmetical processes like addition and subtraction and (b) to unify a multitude of different systems of measurement such as yards, stadia, leagues and so on. Studies, like those carried out last century (doesn't that sound a long time ago!) by the Assessment of Performance Unit for the National Foundation for Educational Research in the United Kingdom, suggest that decimals are far from easy to learn and aren't understood by great swathes of the population. Apparently about 40 percent of us do not understand decimal processes beyond addition.

Here are some of responses made by fifth formers to the question 'What is a decimal?'

> A number less than one.
> A fraction of a number.
> It's less than zero and has a decimal point in front of it.
> A smaller number between two numbers.
> A number that is placed after the 'ones' column.
> A broken-down fraction.
> A number with a remainder after it.

63 is one of the numbers in the Kaprekar process for 2-digit numbers. The Kaprekar process takes the digits and writes them in ascending and descending order before subtracting the smaller from the greater. The process is repeated with the result and so on until a pattern emerges. The Kaprekar process for 2-digit numbers, the digits have to be different of course, leads to the cycle $63 \rightarrow 27(=63-36) \rightarrow 45 \rightarrow 09 \rightarrow 81 \rightarrow 63 \rightarrow \ldots$. You might like to explore the process for integers with higher numbers of digits. (For more on this see 'Afterthought'.)

On a more positive note than the death's one that we are used to, 63 was a good age for some like Samuel Beckett who won the Nobel Prize for Literature at that age. It was also the age of Benjamin Disraeli when he became Prime Minister of Great Britain in 1868, Nikita Kruschev's age when he became Premier of the USSR in 1958 and Richard Wagner's age when his greatest masterpiece The Ring of the Nibelung was first performed in 1876.

But then ... William Bligh, Cicero, Ulysses Grant, Charles Laughton, Alfred Nobel and Franklin Roosevelt all died aged 63.

## $\operatorname{IN} \mathcal{D E} X$

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## What's ne w on nzmaths.co.nz

The interface to the Learning Objects has been updated to include over 150 new links. A large number of these are in the statistics strand. If you are a teacher in a New Zealand school you should have access to these. If not, it is worthwhile registering as there is now a large collection of Leaning Objects, with objects available in all 5 strands of the curriculum at levels 1 to 5 .

Two new units have been added to the Number section of the site, both relating to the 'Difference Bar' Learning Object. Look out for two more units relating to the 'Takeaway Bar' Learning Object, as they should be on the site in the next couple of weeks.

## Diary Dates

ESOL Online invites secondary teachers of mathematics to participate in Language and Learning in Secondary Mathematics. This online workshop will focus on how language strategies can improve content learning in mathematics.
You can find out more about the workshop or subscribe to participate from this page: http://english.unitecnology.ac.nz/esol/maths/

## Tom Lefirer

I have long admired mathematician, satirical songwriter and performer Tom Lehrer so I thought I'd tell you a little about him.

He was born in New York City in 1928. As a child he took piano lessons and at the age of 15 entered Harvard University where he majored in mathematics. At the same time, he began writing and performing. His influences were mainly musical theatre and his style consisted of parodying then-current forms of popular song. For example, his liking for lists led him to set the names of the chemical elements to the tune of Gilbert and Sullivan's 'Major General's Song'. He became a popular fixture at Harvard parties and received his BA in 1947 (Magna Cum Laude, Phi Beta Kappa).

Inspired by the success of his performances, he paid for some studio time to record an album, Songs by Tom Lehrer, which he sold by mail order. Self-published and unpromoted the album
became a great success via word of mouth. With a cult hit, he embarked on a series of concert tours to Canada, Australia, New Zealand and Great Britain and released a second album, which came in two versions - the songs were the same but More Songs by Tom Lehrer was studiorecorded, while An Evening Wasted with Tom Lehrer was recorded live in concert.

During the 50's he worked at the Los Alamos scientific laboratory in New Mexico. Despite the fact that many of his songs had been quite critical of the work being done there, he was still able to get security clearance (McCarthyism missed him). In 1955 he joined the army. The reason, he said, was that "I figured I'd better do it while there was a hiatus between wars." While in the army he worked for the National Security Agency.
In 1960 he stopped touring which he disliked and devoted himself to his academic calling, returning to Harvard and not quite completing his Ph.D. While there his income went from $\$ 1,500$ a week (performing) to $\$ 3,000$ a year (teaching).

In 1964 he returned to performing as resident songwriter for the NBC programme 'That Was The Week That Was' starring David Frost. Tom Lehrer wrote many of the parody songs on that show. During that year he also recorded his 3rd, and most popular album, That Was The Year That Was and toured England and Scandinavia ("because I got a trip out of it"). He then retired from show business once again only coming out to do an occasional benefit for political figures who he felt were going to lose (he did not want to take responsibility for someone if they won and he helped them). In 1971 he wrote several songs for the children's show 'The Electric Company'.

In 1972 he joined the faculty of UC Santa Cruz. He taught 'The Nature of Mathematics', an introductory course for liberal-arts majors which he described as 'maths for tenors'. He also taught a class in Musical-Theatre.

In the early 1980s 'Tom Foolery', a revival of his songs on the London stage, was a surprise hit. Although not its instigator, Lehrer eventually gave it his full support and updated several of his lyrics for the production. On 7 June and 8 June 1998 he performed in public for the first time in 25 years at the Lyceum Theatre, London as part of the gala show 'Hey Mr. Producer!' celebrating the career of impresario Cameron Mackintosh (who had been the producer of Tom Foolery). The 8 June show has been his only performance before the Queen and he sang "Poisoning Pigeons In The Park"
$\mathcal{F r o g s}$ I I
Last month I started on an exploration of the Frogs' Problem. Just to bring you up to date, the idea is that some green frogs are each sitting on a lily pad that happens to be in a straight line with the lily pads that the same number of brown frogs are sitting on. Between the two groups is an empty lily pad - there are no frogs on it. The question is, can you get the green frogs to where the brown frogs are and the brown frogs to where the green frogs are. But there are rules. A frog can only move to a neighbouring vacant lily pad or to a vacant lily pad on the other side of a neighbouring frog.

The reason I like this problem is that everyone can be involved; it seems to be entertaining; and there is some very good maths to be achieved from it. On this last point, the maths is more about the research side of maths - how one conjectures and proves things and generalises.

OK, so last month we started to think about how many moves were needed on the assumption that we could always swap 1 green frog with 1 brown frog; 2 green frogs with 2 brown frogs; 3 green frogs with 3 brown frogs; and so on.
As a result of all that moving around among the lily pads we can draw up the following table, where $f$ is the number of frogs of each sex, $t$ is the total number of moves, $s$ is the number of slides and $j$ the number of jumps. I should note that these are the numbers associated with the fewest possible moves that are required to interchange the two groups of frogs.

| $\boldsymbol{f}$ | $\boldsymbol{s}$ | $\boldsymbol{j}$ | $\boldsymbol{t}$ |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
| 1 | 2 | 1 | 3 |
| 2 | 4 | 4 | 8 |
| 3 | 6 | 9 | 15 |
| 4 | 8 | 16 | 24 |

So what are the patterns here? If we have $f$ frogs on each side can we find formulae for $s, j$ and $t$ ? Now it may be (relatively) easy to see a pattern but in maths that's not enough. We also have to be able to justify these formulae. We have to be sure that if we did indeed manage to line up 78 green frogs with 78 brown frogs that the number of moves would accurately be foretold by the formulae.

So what are the formulae and how can we be sure (prove them)?
Before I go down that track I need to go back a step or two and think about what happens when we are trying to get the green frogs to the brown frogs' side and the brown frogs to the green frogs' side. What is it that messes up the interchanging? When you do it with your class, what moves cause someone to step in and undo what has just happened? What are bad moves?

A little bit of observation shows that you don't want two frogs of the same colour next to each other 'in the middle'. Of course you can't help having two together at the start. And you have to have two together towards the end. But along the way they form a barrier. You can see this from the first (bad) move for $\mathrm{f}=3$.
B_BBGGG

If one of the browns jumps over a brown, then there is an impenetrable barrier over which no green can pass. You can resolve this by moving one of the brown frogs back.
BB_BGGG

And now things can proceed, but you've wasted a move doing this. You could have got to this position by just sliding one brown frog to the right right at the start!

This always happens if you get two of one colour together while still having a frog F of the other colour wanting to pass. F cannot get past the barrier because there is neither a slide nor a jump move open to $F$. This means that it is never a good idea for frogs to jump over frogs of the same colour if we are trying to do all the interchanging in the minimum number of moves.

But back to the table, it looks as if $s=2 f, j=f^{2}$, and so $t=f^{2}+2 f$. How can we be sure that these patterns still hold for $f=78$ though?

Can you wait for a month for the next exciting episode?
Experiences in India by Christopher Gall

I've had yet another great trip to India over the Christmas period. First off I spent 3 weeks at the Samskar School in Andra Pradesh teaching English. The school provides education for children-at-risk and from very disadvantaged backgrounds. Its roll is 290. The children are all happy, secure and well-fed. I wasn't involved with as much classroom work this year as it was the runup to the exam season, which starts at the end of March. Years 7 and 10 both have big public exams (after Year 10, they move on to Intermediate College).

Perhaps I can tell you a bit about their school day. All children (aged 4 to $15 y r s$ ) are expected to get up at 4:00 a.m. to study from 4:30 to 6:30 before taking a bath (a wash under a cold tap). At other times of year there may be sport from 6:00 to 7:00 a.m. with study from 8:00 to 8:30 but at this time of year there is study from 7:30 to 8:30 and then breakfast. School Assembly is at 9:45 with lessons from 10:00 to 1:00 p.m. and again from 2:00 to $3: 45 \mathrm{p} . \mathrm{m}$. There are more study periods until supper at around 5:30. Homework is from 7:30 to 10:00 p.m. Anyone caught sleeping or 'head-nodding' during study time is forced to stand up to study! Classes 7 and 10 have a practice exam every day at 6:00 a.m. The regime wouldn't work in NZ but its normal in India!

The children enjoy many Indian Festivals. Whilst I was there we celebrated Shiva Ratri (Shiva's Night) with a day off school as well as Holi (colour) Festival when the order of the day is to daub colour over everyone else. I spent a fortune on colours and got most of them thrown over me. It starts with powders, moves forward with fabric dye in water bottles and finishes with a mixture of powder, dye, mud and water daubed on faces. One girl was horrified when I appeared in a newish T-shirt and lent me one of hers which got pretty much ruined - I gave her a new one later!

The school actively encourages sport and is highly successful in getting their children representative honours. This year I arrived during the All-India Under-14 Kho-Kho Championships held in Visak. The school had two boys and two girls in the Andra Pradesh State Team (a state of population 70 million ). One girl, Sridevi was the girls' captain. What makes it more remarkable is her background. Her mother is a Jogin (a girl, who at birth effectively is sold into a life of prostitution). Jogins and their offspring are the lowest of the "untouchable" castes. Samskar set about reforming the custom about 20 years ago and the school was initially established to give an education to these children, it not being available from the state for such a low caste. Fifteen years ago a girl like Sridevi wouldn't have been allowed in the stadium let alone captain the team. Such is the success of Samskar in changing ideas. For the record, the boys bagged the gold and the girls the silver medal in their competitions.

## Solution to April's problem

The diagram below shows the situation of last month's problem. Let the houses Arcadia, Birdhaven, Cosy Nook, Dellbrook and Erewhon be denoted by A, B, C, D and E, respectively. We need to show that AE is less than $\mathrm{AB}+\mathrm{BC}$.


Below we give the official answer that was obtained in one way or another by several entrants.
By Pythagoras' Theorem we have that the journey $\mathrm{AB}+\mathrm{BC}=\mathrm{v} 10+\mathrm{v} 17$, while $\mathrm{AE}=\mathrm{v} 53$. Which is greater? Set up two columns:

Square the distances
Subtract 27 from each
Divide each of these by 2
Square each
$10+17+2 \mathrm{v} 10 . \mathrm{v} 17$
53
2v170 26
v170 13
170
170
169

Hence $A B+B C$ is greater than $A E$.
However, we are giving this month's prize to Daniel Snethlage of Lower Hutt because he did it a different way. (We ignored the fact that he had interchanged D and E.) His proof relied on a geometrical idea, that the shortest distance between two points is a straight line.

SOLUTION: The distance between Arcadia and Dellbrook is LESS than the distance between Arcadia and Cosy Nook via Birdhaven

## WORKING

The information given can be represented on a diagram.


The question can be answered by finding the difference between the length ABC and the length AD.

Let's say that the length of the straight line AD is equal to the length of any simple vector with a horizontal component of 2 km and a vertical component of $7 \mathrm{~km}(3+4)$.

We can also say that if F is a point 2 km due west of A , the distance FBC is equal to the distance ABC . This is because the vectors AB and FB have the same length as they both have horizontal components of 1 km and vertical components of 3 km .


Now we can look at it upon the idea that now FBC, like AD, has a total horizontal component of 2 and a total vector component of 7 and hence they should be equal.

HOWEVER, FBC is not a straight line and therefore AD MUST be shorter than FBC and hence ABC.

This can be proved by drawing a diagram whereby we translate FBC 2 km to the east so that F lies on A and C lies upon D .

Firstly, because the triangle ACD is right angled, we know that line AD is 1 km east of AC 3.5 km north of A.

Similarly we know that $\mathrm{DB}^{\prime} \mathrm{A}$ is 1 km east of AD at the point B ' which we know to be 3 km north of A.


As the points at which AD and $\mathrm{F}^{\prime} \mathrm{B}^{\prime} \mathrm{C}^{\prime}$ are 1 km east of AC differ we can show that $\mathrm{DB}^{\prime} \mathrm{A}$ is a triangle.

A, F'


Because a fundamental rule of triangles is that the length of each side must be less than the sum of the lengths of the other two lines, AD is less than $\mathrm{AB}{ }^{\prime} \mathrm{D}$ and hence is also less than ABC .

## This Month's Problem

Sudoku is a very popular game at the moment. For those who haven't come across it, the game consists of putting nine different symbols (often the digits 1 to 9 ) into each row and column of a nine-by-nine square such that every row and column contains only one of each symbol. In addition, if the nine-by-nine square is divided into nine non-overlapping three-by-three squares, each of these smaller squares must contain one and only one of each symbol.

We will call the standard game a 9-Sudoku. Another version of the game is the 16-Sudoku. It is still a game of trial and error except that it takes a lot longer to complete!

Our question this month is: How many different solutions does the game 4-Sudoku have? We will assume that two solutions are different if, when you put one directly on top of the other, you get two different numbers in at least one place. Here's one of the solutions.

| 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- |
| 4 | 3 | 2 | 1 |
| 2 | 4 | 1 | 3 |
| 3 | 1 | 4 | 2 |

We will give a book voucher to one of the correct entries to the problem. Please send your solutions to derek @ nzmaths.co.nz and remember to include a postal address so we can send the voucher if you are the winner.

## Solution to April's I unior Problem

Last month's question was a flop. I had no answers for my attempt to produce a different type of problem. I'll know better next time. I have given the problem again below in the hope that I might still get an answer from someone.

What is an interesting number? So the old story goes, one is interesting because it is the only number that has one factor; two is interesting because it is the only even prime; three is interesting because it is the first prime; four is interesting because ... So is there a uninteresting number? Surely not! Again the story suggests that if 54, say, was the first uninteresting number it would be interesting because it is the first number that's not interesting.

So this month I'm interested in what's interesting or uninteresting. Choose a number that you think is interesting. Let me have three reasons (more if you like) as to why your chosen number is interesting. The prize will go to the most interesting and novel answer that I get.

## This Month's I unior Problem

This section contains a monthly problem competition for students up to Year 8 with a $\$ 20$ book voucher available for the winner. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so that we can send you the voucher if you are the winner.

So what have we got this month? It's not too difficult to see that $9 \times 1089=9801$. Here, multiplying by 9 we manage to reverse the digits of 1089 . Are there any 4-digit numbers abcd so that multiplying by 4 reverses their digits? In other words $4 x$ abcd $=$ dcba?

Note that if you think there are you need to find them all. If you think there aren't you need to convince me that there aren't any. Good luck.

## Afterthought

To see that the Kaprekar process always ends in the cycle starting with 63, take any 2-digit number ab and assume that $\mathrm{a}>\mathrm{b}$. Then what is $\mathrm{ab}-\mathrm{ba}$ ? I don't know but I do know that it is the same as $(10 a+b)-(10 b+a)=9 a-9 b=9(a-b)$. So $a b-b a$ is divisible by 9 . But there are not too many 2 -digit numbers that divisible by 9 . In fact, the only ones are the ten numbers 09,18 , $27,36,45,54,63,72,81$, and 90.

But the cycle of numbers is $63 \rightarrow 27 \rightarrow 45 \rightarrow 09 \rightarrow 81 \rightarrow 63$. And each one of the ten numbers is either in the cycle or the reverse of a number in the cycle. So when we consider any number $a b$ with $\mathrm{a}>\mathrm{b}, \mathrm{ab}-\mathrm{ba}$ either takes us straight into the cycle or it takes us to one of the remaining ten numbers. From here we get to the cycle on the next move.

