

Newsletter No. 57

September 2006



A mathematical acquaintance mentioned that he'd recently been awarded an honorary doctorate. Since he was already a doctor and is German he's now known in academic circles in his own country as Doktor Doktor Schmidt. He added, in passing, that it is not uncommon for some of the more high profile academics to obtain a number of honorary doctorates and more than one academic prefixes his name with Doktor Doktor Doktor Doktor Doktor Whoever. Apparently there's some discussion taking place at the moment in German academia about whether this odd state of affairs could not be circumvented in some way, perhaps by using a form of shorthand like Doktor (mult.), indicating that the person holds multiple doctorates. There is a general consensus that such a course would be acceptable to most people, although there is one little point about which there is considerable debate. No-one can agree whether the word multiple should apply for two doctorates or be kept for more than two. Whether there should be the title Doktor Doktor Doktor Doktor Doktor in both cases.

If more honorary doctorates were held by mathematicians I think the problem would have been solved long ago. We'd have Doktor<sup>2</sup> Schmidt for example and Doktor<sup>4</sup> Whoever. Anyway, it's all academic as far as I'm concerned.

I've been offered titles but I think they get one into bad company.

**George Bernard Shaw** 

If you like the Simpsons and you like to mix humour with your maths then you've just got to check out the website:

http://homepage.smc.edu/nestler\_andrew/SimpsonsMath.htm

Here's some examples from it:

Homer [sickly]: Answer:	What are the odds of getting sick on a Saturday? A thousand to one.
Friend:	Don't you think you deserve to earn just as much as a man who does the same job?
Marge:	Well, not if I have to do heavy lifting or maths.

Bart's dog spits out his chewed-up homework – it reads " $9 \times 9 = 100$ ."

Michael Jackson:	Homer, this is Floyd. He's an idiot savant – give him any two numbers and he can multiply them in his head just like that
	nis nead, just like that.
Homer:	OK, 5 times 9!
Floyd:	45.
Homer:	Wow!

And since this is the 57<sup>th</sup> issue of the newsletter here are some things you may not have known about the number 57:

It is the number of degrees in a radian to the nearest degree (a closer approximation is 57.296 degrees). George Pompidou became President of France at age 57. 'Passenger 57' was a movie about an airline security agent trapped on board an aircraft which had been seized by terrorists and then, of course, there's the 57 varieties invented in 1892 by Henry J. Heinz as a marketing technique. Niccolo Paganini, Humphrey Bogart, Vincent Schiavelli, Michael Piller, Sergei Diaghilev, Horace (the poet) and John Logie Baird all died aged 57.

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#### What's new on nzmaths.co.nz

This month we have again added a new jigsaw puzzle piece to the site. This time the new section is aimed at families and the community. We have included some general information about the Numeracy Projects, as well as a large collection of activities suitable for families to use to support the development of children's number knowledge.

We have also added links to two feedback surveys, collecting information on people's response to the mathematics learning area in the new draft curriculum. These links are available on the front page of the website.

#### **Booke Review**

#### **Treviso Arithmetic**

This is the first printed work devoted entirely to mathematics. It has no title page and simply begins: 'Here beginneth a Practica, very helpful to all who have to do with the commercial art commonly known as the abacus.' Treviso at that time was a small commercial town 30 kilometres north of Venice. The book was written anonymously in about the middle of the fifteenth century. It was not particularly popular because it didn't cover rates of exchange among the many monetary systems in use at the time. It did cover the use of the Hindu-Arabic numeration system which was then fairly new. It was written in the Venetian language.

As an example of its content, the product of 314 and 934 was done in four ways, two being variations of the method of long multiplication used today. Although the Treviso Arithmetic was the first printed book on mathematics many more were produced in the remaining years of the fifteenth century. By 1480, for example, there were 38 such works, over 100 by 1490.

#### Queues of 19 or More

Here is a problem that you can do with your students as a joint exercise.

You may remember that last month we posed this problem:

A class at school contains 19 pupils. Before they go into maths each day, the teacher insists they line up in a straight line outside the door. She also insists that every day the pupils must be in a different order so that for each pupil they never have the same person either in front or behind them. To make this clearer, if there were 6 pupils A B C D E and F then the following would be OK:

> Day 1 ABCDEF Day 2 BDAFCE

But this would not be OK:

*Day 1 ABCDEF Day 2 BDAFEC (F and E are still next to each other)* 

Now back to the problem.

For the 19 pupils in this maths class, what is the maximum number of days that they can comply with the maths teacher's rule?

Before we even think of looking for a solution, the problem above is one that I have used with children of 9 or 10 as well as with secondary students. Don't start with 19 it's too big. I use 5 first and then move on to 7. Get some of the students to actually try

forming the different queues with the help of the rest of the class. Can you get 3 queues with 5 students? Why? Why not? What about 4 queues with 7?

With these smaller numbers it's easy to produce some quite nice logic to show that there are only 2 possible queues with 5 pupils and 3 with 7.

So let's have a look at 19. First of all let's show that there can be no more than 9 days of queuing. To do this, suppose that there are 10 queues. The next thing to think about is the number of pupils any one pupil can be next to. If that pupil is at the end, then they're only next to 1 other pupil; if they're in the middle they are next to 2 pupils. So in every queue we have seventeen students next to 2 students and two next to 1. That makes a total of  $17 \times 2 + 2 \times 1 = 36$  "next to's". But there are 10 queues so we have  $10 \times 36 = 360$  "next to's" altogether.

Since there are 19 students, on average each one is next to 360/19 = 18.95. Now because this is the average, there must be one student who is next to at least 19 others over all 10 queues. But there are only 18 others that that student can be next to. So we can't have 10 queues. The most we can possibly get is 9.

Now the next difficulty is that we may not actually be able to make up 9 queues. So our second task is to show 9 proper queues. We do this by labelling the students 1, 2, 3, ..., 19. So here are 9 queues.

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19 2, 5, 8, 11, 14, 17, 1, 4, 7, 10, 13, 16, 19, 3, 6, 9, 12, 15, 18 3, 8, 13, 18, 4, 9, 14, 19, 5, 10, 15, 1, 6, 11, 16, 2, 7, 12, 17 4, 11, 18, 6, 13, 1, 8, 15, 3, 10, 17, 5, 12, 19, 7, 14, 2, 9, 16 5, 14, 4, 13, 3, 12, 2, 11, 1, 10, 19, 9, 18, 8, 17, 7, 16, 6, 15 6, 17, 9, 1, 12, 4, 15, 7, 18, 10, 2, 13, 5, 16, 8, 19, 11, 3, 14 7, 1, 14, 8, 2, 15, 9, 3, 16, 10, 4, 17, 11, 5, 18, 12, 6, 19, 13 8, 4, 19, 15, 11, 7, 3, 18, 14, 10, 6, 2, 17, 13, 9, 5, 1, 16, 12 9, 7, 5, 3, 1, 18, 16, 14, 12, 10, 8, 6, 4, 2, 19, 17, 15, 13, 11

You can check them all out of you like but the key here is that in the first queue everyone is either +1 or -1 away from their neighbours; in the second queue everyone is either +3 or -3 from their neighbours; in the third queue they're +5 or -5; fourth queue it's +7 and -7; and so on. None of these differences is ever the same so no pupil is ever next to the same pupil again over the 9 days.

And if n is a prime, you can do the same again to show that there are precisely (n - 1)/2 different queues. But does this formula hold for any n, prime or not?

### Solution to August's Problem

We had to find the smallest natural number with 24 divisors. Suppose it is N. Does it help to express N in prime factors?

There are two things to consider:

- 1. If N only had only two different two prime factors they would be 2 and 3 since we are looking for the smallest N. This idea generalises.
- 2. If N were given as the product of prime factors we could deduce the number of divisors. A couple of examples will make the method clear.

For  $N = 30 = 2 \times 3 \times 5$ , there are eight divisors  $(2 \times 2 \times 2)$  given by the routes through the factor tree;



For  $N = 72 = 2^3 \times 3^2$ , the number of divisors is 12 (4 x 3) given by the routes through this (incomplete) factor tree;



Together these two ideas show a way of solving the problem of what is the least value of N that has 24 divisors. The number of divisors of numbers like  $2^a \times 3^b \times 5^c \times 7^d$  is (a + 1)(b + 1)(c + 1)(d + 1). You can see this by noting that in the factor tree, at one stage there have to be a + 1 branches coming out for the contribution of the 2; b + 1 branches because of the 3; c + 1 for the 5; and d + 1 for the 7. This gives (a + 1)(b + 1)(c + 1)(d + 1) = 24 ends/factors.

This means that  $2^{23} (23 + 1)$ ,  $2^{11} \ge 3 (11 + 1)(1 + 1)$ ,  $2^7 \ge 3^2 (8 \ge 3)$ ,  $2^5 \ge 3^3 (6 \ge 4)$ , and  $2^3 \ge 3^5 (4 \ge 6)$  all have 24 divisors. However, you can check that these numbers decline until the last one and that by looking at  $2^a \ge 3^b \ge 5^c$  we get other possibilities that lead to 360.

Working these tricks with  $2^a \times 3^b \times 5^c \times 7^d$ , etc., doesn't get you a smaller N than 360. So the smallest number with 24 divisors is 360.

This month's winner is B Moody from Rotorua.

By the way, is 72 the smallest number with 12 divisors? (Answer below.)

#### This Month's Problem

Here's a simple little problem of combinatorics. In how many ways can the numbers 1 to 6 be placed on a regular six-sided die such that the 1 is opposite the 6, the 2 opposite the 5 and the 3 opposite the 4? This isn't one of them!

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We will give a book voucher to one of the correct entries to the problem. Please send your solutions to <u>derek@nzmaths.co.nz</u> and remember to include a postal address so we can send the voucher if you are the winner.

# Solution to August's Junior Problem

Last month we asked this problem:

	1	9	3	2	4	8
2	3	11	5	4	6	10
7	8	16	10	9	11	15
3	4	12	6	5	7	11
5	6	14	8	7	9	13
8	9	17	11	10	12	16
7	8	16	10	9	11	15

Here we have a table of numbers where each number is the sum of the pink number at the top of the column it's in and the yellow number at the start of the row it's in.

Now the green numbers are chosen so that there is one green number in each row and in each column. These green numbers add up to 3 + 9 + 12 + 13 + 12 + 10 = 59.

Call a set of numbers *nice*, if they are like the green numbers in that there are six of them and there is one of the nice numbers in each row and column. If you sum all the numbers in a nice set, what is the biggest total you can get and why?

Michael Catterall of Upper Hut came up with this very thorough solution.

The highest possible 'nice' number is fifty-nine. This is because on the diagram

	1	<mark>9</mark>	<mark>3</mark>	2	<mark>4</mark>	<mark>8</mark>
2	3	11	5	4	6	10
7	8	16	10	9	11	15
<mark>3</mark>	4	12	6	5	7	11
<mark>5</mark>	6	14	8	7	9	13
8	9	17	11	12	16	16
7	8	16	10	9	11	15

when you have a number in every row, no matter how far along each number is in the row you must add all the numbers at the start of the row e.g.

		$\frown$	1	<mark>9</mark>	<mark>3</mark>	2	<mark>4</mark>	<mark>8</mark>
No matter how far along in the row all of the numbers at the start must be added.		2	3	<mark>11</mark>	5	4	6	10
		7	<mark>8</mark>	16	10	9	11	15
		<mark>3</mark>	4	12	6	5	7	11
		5	6	14	<mark>8</mark>	7	9	13
		8	9	17	11	12	16	<mark>16</mark>
	×	7	8	16	10	<mark>9</mark>	11	15

And when you have a number in every column you must add all the numbers at the top of the column no matter how far down each number is, e.g.



No matter how far down the numbers in the column, all the numbers at the top must be added.

Since there is both a number in each row and a number in each column we must add all the numbers down the side and along the top together (2 + 7 + 3 + 5 + 8 + 7 + 1 + 9 + 3 + 2 + 4 + 8 = 59) which equals a total of fifty-nine, the highest 'nice' number possible.

#### This Month's Junior Problem

This section contains a monthly problem competition for students up to Year 8. What's more the usual \$20 book voucher is available for the winner. Please send your solutions to <u>derek@nzmaths.co.nz</u> and remember to include a postal address so we can send you the voucher if you are the winner.

Look at the whole numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, ... where we have put the prime numbers in red. Note that the longest string of consecutive composite (non-prime) numbers so far in our list consists of the five numbers 24, 25, 26, 27, and 28. Can you find a string of 10 or more consecutive composite numbers? Is it possible to find a string of 100 or more consecutive composite numbers?

# Afterthoughts

The smallest number with 12 divisors is 60.

# A Little More Humour (?)

This little funny (even if you don't know what a limit is) has filtered down from somewhere!

After explaining to a student through various lessons and examples that:

$$\lim_{x \to 8} \frac{1}{x-8} = \infty$$

I tried to check if she really understood that, so I gave her a different example. This was the result:

$$\lim_{x \to 5} \frac{1}{x-5} = in$$