Newsletter No. 56 **August 2006**

The problem that we featured in the newsletter last month, about sheep and goats, has appeared over the years in a number of different forms. Since the answer is perhaps counter-intuitive it gives rise to a great deal of interest (most would-be solvers insist the answer is 1/2). As far as I can tell the problem appears in its original form in Lewis Carroll's book *Pillow Tales* published in 1893. Lewis Carroll was the pen-name of the Oxford University mathematician James Dodgson who was better known for his children's' books *Alice in Wonderland* and *Through the Looking Glass*. *Pillow Tales* contains 72 mathematical posers that most school students would find very difficult these days as they require a fluency in Euclidean Geometry, not to mention high skill in algebraic manipulation. This sort of mathematics is no longer taught in schools. Less well known is that Dodgson tried out as a school teacher but gave up very quickly, in hours rather than days, with the comment that the profession was likely to lead him to an early death.

> *Teaching school is but another word for sure and not very slow destruction.*
>
> **Thomas Carlyle**

The triangular numbers are 1, 3, 6, 10, 15, …. and so on. Successive sums of these; 1, 4, 10, 20, 35, … , are called tetrahedral numbers. The sixth tetrahedral number is 56.

And did you know that in 56 days a silkworm can eat 86,000 times its own weight and that there are 56 short stories featuring Sherlock Holmes? Francis Drake, Albrecht Durer, Douglas Fairbanks, Ian Fleming, George Formby, George VI, Betty Grable, Dag Hammarskjold, Adolph Hitler and Abraham Lincoln all died aged 56.

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What's new on nzmaths.co.nz

There have been several significant additions to the site in the last month:

- We have added a search feature to allow users to more easily find resources on the site. You can access the search either from the front page or by clicking on 'Search' in the menu bar.

- A collection of resources relating to the release of new coins is available from a link on the front page of the site. See the review later in this newsletter for more information. The existing units on money have been edited to reflect the changes to the coins.

- A compendium of research findings from the 2005 Numeracy Development Projects has been released. You can download it from: http://www.nzmaths.co.nz/numeracy/References/compendium05.aspx

- Ten new equipment animations have been added in the Numeracy Project section: http://www.nzmaths.co.nz/Numeracy/Animations/animations.aspx
  They are also available in Māori language versions: http://www.nzmaths.co.nz/Numeracy/Animations/maorianimations.aspx

- Additions have been made to the Lead Teacher material: http://www.nzmaths.co.nz/Numeracy/Lead_Teacher/index.aspx

Diary Dates

31 July
The New Zealand Curriculum: Draft for consultation 2006, was released by the Minister of Education, Hon. Steve Maharey, this week, July 31, 2006. Copies of the draft and feedback questionnaires will be sent to all schools.

The New Zealand Curriculum: Draft for consultation 2006 will be available online from mid afternoon. An online feedback questionnaire is also available on this page.

URL: http://www.tki.org.nz/r/nzcurriculum/

14 -18th August

2006 Maths Week
http://ww.mathsweek.org.nz
We are assured that the Maths Week site will be online this week, with more additions as Maths Week approaches.

26 - 29 September

Australian Mathematical Society Annual Conference
http://www.austms.org.au/People/Conf/

Booke Review

Chiu Chang Suan Shu
Written by Chang Tsang in about 170 BCE, the title translates as 'Nine Chapters on Mathematical Art'. It is often described as the greatest mathematical work of antiquity but is relatively unknown in the West. It is a collection and restoration of earlier work that is traditionally placed in the third millennium BCE. The text contains 246 problems classified into nine sections and is the earliest mathematical work that mentions negative numbers. Here's an overview by chapter (some of the more cryptic parts of this review are considered in the Afterthought section at the end of the newsletter):

Chapter 1: Land Surveying
Uses $\pi = 3$ for areas of circles and segments. Includes the arithmetic of fractions employing lowest common denominator for addition.

Chapter 2: Millet and Rice
Questions involving simple percentage and proportion.

Chapter 3: Distributions by Progressions
Taxation, partnership, arithmetical and geometrical progressions.

Chapter 4: Diminishing Breadths
24 problems on land mensuration; includes the method of finding square and cube roots taught in English (and New Zealand) schools in the late 1800s.

Chapter 5: Consultations of Engineering Works
Formulae for volumes of prisms, cylinders, cones and others.

Chapter 6: Impartial Taxation
Mostly pursuit problems.

Chapter 7: Excess and Deficiency
Solving equations of the form $ax - b = 0$ by 'false position'. This method was used because algebra had not been developed.

Chapter 8: Calculating by Tables
18 problems on solving systems of simultaneous equations.

Chapter 9: Right Angles
24 problems using Pythagoras' Theorem.

Money, Money, Money

You mean that you don't remember where you where when you first heard that ABBA song? Well it has new meaning now when you recall that as we speak, some of our coins are being eliminated and others reduced in size and weight. The Reserve Bank has been publicising the demise of the 5c coin for a while now. As their pamphlet says new coins are in circulation from 31 July; during August, September and October both old and new coins are acceptable but the old ones will be gradually taken out of circulation; and from 1 November any old coins you have won't be accepted in normal trading.

Now if you want to find out all there is to know on the subject you only have to go to our home page and click on the coins. But then I know that all but one of you have gone down that path already. So I'll just talk to the one person who hasn't (shame).

Fortunately, the Reserve Bank in cooperation with Ian Stevens and Anne Brunt has developed resources which are hosted on our site for use in your classroom and all the information that you wanted to know and more about the new coins. For instance, did you know how many coins get taken out of circulation each year? I suppose that they are lost down cracks in the floorboards, taken to the Cannes Hilton and used as tips, stolen by foreigners, and put into the baby's piggy bank. But it's hard to believe that 27 million of them bite the dust each year. You might guess that it's the 5 cent coin that disappears most. Presumably this is partly because it's the smallest in size and so is more easily overlooked and partly because it's the least value and so least valued. (Is it worth grovelling on the pavement if one drops out of your purse?)

Now you can also find the answers to the frequently asked questions below by clicking on the question (not here, you'll have to go to the web site).

- When will the new coins be introduced?
- When will the new coins be available?
- Why did the Reserve Bank decide to make the coins smaller?
- Why will the coins be made of plated steel?
- Will my current coins lose their value?
- Why will the Bank "demonetise" the current coins?
- What should I do with my current coins?
- Will the general public and those in cash handling jobs be able to distinguish the new coins easily?
- Will blind and visually impaired people be able to distinguish the new coins?
- How will this affect the coin vending industry?
- Can retailers and other businesses decline to accept payment in old coins once those coins are demonetised?
- Why did the Bank decide to withdraw the five cent coin?
- Will the withdrawal of the five cent coin cause prices to rise?
- Why do other countries have very low denomination coins?
- What was the result of the public consultations?
- Why were the one and two dollar coins not changed?
- Why did the Reserve Bank leave the designs unchanged?
- Did the Reserve Bank consider introducing a 25 cent coin?
- Did the Reserve bank consider introducing a five dollar coin?
- Did the Reserve Bank consider introducing irregular shaped coins to assist identification?
- How will rounding work?
- What are the magnetic properties of the new coins?

For the classroom with a computer, there are games to play that enable students to recognise the new coins, to make up different amounts of money, to produce change (even requiring them to round amounts in the process) and to do various other calculations with money in a novel setting. There is also information on how the coins are made (and where) and a CPI inflation calculator so you'll know what $2 in 1946 would be worth today. And you might like to work out how many new 50c coins would balance you on a set of scales? Or is it easier to see how many 10c coins laid side by side would be needed to make a chain 1 km long?

If you really haven't seen the web site I think that you'll find several things of interest to your class and a load of activities for them to be involved in.

Queues of 19 or More

A little while ago Derek Smith sent me this problem.

A class at school contains 19 pupils. Before they go into maths each day, the teacher insists they line up in a straight line outside the door. She also insists that every day the pupils must be in a different order so that for each pupil they never have the same person either in front or behind them. To make this clearer, if there were 6 pupils A B C D E and F then the following would be OK:

Day 1 ABCDEF
Day 2 BDAFCE

But this would not be OK:

Day 1 ABCDEF
Day 2 BDAFEC (F and E are still next to each other)

Now back to the problem.

For the 19 pupils in this maths class, what is the maximum number of days that they can comply with the maths teacher's rule?

This is an interesting problem for a number of reasons. First of all it reminds me a bit of the golfing problem we had in February this year. It looks like a combinatorial problem. All you have to do is to fiddle and with a bit of luck you'll get the thing out one way or another.

Second, you'll note that it doesn't say that we have to show what the queues are. We only have to establish the maximum number of different queues we can have. So is it possible to say this without knowing what the queues actually are? Or could we aim a bit lower and find a bound to the possible number of different queues? Is there a good reason why you can't have 19 queues, say, or 15 or 10 or whatever? There may be room for playing with inequalities to get some handle on what's going on.

Third, if we're any good at writing programs we can probably knock off a set of queues in no time.

Fourth, the problem has room for us to play. We can get in there and get our hands dirty and have a bit of fun. But to get anywhere with a head-on attack we're going to have to be lucky. So a bit of guile might be in order. To get some feel for what is happening we might try a good old problem solving heuristic: try smaller cases. So what if there were only 3 in your class (dream on!)? Could you do that? Or maybe 7 or 10 or what?

And fifth, if we're lucky we might be able to generalise the problem and work out an answer for q queues.

I'll let this simmer here and come back to it next month. If you make any progress with classes of size 9, 19, 90 or anything above or below email derek@nzmaths.co.nz. He'll make sure that whatever he gets will appear in what I produce next month.

Solution to July's problem

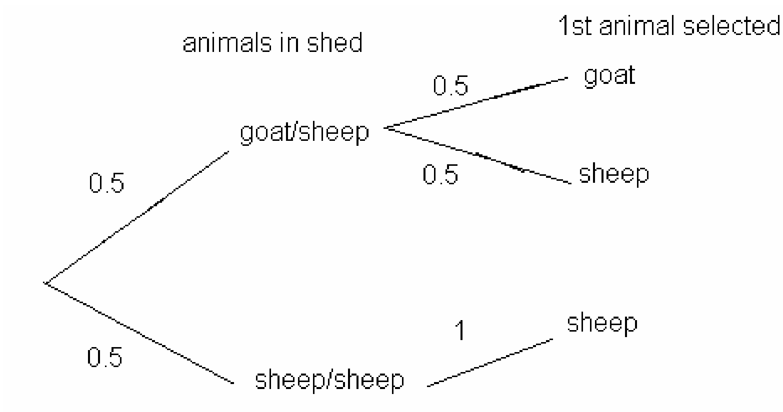
July's problem was the following:

Old McDonald had 100 sheep, 100 angora (white) goats and a dipsomaniac son Ralph. The fences on his farm were in poor repair and the animals were thoroughly mixed. One evening Old McDonald decided he'd put a sheep in the shed for butchering next day. Being a dutiful, if inebriated son, Ralph decided to help his dad out and do the job for him. Not being quite himself he grabbed the first animal he saw and shoved it in the shed. Not knowing that his son had been so helpful Old McDonald went out later, caught a sheep and put it in the shed completely oblivious to the fact that there was an animal in there already. Next morning Old McDonald asked Ralph to get the animal from the shed. Ralph, still being rather bleary-eyed, went to the shed and grabbed the first animal he saw to take to his dad. Fortunately it was what his dad wanted, a sheep.

If you're of a pastoral bent you might like to work out the probability that the animal left in the shed is a sheep.

Marnie Fornusek sent in this answer to win the $50 book voucher. Well done Marnie.

The probability of the Ralph selecting a sheep is $\frac{100}{200} = 0.5$. The probability of the son selecting a goat is also 0.5. Old McDonald catches a sheep and adds it to the shed. This means that in the shed there could either be a goat and a sheep P(goat/sheep) = 0.5 or 2 sheep P(Sheep/sheep) is 0.5 .



The probability that the next day that Ralph grabbed a sheep is 0.5 x 0.5 + 0.5 x 1 = 0.75.

The probability that there is a sheep left behind can only occur if there were two sheep in the shed. Therefore the P (sheep left behind if first animal was a sheep) = $\frac{0.5}{0.75} = 0.\dot{6}$ or $\frac{2}{3}$.

This Month's Problem

This week we're much briefer with our problem. What is the smallest (natural) number with 24 divisors?

We will give a $50 book voucher to one of the correct entries to the problem. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so we can send the voucher if you are the winner.

Solution to July's Junior Problem

Last month we asked this problem:

Let's try some more logic. Alice, Beryl and Clair all play different ball games.

(i) If Alice plays netball, Beryl plays golf.
(ii) If Alice plays hockey, Clair plays golf.
(iii) If Beryl does not play hockey, Clair plays netball.

Now the best solution I got came from Michael Catterall of Hutt International Boys' School. He told me that

Alice plays golf
Beryl plays hockey
Clair plays netball

Now there are a number of ways of looking at this. Many of you will do it using a table but I'll write it all out.

Make an assumption. Suppose that Alice plays netball. So by statement (i) Beryl plays golf. But this means that she doesn't play hockey so by statement (iii), Clair plays netball. Since they all play different games we have a problem as Alice was assumed to play netball. That assumption just has to be wrong.

So suppose that Alice plays hockey. Statement (ii) then tells us that Clair plays golf. The only game that is left is netball so Beryl plays netball. Statement (iii) then tells us that Clair plays netball. But Beryl is playing netball too so we have another contradiction.

By a process of elimination, Alice has to play golf. If Beryl doesn't play hockey, statement (iii) gives Clair playing netball. So poor Beryl suddenly has no game to play as Alice has got golf and Clair netball and Beryl doesn't play hockey! That can only mean that Beryl does play hockey which leaves Clair to play netball – it's the only option left.

Congratulations to Michael. Your book voucher is in the mail.

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Disappointingly I didn't get any nibbles at my attempt to make the problem from May harder. But, ever optimistic, I'll put it in again. Here it is.

All this makes me wonder if we could get a similar problem with three marbles. Say the marbles were Red, White and Blue. We would then have boxes labelled, BBB, BBW, BWW, and WWW. If all of the labels were wrong how many marbles would have to be looked at before you could tell which box was which? I don't think that there is a nice answer to this. Can anyone help me out here?

This Month's Junior Problem

This section contains a monthly problem competition for students up to Year 8. What's more the usual $20 book voucher is available for the winner. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so we can send you the voucher if you are the winner.

	1	**9**	**3**	**2**	**4**	**8**
2	3	11	5	4	6	10
7	8	16	10	9	11	15
3	4	12	6	5	7	11
5	6	14	8	7	9	13
8	9	17	11	10	12	16
7	8	16	10	9	11	15

Here we have a table of numbers where each number is the sum of the pink number at the top of the column its in and the yellow number at the start of the row its in.

Now the green numbers are chosen so that there is one green number in each row and in each column. These green numbers add up to 3 + 9 + 12 + 13 + 12 + 10 = 59.

Call a set of numbers ***nice***, if they are like the green numbers in that there are six of them and there is one of the nice numbers in each row and column.

For the $20 book voucher, if you sum all the numbers in a nice set, what is the biggest total you can get and why?

Afterthoughts

I want to consider some of the words and problems from the Booke Review. Note that by today's standards, the problems are not that well defined. I can imagine some mathematicians insisting the reed doesn't bend (third example below) or that the word 'pace' in the example after needs to be better defined (it doesn't of course, if some common sense is applied). In the book, methods, when given, are given as a set of specific instructions.

So, what is partnership in Ch 3? This refers to problems that relate to the sharing of capital or risk, etc. For example:

'When buying things in companionship, if each gives 8 pieces, the surplus is 2; if each gives 7, the deficiency is 4. Find the number of persons and the price of things bought.'

Solutions are given along the lines of 'add the surplus to the deficiency, which makes the *shih*. The difference of the rates is the *fa*. The *shih* divided by the *fa* .... and so on, as I mentioned, a set of instructions!

Chapter 8 has a series of 18 problems on what we would call simultaneous equations. Here is one of the simple examples.

'If 5 oxen and 2 sheep cost 10 taels of gold and 2 oxen and 5 sheep cost 8 taels, what are the prices of the oxen and sheep respectively?' (I hope that they had a way to divide taels up into 21sts.)

Here's an example from chapter 9:

'There is a pool 10 feet square with a reed growing in the centre which rises a foot above the surface. When drawn towards the shore it reaches exactly to the brink of the pool. What is the depth of the water?'

Then you may ask: how does 'Mostly pursuit problems' get into Ch 6 on Impartial Taxation and what are pursuit problems. I can't answer the general question – I can't get into the mind of an ancient Chinese! But here's an example of a pursuit problem.

'A hare runs 100 paces ahead of a dog. The latter pursues the former for 250 paces when the two are 30 paces apart. In how many further paces will the dog overtake the hare?'

And then: how do you solve $ax - b = 0$ by false position? And what is false position? I guess this would be called 'trial and error' today. Rather than solve b divided by a, they would try a value for x and if it proved to be too big try a smaller value. If this was too small they could take the average of the two answers to get a closer 'approximation' of the answer, and so on. Actually this is not a bad way to solve more complicated equations. These days you could do this using a spreadsheet.

And does anyone nowadays know how to find square roots and cube roots by hand? To answer that question you can resort to the fount of all knowledge – the web. A search on square+root+algorithm got many responses. There are some good explanations out there.