Newsletter No. 59 **November 2006**

At the recent Otago Festival of the Arts one of the productions was a play set in the old Dunedin Railway station. *Lines of Fire*, specially commissioned for the festival and written by Gary Henderson, was a melding of three stories about women setting out on journeys from the station during its 100-year history. The first story concerned a nurse heading off to the front during WWI. The second was about a mathematician off to join Alan Turing's group at Bletchley Park in England where they broke the German Enigma codes during WWII and the third was set on 10th February 2002 - the last day on which the Southerner took passengers from Dunedin.

As it was a promenade production we were not only able to enjoy a good play but also a tour of the beautiful station. Since one of the characters was a mathematician we also got to hear about some of the mathematical aspects of the building. There was a raft of statistics that would have provided any number of students with project material for weeks. We were also to hear about tessellations as they related to the mosaics of the foyer floor, golden ratios in the architecture of the columns and arches with a sidestep into Fibonacci numbers, perspective, Florence Nightingale's contributions to the study and history of statistics and a swag of related ideas. Mathematics was seen as a vital and alive subject as, of course, it is.

Hey, this is the last newsletter of 2006! We hope that the end of the year goes smoothly for you and that you have a complete rest over the summer and return to us fresh and raring to go in February.

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Euler posed the problem in 1772 to find a number that is the sum of two fourth powers in two ways. He found the smallest solution which is, $59^4 + 158^4 = 133^4 + 134^4$. Apparently only 59% of the moon's surface is ever visible from Earth and the planet Mercury turns once every 59 (Earth) days. On the usual more sombre note: Oliver Cromwell, Clark Gable, Sonja Henie, Charles Lamb, Dusty Springfield, John Osgood, Virginia Woolf and Constance Bennett all died aged 59.

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What's new on nzmaths.co.nz

This month we have added audio links to two of our Māori-medium resources – the equipment animations and the representational framework.
Keep an eye on the site over the next few months – we have eight new units due to be completed over the summer, as well as another new learning object.

Booke Review

Algebra for Beginners - Isaac Todhunter, 1863
The book was designed for teachers laying the basics of algebra and was part of a larger treatise intended for older high school students. It was written in the style of the time with definitions laid and copious exercises given. The first ninety pages cover the concept of algebraic terms, their sum, difference, product and quotient, common denominators, multiples and algebraic fractions. Then Todhunter tackles equations, very quickly getting into those with algebraic fractions. On page 112 we at last come across practical, worded problems related to solving equations and these continue as the book progresses through simultaneous and quadratic equations. The remaining chapters cover indices, ratio and proportion, progressions (arithmetic, geometric and harmonic) and the Binomial Theorem.

The book is obviously designed for able students. The rate at which the level of difficulty in handling algebraic terms increases is mind-boggling compared with current practices and assumes high skills in arithmetic. In addition, students will need to fully understand how to handle fractions before they begin. The words coefficient, dimension, exponent and homogeneous, are first used on page six! Here's a typical early problem you might like to have a go at:

> A horse-dealer buys a horse and sells it again for £144 gaining as many pounds percent (%) as the horse had cost him. How much did he pay for the horse? [Answer below]

Grigori Perelman

In some ways I'm loath to talk about Grigori Perelman. Partly this is because many of you will have read about him already and the fantastic piece of work he did in solving Poincaré's Conjecture. It doesn't matter what the Conjecture was, what is more fascinating is the person who solved it.

Perelman is a Russian mathematician who went through the normal sequence of undergraduate, PhD, lecturer but roughly a year ago he gave up his job at the Steklov Institute of Mathematics. You can find out all of these details by typing his name into any search engine.

But he seems to have done something that mathematicians rarely do, he shut himself away and worked exclusively on a major major problem. Andrew Wiles had done this a few years earleir when he solved Fermat's Last Theorem. When you think about it you either have to be extremely confident in your own abilities or crazy because the stakes are so high. Universities these days like their mathematicians to be writing papers. Confining yourself to one deep problem is not conducive to producing lots of papers. The one or two that might finally be produced had better be good.

A fictional account of someone who failed to grasp his prize can be found in the excellent book by Doxiadis called Uncle Petros and Goldbach's Conjecture. Goldbach's Conjecturer is easy to explain. It is teasingly simple to state. Every even number is the sum of two prime numbers. This has been an open problem since 1742 and Uncle Petros devoted his life to solving that conjecture. Why would he do that? Because he was a good mathematician but he wanted to be the best and he wanted everyone to know it.

So you have to wonder why mathematicians do what they do. What motivates them? Is it the discovery of truth? Is it fame and fortune? G.H. Hardy the famous English number theorist who did his best work in the first half of the 20th Century said the following:

> There are many highly respectable motives which may lead men to prosecute research, but three which are more important than the rest. The first … is intellectual curiosity, desire to know the truth. Then, professional pride, anxiety to be satisfied with one's performance, the shame that overcomes any self-respecting craftsman when his work is unworthy of his talent. Finally, ambition, desire for reputation, and the position, even the power or the money, which it brings. (A Mathematician's Apology)

Uncle Petros is much more blunt.

> Although the more spiritually inclined members of the scientific community may indeed be indifferent to material gains, there isn't a single one among them who isn't mainly driven by ambition and a strong competitive urge.

So now let us go back to Perelman. Having knocked off the Poincaré Conjecture he is offered the Fields' Medal. This is a prestigious prize for mathematicians as we mentioned in an earlier newsletter this year. He was also offered a prize that had been established for settling this Conjecture. This came to a mere million US dollars. And Perelman's response? He turned down the Fields' Medal and did not accept the cash prize either.

Perhaps there are still a few pure souls out there after all.

Christmas Quiz

For your Christmas entertainment and edification we include this multi-choice quiz. In each case state which you think is the best or most likely answer.

1. What are Napier's Bones?
    - (a) What Napier left on his plate.
    - (b) A calculating device.
    - (c) A musical instrument.
    - (d) Truncated logarithms.
    - (e) Stellated logarithms.

2. What is a frustrum?
    - (a) A pack of rugby forwards.
    - (b) The maximum possible friction opposing an object on an inclined plane.
    - (c) The point about which a lever is pivoted.
    - (d) A guitar technique.
    - (e) The part of a solid cut off by two planes parallel to the base.

3. Which of the statements (a) to (e) is/are true?
    - (a) Exactly one statement on this list is false.
    - (b) Exactly two statements on this list are false.
    - (c) Exactly three statements on this list are false.
    - (d) Exactly four statements on this last are false.
    - (e) Exactly five statements on this list are false.

4. Which best describes the shape of the Earth?
    - (a) A saucer.
    - (b) A sphere.
    - (c) An oblate spheroid.
    - (d) A prolate spheroid.
    - (e) A cube.

5. What is a Turing machine?
    - (a) An open-seater car.
    - (b) An efficient form of lever.
    - (c) A pulley system.
    - (d) A machine with mechanical advantage close to 100%.
    - (e) An idealised automaton capable of performing any devised calculation.

6. Which mathematician was born first?
    (a) Derek Holton.
    (b) Descartes.
    (c) Fermat.
    (d) Newton.
    (e) Gauss.

7. What is a harmonic progression?
    (a) A marching band.
    (b) A sequence of unit fractions whose reciprocals are in geometric progression.
    (c) A sequence of unit fractions whose reciprocals are in arithmetic progression.
    (d) The consecutive terms of a Fourier series.
    (e) A null sequence which has no sum to infinity.

8. What was Fermat's last theorem?
    (a) Everyone dies.
    (b) Equivalent to Pythagoras' Theorem in three dimensions.
    (c) Equivalent to Pythagoras' Theorem in n dimensions, $n>2$.
    (d) $x^n + y^n \neq z^n$; $x, y, z, n \in N$; $n > 2$.
    (e) $n^p - n$ is divisible by p; $n \in N$, p prime.

9. Which of the following is not prime?
    (a) 2 (b) 101 (c) 199 (d) 4001 (e) 4891

10. What is Mercator's projection?
    (a) A cheap version of Old Moore's Almanack.
    (b) A plane formed by cutting a sphere along its meridians and flattening the shape formed.
    (c) The net of a cube.
    (d) A plane obtained by cutting a cylinder along a generator and rolling it out.
    (e) Four points with equal internal and external ratios.

11. What is a cusp?
    (a) A small cup for expectorating into.
    (b) A double point on a curve where two distinct tangents occur.
    (c) A double point on a curve where two tangents coincide.
    (d) A U-shaped curve.
    (e) A network with 100 edges.

12. What is a node?
    (a) Yesterday's negative.
    (b) A double point on a curve where two distinct tangents occur.
    (c) A double point on a curve where two tangents coincide.
    (d) A U-shaped curve.
    (e) The end point of a curve.

13. What is a lemma?
    (a) The Latin plural of a lemon.
    (b) One of the two or more values that a function tends towards.
    (c) An exception to a theorem.
    (d) A proposition which is an afterthought to a theorem.
    (e) A preliminary proposition to a theorem.

14. What are transcendental numbers?
    (a) Numbers to meditate on.
    (b) Some of Cantor's infinite numbers.
    (c) Any number not the root of an algebraic equation with rational coefficients.
    (d) Ones like $\sqrt{2}$ or $\sqrt{3}$ or combinations of them.
    (e) Any non-repeating decimal.

15. What are irrational numbers?
    (a) Ones that don't make sense.
    (b) Non-fractions.
    (c) Solutions of quadratic equations for which the discriminant is not square, zero or negative.
    (d) Any decimal.
    (e) Any non-repeating infinite decimal.

16. Which of the following cannot be expressed as a power series in x?
    (a) e^x (b) logx (c) sinx (d) cosx (e) $\tan^{-1}x$

17. How many faces has an icosahedron?
    (a) 100 (b) 24 (c) 20 (d) 12 (e) 10

18. What is a locus?
    (a) An Australian flying insect.
    (b) The focus of any conic.
    (c) The focus of a particular conic.
    (d) A path traced out by a point.
    (e) A point which temporarily takes the place of another.

19. In which century was the infinitesimal calculus invented?
    (a) 15^{th} (b) 17^{th} (c) 14^{th} (d) 16^{th} (e) 18^{th}

20. What is a catenary?
    (a) A holiday home for feline pets.
    (b) A curve belonging to the family of conics.
    (c) One of the cycloid curves.
    (d) The curve assumed by a freely hanging chain.
    (e) The curve of an arch bridge.

[Answers and comments below]

Solution to October's problem

We are looking for an arrangement of the digits 1 to 9 taken three at a time where the second is twice the first and the third is three times the first. An example was given: 327, 654, 981. Our solution must have the first number even.

There is a deal of trial and error involved with this problem but there are ways to reduce it. For example, the first digit can only be a 1, 2 or 3, anything more results in too big a third number (i.e. it would be of four digits). Since the smallest number is even its final digit can be 2, 4, 6 or 8. Suppose, for example, the smallest of the three numbers is 1x2 where x is less than 5. When this is doubled we get 2?4 but the digit 2 has already be used so x must be 5 or more. Continuing in this way we fairly quickly arrive at a solution 192, 384, 576 for the three three-digit numbers. I guess it only remains to check that this is the only solution to the problem but you could be forgiven for stopping there since you were told there **is** only one.

Besides 192 and 327, there are two other. Can you find them? [Answers below]

This month's winner is Derek Smith of Lower Hutt. Though others got the right answer, Derek's solution was about as complete as it could be. Congratulations Derek!

This Month's Problem

Colin reckoned he was a bit of a painter, a house painter that is, and was decorating his house after work. He needed six litres of light grey paint for the roof that his wife Alice had told him to mix as two parts of white to one part black. Sometimes though, Colin's left hand didn't know what his right hand was doing and today was such an occasion. While his left hand was mixing in two litres of white paint his right hand was busy mixing four litres of black.

"Oh dear", said Alice, "I asked him to mix me a light grey and he's mixed me a dark grey instead".

What can Colin do? He could throw away all the dark grey paint and begin again. On the other hand, he could add six litres of white to the mixture and then throw half of *that* away, again wasting six litres. What should he do so as to waste as little paint as possible?

We will give a book voucher to one of the correct entries to the problem. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so we can send the voucher if you are the winner.

A Note on the Solution to September's Junior Problem

In the last newsletter, Kieran Liddington of Mount Wellington gave us the numbers 199 to 211 inclusive as an example of more than 10 consecutive composite numbers. Richard Catterall, a teacher in the Hutt Valley, has pointed out to us that there is an earlier and longer string between 113 and 127.

Solution to October's Junior Problem

Last month we were asking for you to think logically with the problem below.
I have a set of cubes.
They are each coloured red or blue and there is at least one of each colour.
They weigh either 1 kg or 2 kg and there is a least one of each weight.
Is it true that there are two cubes that have different colours ***and*** different weights?

I'm sorry but we didn't get a correct answer this month but here is a solution. We can first take any cube which has a given colour and weight. We'll assume that it is red and 1kg but this doesn't make any difference to the argument. We know then that there exists a cube that is 2 kg because there is one of each weight. If this cube is blue we have found the two cubes we were looking for.

So suppose that we now have a 1 kg and a 2kg cube that are both red. However, we are told there is a cube that is blue too. So take this cube. If it is 1kg we can pair it with the red cube that weighs 2 kg; if it is 2kg we can pair it with the red cube that weighs 1kg. Either way we have found two cubes that differ in colour and weight.

This Month's Junior Problem

This section contains a monthly problem competition for students up to Year 8 with a $20 book voucher is available for the winner. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so we can send you the voucher if you are the winner.

Over the period from now until February you have time to work on this one.

Below is a configuration of five circles three of which are in a horizontal line and three in a vertical line. Put the numbers 1, 2, 3, 4, 5 in the configuration, so that each circle has a different number. Also make sure that the three numbers in the two lines add up to the same sum.

O O O
O
O

You are allowed to use symmetry to reduce the number of extraneous possibilities.

Acknowledgements

A lot of people played a hand in putting this newsletter together every month this year. In particular we would like to thank: Russ Dear, Jenny Ward and Andrew Tagg. We would also like to thank all of those people who sent in solutions to our monthly problems. We hope to hear from you all again next year.

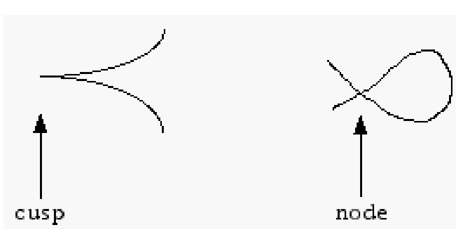
Afterthoughts

From Todhunter's Algebra:
The horse cost £80.

From Solution to October's Problem:
The other two partitions of the digits 1 to 9 satisfying the conditions of the problem have their smallest numbers 219 and 273.

Answers to Christmas Quiz:

1. (b), get your students to make a set.
2. (e); (c) is frustrating but a fulcrum.
3. (d)
4. (c), an oblate spheroid is the surface formed when an ellipse is rotated about its minor axis. The earth's diameter between the poles is less than that across the equator. If you want to find out a little more on this read Bill Bryson's A Short History of Almost Everything. But you should read it anyway.
5. (e), named after its inventor the British mathematician A. M. Turing. Turing was a major figure at Bletchley Park. Britain could have led the world in computer technology immediately after the war if the British government had not destroyed all of Bletchley's calculating machines.
6. (b), deduct ten points if you said (a)! But note that the contenders Descartes (1596-1650), Fermat (1601-1665), Newton (1642-1727), and Gauss (1777-1855), are relatively tightly bunched.
7. (c), the sequence 1, 1/2, 1/3, 1/4, …. Is often referred to as <u>the</u> harmonic sequence but it is only one example. It is true that the harmonic sequence is null and has no sum to infinity but so do other sequences which are not harmonic.
8. (d); actually in three dimensions the square of the length of the diagonal to a box of sides, x, y, and z, is $x^2 + y^2 + z^2$. And (e) is in fact Fermat's Little theorem.
9. (e), it is 67×73.
10. (d); it's better thought of as a projection of the earth in which lines of shortest distance are straight. This used to make it valuable for ships at sea.
11. (c)
12. (b)



13. (e); (d) is usually called a Corollary.
14. (c), e and π are obvious examples. e^{π} is also transcendental but it is not known whether the same is true for π^{e}. Although we only ever seem to run into a few transcendental numbers there is an infinite number of them. In fact there are more of them than there are whole numbers!
15. (e), it is not known if πe, π^{π}, e^{e} or π^{e} are irrational.
16. (b); strangely you can express logx as a power series in x-1!
17. (c)
18. (d)
19. (b); used by Newton in or before 1666 but no account was published until 1693. The earliest known use by Leibniz, who worked independently of Newton, was in about 1675.
20. (d); if its lowest point is taken as the origin and its line of symmetry the y-axis, its equation is $y = c(\cosh x/c - 1)$

Never mind about your score just have a great Christmas!

Here's another filtered down funny: