Newsletter No. 54 **June 2006**

Occasionally I come across old mathematics textbooks in secondhand shops. Often they are written by Clement Durrell or Hall and Knight, all of whom wrote best sellers in the early-to-mid 1900s. It got me thinking about writing reviews for old textbooks. I think it was Reg Alteo who started the idea back in the 1980s. He called them Booke Reviews and there's an example below.

Durrell's books were much in evidence when I was at school and I still occasionally see them among teacher's reference books. They all seemed to us as young students to be inappropriately titled. *Elementary Geometry* for example, was to us anything but elementary and *A New Geometry*, well it was Euclidean geometry and how could a subject taught for over 2000 years be described as new! Hall and Knight were as bad with one of their weighty tomes entitled *A Shorter Algebra*. Having said that, it's certainly true that many a professional mathematician gained his or her first flicker of interest in the subject from being made to work through some of Durrell's versions of Euclidean proofs.

> *Anyone who has studied geometry is infinitely quicker of apprehension than one who has not studied it.*
>
> **Socrates**

Did you know that the largest pyramid ever built is 54 metres high? It is located at Cholula di Rivadabia in Mexico. Agatha Christie mentions 54 kinds of poison in her crime books! '54' is also the name of a movie about a famous 70s New York City nightclub told through the eyes of a young employee. In some countries the movie was known as 'Studio 54' and a nightclub of that name flourishes in Las Vegas today. Paul Gauguin, Peter Sellers, Gertrude Lawrence and Laventry Beria all died aged 54.

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What's new on nzmaths.co.nz

As mentioned in last month's newsletter, we have now added a new jigsaw piece to the front page. You can use the Learning Objects section of the site to select appropriate online digital resources from the Digistore for your students to use, provided you are in a New Zealand school which has registered for access.

Nine new units are available in the Number section of the site.

We have started running an online user survey. There is a link from the front page of the site to this brief survey which is designed to collect information for us to use to help inform development of the site. The survey will change several times a year so that we can collect a variety of information from our users.

Diary Dates

July

8th Australasian Conference on Mathematics and Computers in Sport
http://www.anziam.org.au/

27 July

The Australian Mathematics Competition
http://www.westpac.co.nz/

14 -18th August

2006 Maths Week
http://ww.mathsweek.org.nz

26 - 29 September

Australian Mathematical Society Annual Conference
http://www.austms.org.au/People/Conf/

Numeracy Conference Plenary Sessions

In the April and May newsletters we presented a summary of the talks that Paul Cobb and Peter Hughes, respectively, gave at the Numeracy Conference in Auckland in January. This month we do the same for Marj Horne's plenary session.

It's important here to note that although Marj starts to talk about algebra at secondary level, what she is concerned about is a sound algebraic foundation being laid in primary school. This is underlined by her later discussion and by some of her bullet points at the end.

Once again we are thankful to Jenny Ward for organising this material for us.

The power point presentation, including full references, for this address is available online at http://www.nzmaths.co.nz/numeracy/References/keynotes06.aspx

Marj Horne: Algebra: Directions for the Future

The introduction of Computer Algebra Systems (CAS) provides a challenge to teachers of algebra in secondary schools. There is a need for professional development to enable teachers to use CAS creatively as a learning tool, rather than just as a tool for getting answers. CAS extends the range of problems and applications readily accessible to students, enabling more complex investigations and problem solving tasks to be carried out as well as providing a tool for learning.

In using CAS teachers need to focus on developing students' understandings rather than practising routine skills. Real problems can be set as classroom challenges with students more in control of their own learning, working in groups to carry out investigations. The skills that have more traditionally been covered in the study of algebra at this level can be introduced and consolidated in a problem solving context.

New Zealand has acknowledged the importance of Algebra in the early years for longer than most other countries. In the early years of schooling students need to be introduced to the connections between arithmetic and algebra, the study of patterns and the use of generalisations.

The understanding of the equals sign (=) develops from the early years on. If teachers focus extensively on calculation outcomes for the equals sign, rather than incorporating a variety of uses of "=" students develop a restricted understanding of the arithmetic operations, seeing them as combining only, rather than as relational. For example, in the problem "7 + 8 + 9 =" the equals can be interpreted as 'makes' or 'now work it out'. In contrast, the equation "5 + 4 = 15 – 6" develops an understanding of the "=" as 'the two sides balance'. One way to develop this relational understanding of the equals sign in the early years is to use balances and weights of different colours to illustrate the "=" relationship.

2 + 3 = 4 + 1

In the early years it is also important for students to develop skills in relational thinking. This can be done using questions such as "If I know that 78 + 34 = 112 what else do I know?" Students can then identify a list of related facts such as

79 + 34 = 113
178 + 34 = 212
77 + 33 = 110

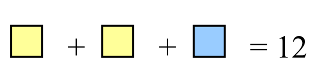
One reason for students' difficulty in algebra is a lack of understanding of operation symbols. Many students see "+" as a sign meaning to combine two numbers, accompanied by an action. For example 5 + 7 is 12. Once the addition has been carried out the parts of 12 are no longer visible. In contrast, in the equation $a + 7$ the "+" sign does not mean actively combine the two parts as it did in $5 + 7$. While $a + 7$ can be seen as a single object, the components maintain their identity. Seeing the operations as an instruction to combine leads to the incorrect $4x + 3 = 7x$. Another way of seeing $5 + 7$ is in a relational way, as 7 more than 5. $a + 7$ then becomes 7 more than a number a. The use of this type of language rather than translating it into words as a plus 7 is one that seems to be of great assistance in making sense of algebra.

To be successful in algebra students need to learn to develop rules from patterns. The extensive use of sequential patterns can focus students' attention on aspects of the numbers which actually limit their understanding of function. For example, in the grid below, if students focus exclusively on the vertical relationships between numbers they can find it difficult to see the horizontal relationship between the numbers.

x	y
1	4
2	7
3	10
4	13
5	
10	
100	

One question teachers of early algebra need to address focuses on the use of symbols: when should letters be introduced? Often teachers delay this as they think the use of letters may be too difficult for students but this delay is not always appropriate. Symbols should be introduced naturally as a way of generalising when the need arises, using the correct approach from the start. Why not use n for any number? Delaying the introduction of letters as symbols can cause confusion. For example:

> Mary has the following problem to solve:
>
> "Find the value(s) for x in the following expression: $x + x + x = 12$."
>
> She answered in the following manner
>
> A. 2, 5, 5
> B. 10, 1, 1
> C. 4, 4, 4
>
> Which of her answers are correct? Circle the letters for each correct answer.
>
> Would your answer have changed if the question was:



The use of x implies the letter's numbers are all the same, but the use of the coloured boxes is ambiguous.

In summary, to meet the needs of the future we need to

- Build algebra sense, making sure the concept of "=" and all of the operations are well developed.
- Build relational thinking rather than concentrating just on calculating.
- Be willing to explore symbols and numbers beyond the syllabus outcomes.
- Recognise that concepts need to be introduced and explored a long time ahead of when we expect them to be well established and connected in a child's mental framework of mathematics.
- Support students to take control of their own learning.
- Create classroom cultures where discussion and debate is an integral part of learning.
- Raise expectations of what is possible and not hide things from students because we think they are too hard for them.
- Include a variety of problem solving and investigations into classroom programmes.
- Utilise appropriate tools to support learning.

Some Thoughts on Inequalities

The quote below is by the mathematician Vera Pless and comes from an article written in 1991. For a full reference and greater biographical data of Vera Pless, see the web site http://www-groups.dcs.st-and.ac.uk/~history/Biographies/Pless.html.

In retrospect, I think I was very lucky. I worked in coding from its beginning and it has developed into a fascinating mathematical topic. I have appreciated the opportunity to work with many wonderful mathematicians, in particular Richard Brualdi, John Conway, and Neil Sloane. I was able to care for my children in their younger years in a low pressure environment. I would find child rearing difficult facing the pressures our assistant professors face. Our discipline is not the only one demanding a great deal. My daughter, a medical resident with a young daughter of her own, has plenty to say about the long hours required of residents. Unfortunately, our society is probably losing valuable contributions from women for these reasons, and many women are paying a great emotional toll either in forfeiting careers or in not devoting as much time to their families as they feel they should.

Have things improved for women over the last 15 years?

Book Review

De Arithmetica, Book I - by Boethius

Boethius was executed as a political prisoner in 525 AD at the age of 43 so it is doubtful that you have an original copy of this book. Boethius made considerable contributions to intellectual thought with original works on philosophy, theology, music and mathematics and translations of, among others, Aristotle and Cicero. Book I of De Arithmetica is very largely a translation of the arithmetic of Nicomachus of Gerasia who flourished about 100 AD although Boethius expanded and refined Nicomachus's ideas.

Book I discusses the classification of numbers, their properties and relationships. For example, number is defined as a collection of units or as a flow of quantity made up of units. The author divides numbers into sets such as even, odd, evenly-even and evenly-odd. Evenly-odd numbers are odd multiples of 2, for example, 6, 18 and 30. He also notes that when numbers obtained from 1 by successively doubling are summed, the total is one less than the next number in the sequence. For example, $1 + 2 + 4 + 8 = 16 - 1$.

Boethius has other ways of classifying even numbers, for example; superabundant, deficient and perfect (see our March 2004 Newsletter). He also defines heteromecic numbers as those that are the product of two consecutive whole numbers like $12 = 3 \times 4$. The book lists dozens of relationships that exist between the many defined sets of numbers.

De Arithmetica gives no rules for computation or practical applications so I guess it would be out of favour as a textbook today. It's also difficult to read in the original but could be a useful reference book for teachers (!)

Solution to May's problem

Deprimes, pronounced 'derprimes', are whole numbers whose digits when taken consecutively in pairs are all prime. Thus 137 is a deprime since 13 and 37 are prime. All two digit prime numbers, of course, are deprimes. Now we asked you to tell us how many deprimes there are between 9,000 and 10,000 and list them. We also asked if any of these are prime.

We got a number of entries and this month's winner is Marnie Fornusek of Rotorua so she'll get the book voucher. Her solution is given below. But I should note first that there was a common error around 91. Now 91 is not a prime because it's divisible by 7.

"I wasn't sure what of the best way to go about finding the Deprimes between 9000 and 10000. I started by setting up a spreadsheet (attached) to work out which of the numbers between 9000 and 10000 were Deprimes. I used the mod function to find out if the 3 pairs (e.g. 90, 01, 12 for 9012) in the number left a remainder, when divided by 2, 3, 5 and 7. I used the COUNTIF function to count the number of times the pairs had remainders. If they had less than a count of 12 then it meant one of the pairs was divisible by 2, 3, 5 or 7. The numbers that weren't divisible were 9711, 9713, 9717, 9719, 9731, 9737, 9797.

"I now realise that it was easier without the spreadsheet. First of all look at the first two digits for numbers between 9000 and 10000. The first two digits can only be between 90 and 99. If you take out the even numbers, 93 and 99 (as divisible by 3), 91 (as divisible by 7), 95 (as divisible by 5), the only prime number is 97. The middle pair of digits must therefore start with 7 and can only be from 70 – 79. Again if you take out the non primes the only options are 71 and 73. The first 3 digits are either 971 or 973. The only 2 digit prime numbers that start with 1 are 11, 13, 17 and 19. The only 2 digit prime numbers that start with 3 are 31, 37 and 39. Therefore the Deprimes are 9711, 9713, 9717, 9719, 9731, 9737, 9797.

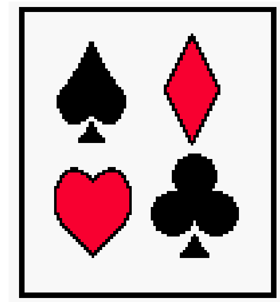
"I used the mod function in the spreadsheet to find out that 9719 is the only prime as it is the only one that has a remainder when divided by all the prime numbers less than 100." (See Note below.)

I have to say though that Marietta Sansom-Gower sent in a very nice solution. But she is from Tasmania. This is the first solution we've had from outside New Zealand. We think that the logistics of organising prizes for winners from another country are too great for us at this point. But many thanks for entering, Marietta, and we hope that you'll continue to take an interest in our site and the newsletter.

Note: It may be just worth saying that to find out if a number is prime, or indeed if it has any factors, you only need to divide the number by the prime numbers up to and including the square root of the number. This is essentially because if the number, N say, has a number has a factor, F, bigger than the square root, then N = F times something. What's more the 'something' will be less than the square root. But we will have found this 'something' as we looked for factors less than the square root. So we will know that F is a factor before we get to the square root of N.

I'm sorry if that all sounds a very complicated explanation for what is a relatively simple idea.

This Month's Problem



Let's try a little problem on probability or proportions for a change. We have an ordinary pack of 52 cards that are separated into two unequal portions A and B.
If a card is drawn at random from A, the odds are two-to-one against it being red. If however a red card had been transferred from portion B to portion A, then the odds would be two-to-one against a randomly drawn card from B being black. Got the idea? Your question is; how was the pack originally divided?

We will give a book voucher to one of the correct entries to the problem. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so we can send the voucher if you are the winner.

Solution to May's Junior Problem

Last month we left you with this problem. Each letter in the expression "GO AT IT" is a non-zero whole number. Each word is a perfect square. Now find ALL of the solutions for me and make sure you have told me why there aren't any more.

I got a very good solution from Jake Boyce-Bacon of Remuera Intermediate. This wasn't the only good solution I got but it was the first. I should note that we've had winners from Remuera for each of the last two months. Come on the rest of the country!

"There are four possible solutions for GO AT IT. They are –

25 36 16
25 16 36
49 36 16
49 16 36

First of all, the only letters that are alike are the T's. Then, the only 2-digit square numbers that have the same last digit are 16 and 36. These numbers could also be either way around, as there is only one each of A and I. Now, for G and O, they could be anything not using the digits 1, 3 and 6. The only square numbers that don't use these digits are 25 and 49. As there are two ways to have the 16 and 36, and two ways to have GO, you can say 2 x 2 = 4. There are four possible solutions, and no more."

This Month's Junior Problem

This month think boxes and boxes containing white and black marbles. In fact we have three boxes and every box contains precisely two marbles. And we should know what is in each box because someone has labelled them BB, BW, and WW. But the labeller has mislabelled every box

To find out what really is in each box you may take one marble at a time from each box without looking at the other marble. What's the smallest number of marbles you need to look at before you know the contents of each box?

Clearly you will need to explain how you got your answer.

Anyway let us remind you that this section contains a monthly problem competition for students up to Year 8. What's more the usual $20 book voucher is available for the winner. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so we can send you the voucher if you are the winner.