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Early in my career I taught in a small country school. There were two classes in each year taking six 40 minute lessons of mathematics per
 week. At least two of the lessons ran consecutively in what was called a 'double period'. The Deputy Principal of the school was one of the old brigade, rather like TVNZ's Gormsby. He had few qualifications but was a firm and effective teacher. He taught junior maths and with each of the first year classes took one of the double periods each week for half a year of 'practical' mathematics. This included a great deal of trigonometry. He knew the exact height of the school flagpole and rugby posts and a nearby hill, how far apart the two towers at each end of the main building were and the area of the hard-surface playground. He had at his disposal sets of clinometers, surveyors' chains and poles, plane tables and the like and each year showed classes how to determine by 'mathematics' what he already knew and what couldn't be obtained by direct measurement. For many he was an inspiration. He showed me that solving practical problems in the classroom gives pupils an inkling of the importance of mathematics - even if they don't all use the skills themselves in later life some people surely would.

Incorporating applications of mathematics in the curriculum substantially assists the acquisition and understanding of mathematical ideas.

Richard Lesh

This is our sixty-second issue which suggests we pass on a few facts about that number. First there is an anonymous verse for 62 which goes: 'At sixty-two I cannot do but may decide what others do.' Still, it is a good age for some. For example, Aristotle Onassis married Jackie Kennedy when he was 62 and Carl C. Magee invented the parking meter at that age in 1935. Furthermore, did you know that the battleship New Jersey BB62 was America's most decorated warship? On a more sombre note Aristotle (not Onassis but the original Greek version!), Sidney Bechet, G.K. Chesterton, Hernan Cortes, Jean Baptiste Fourier, Nostradamus, Robert Oppenheimer, Ludwig Wittgenstein and Emile Zola all died aged 62.
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What's new on nzmaths.co.nz
We have added another new Learning Object, the Fraction Bar, to the collection available through the Learning Object section of the site. Because we have developed this one ourselves, rather than it being developed by the Learning Federation, it is available without a password from this page: http://www.nzmaths.co.nz/LearningObjects/FractionBar/index.swf

Several new activities have also been added to the Secondary Numeracy Project section of the site this month.

Making Links witf Literacy - Numeracy In the Junior Classroom - Some Ideas go Patrick,

While I was at the Numeracy conference in Auckland in February, I asked several people if they could let me have some thoughts from their talk or workshop. The first person to send me in something is Jo Patrick of Long Bay Primary, Auckland (who was once a Numeracy facilitator).

Having spent the last 7 years focusing on maths as a maths adviser and Numeracy facilitator I've recently returned to a school as a teaching AP and have learnt even more about young children's numeracy learning. Here are some ideas I've picked up over the years and recently that I've found quite useful.

## Whole Class Knowledge Sessions: A Useful Guide For Where to Pitch Content:

A useful guide for how to choose a suitable shared book for the class, for teaching concepts about print, word attack, etc, is to pitch it at the level of the top group in the class. This helps challenge the top group whilst gradually building the capability of the lower groups over time. The understanding being that the children will filter the things they need to learn at whatever stage they are at whilst being exposed to more advanced concepts over time and with regular input.

With numeracy knowledge this too is a useful guide. If most of your class is operating at counting from one on materials and you have some at counting from one by imaging and maybe one child is advanced counting, pitch content up to the advanced counter's stage. They need to be learning, amongst other things, to count at least to 100 and back, also exploring counting to 1000 and back. They need to learn to skip count in 2 s 3 s 5 s and 10 s . By counting to at least 100 every day you are also catering for the children who can't yet get to 20 accurately and are exposing them to higher numbers and patterns. You can go beyond 100 with the use of the thousands book. Whisper counting in 2 s , counting in 5 s and 10 s on the hundreds flip board forwards and backwards may seem too advanced for most of the children, but with teacher modelling and support, just as you would with shared book in reading, keeping it brief and regular, this knowledge will gradually make sense and sink in over time.

## A Classroom 'Dripping In Print':

In literacy we know how important a room 'dripping in print' is. In numeracy it is important to have a room 'dripping in print' also - of the numerical kind! Number lines, numeral strips, tens frames, the slavonic abacus and other numerical representations can be displayed and
their use modelled regularly. Having children share their creations and independent activities can snowball the ideas of the other children.

Interactive displays are visually repetitive and content-repetitive. Why not 'mathsing around the room' as the numeracy version of 'reading around the room'?

## Keeping It Repetitive Yet Motivating:

"A little bit every day grows the concepts they say." How can we make doing the same thing day after day (ie counting to at least 100 forwards, and back from numbers to other numbers within that range) interesting for both the children and us? Vary the materials used (eg hundreds board/thousands book/number line/numeral flip strip, etc) and the contexts for counting (eg counting how many girls and boys are at school every day, counting down from a 10 stretch to a 0 sitting as an attention technique on the mat), 'counting around the room', class counting while a child completes an activity - race (eg putting the pencils out/stacking chairs), counting steps to another room or the library, etc. The children themselves may come up with some interesting counting activities, particularly during independent activities.

We keep interest in reading up by changing the books we are reading, yet we are using the same knowledge and strategies as we read and consolidating them. This teaching/ learning technique also has relevance in numeracy. We can simply vary the activities and games.

## Booke Review <br> Mathematical Puzzles and Diversions - Martin Gardner, 1959

I find it almost impossible to believe that it is over 50 years since Martin Gardner began writing his monthly column on recreational mathematics for Scientific American magazine and 20 years since he stopped. He just about reinvented the genre and for many his writings were the highlight of the magazine. Mathematical Puzzles and Diversions was the first of the 15 titles he wrote based on the column and the mathematical insights they reveal are as fresh today as when he first wrote them. All the books include not only the original articles but reader feedback and Gardner's hindsight annotations.

Mathematical Puzzles and Diversions opens with a chapter on hexaflexagons. Like all topics Gardner writes about this one can be simply read or better still actively explored.
Hexaflexagons can be made and their properties examined. The simpler ones have worked well for me in the classroom and encouraged a proportion of students to explore more complicated flexagons.

Other chapters in this ground-breaking book include probability paradoxes, polyonimoes, a mathematical look at various well-known games like noughts and crosses, Hex, the Tower of Hanoi and Nim, a brief biography of America's greatest puzzlist Sam Lloyd and a collection of mathematical fallacies. What was to become a regular in his books are a couple of chapters Gardner calls Nine Problems. They're more what one would expect from a weekly puzzle corner in a newspaper or magazine and require insight as well as persistence to solve.

The whole book is a delightful ménage of mathematics approached through games and puzzles - the accent is on taking up the challenges laid down.

If you come across Mathematical Puzzles and Diversions, perhaps in a second-hand shop, and you're interested in mathematics, buy it at once. If you already have it why not look out for the other 14 that make up the set.

## Frogs I

If this looks as if it is the beginning of a series, then it looks right. What I want to do is to take the Frogs' Problem and see what deep mathematics lies in such an apparently insignificant situation.

The Frogs problem was the one of the first animated problems that we put on our Bright Sparks section of the web. You can find it on this page: http://www.nzmaths.co.nz/brightsparks/frogs.asp?applet
You will need Java installed on your computer to use the Bright Sparks.
But I'll also say what the Frogs' Problem is in case you don't know it. The idea is that some green frogs are each sitting on a lily pad that happen to be in a straight line with the lily pads that the same number of brown frogs are sitting on. Between the two groups is an empty lily pad - there are no frogs on it. The question is, can you get the green frogs to where the brown frogs are and the male frogs to where the female frogs are. But there are rules. A frog can only move to a neighbouring vacant lily pad or to a vacant lily pad on the other side of a neighbouring frog.

In class I always do this using students as frogs. You can keep on adding on to the number of frogs on each side so that eventually you get a whole lot of data and at the same time, give every one a turn at being a frog. To keep the non-frog students busy, I appoint a Frog Master who is in charge of frog movements. You can also appoint an official scorer to write the data on the board. Everyone else has to help in the counting as well as check that nothing goes wrong with the frogs' movements.

So let's make sense of this problem by looking at one frog of each kind and two frogs of each kind.

One frog:


Two frogs:


OK, so you can see how we can get one frog per side and two frogs per side to change position. By now you have also probably worked out how to interchange three frogs. What are the chances of four and five aside working? How about 100 ? How can you be sure?

Let's leave that alone for a while and try something else. Assuming that you can move 100 aside across, what is the minimum number of moves you need?

If you look at the 1 case it seems to be possible to do it in 3 moves. For 2 it looks like 8 and for 3 aside, 15. Are there any obvious patterns here? Can we guess the minimum moves for 4 or 5 or 100?

Would it help if we looked a bit closer into the numbers of slides and jumps? Say that a slide occurs when a frog just moves on to the lily pad next to it and a jump is a move over another frog. How many slides and jumps are there for $1,2,3,4,5$ frogs aside?

For the last several questions we've just been pattern chasing. Sure chase the patterns. But ..., to be doing real mathematics on this you are going to have to say why the patterns occur. Why is it so?

I'll come back to that next month unless I have another senior moment.

## Solution to March's problem

If you read either of the items on magic squares in the issues of the newsletters we mentioned last month you will know that the centre number of a three-by-three magic square is one third of the magic constant. Since we know the magic constant is 123 the centre number is therefore 41 and the square can be readily completed.


It turns out that we got a lot of solutions to this problem. Actually two of these solutions came from overseas, one from Germany and one from the States. I'm sorry that I haven't yet worked out how to give prizes to people not living in New Zealand but I'm glad to have you send them in. Congratulations for correct answers to Courtney Cochrane of Bancroft School in Worcester, Massachusetts, USA and Christiane Schicke, from Braunschweig (the home of Gauss), Germany.

The first solution that I got, that was essentially the one given above, came from Bruce Moody of Rotorua, so he gets this month's voucher.

Can I say that many of the people who sent in something sent in what I call answers, rather than solutions? What I mean is that they gave me the answer but didn't explain how they got it. I'm really keen to give the prize to a solution, where the method of achieving the answer is also given. So I'm looking for the best solution as well as the right answer.

## This Month's Problem

Five houses are scattered round the village, Dellbrook is 3 kilometres due north of Arcadia but 4 kilometres south of Cosy Nook. Birdhaven is 1 kilometre west of Dellbrook while Erewhon is 2 kilometres east of Cosy Nook. Assuming you travel in straight lines which journey is the longer; from Arcadia to Birdhaven and on to Cosy Nook or from Arcadia to Erewhon? You are to solve the problem without calculators or tables.

We will give a book voucher to one of the correct entries to the problem. Please send your solutions to derek @ nzmaths.co.nz and remember to include a postal address so we can send the voucher if you are the winner.

## Solution to March's I unior Problem

Following up from February, this month I asked "How many 5-digit numbers are there that only use even numbers but in such a way that no digit occurs more than once?" Of course I also wanted to know how the students got their answer.

Unfortunately there was no correct answer. I guess this question might have been too hard so let me answer it in full along with a strategy or two that can be used to tackle many such problems.

What we want to do here is to use the numbers $0,2,4,6$, and 8 to make up as many 5 -digit numbers as possible. Of course the first thing that you notice here is that you can't use 0 as the first digit.

Now you can always try to write them all out. If you take this approach you will really have to be very systematic. But being systematic is often a way to go about solving maths problems. Even if you don't succeed in putting all the numbers down you might at least discover another idea that will work.

So here goes. Start with 2 in the 10,000 s column.
20468; 20486; 20648; 20684; 20846; ...
You see that I am keeping the first few positions fixed for as long as I can. When I've found all the numbers starting with 204, say, I change the 4 to the next highest number and continue. Eventually I'll run out of numbers that start with 20 and will then move on to $24 \ldots$, then $26 \ldots$, then $28 \ldots$. After that I'll need to start with 4 and 6 and then 8 .

And at this point there are some questions to ask yourself. Are there as many numbers that start with 2 as there are numbers that start with 4 (or 6 or 8 )? Are there as many numbers that start with 20 as there are numbers that start with 24 (or 26 or 28)? Are there as many numbers that start with 204 as there are numbers that start with 206 (or 208)? If so, how many start with 20 and how does this help?

Another strategy that's frequently worth trying is to look at a smaller case. Is it easier to find all 4 -digit numbers that use only $0,2,4,6$, and 8 once? Or maybe we should try 3 -digit numbers or even 2 -digit ones?

Let's write out the 2 -digit numbers that start with 2 . They are

$$
20 ; 24 ; 26 ; \text { and } 28 .
$$

There are four of them. What about 2-digit numbers that start with 4. Don't we get four again? So that looks a promising way to go. Do we get $4 \times 42$-digit numbers?

It now looks worth trying 3-digit numbers. How many start with 2 and 4 and 6 and 8? Is it $4 x$ $4 \times 3$ ? And, if so, why?

So are there $4 \times 4 \times 3 \times 2$ (96) 4-digit numbers and $4 \times 4 \times 3 \times 2 \times 1$ (96 again) 5-digit numbers?

Where do these 4, 3, 2 and 1 come from? Go back to the 2- and 3-digit cases to find out. That shows that there are 4 choices for the first digit. Even though we seem to have five numbers to choose from we can't use the 0 so there are really only four choices. And for the second digit we still have four numbers to choose from as 0 is allowed to be used but one number has gone
on the first digit. It ought to be OK from there as we lose one number at each succeeding digit.

But doesn't it worry you that there are as many 4-digit numbers as there are 5 -digit ones?

## This Month's I unior Problem

This section contains a monthly problem competition for students up to Year 8 with a $\$ 20$ book voucher available for the winner. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so that we can send you the voucher if you are the winner.

What is an interesting number? So the old story goes, one is interesting because it is the only number that has one factor; two is interesting because it is the only even prime; three is interesting because it is the first odd prime; four is interesting because ... So is there a uninteresting number? Surely not! Again the story suggests that if 54, say, was the first uninteresting number it would be interesting because it is the first number that's not interesting.

So this month I'm interested in what's interesting or uninteresting. Choose a number that you think is interesting. Let me have three reasons (more if you like) as to why your chosen number is interesting. The prize will go to the most interesting and novel answer that I get.

