Newsletter No. 53 **May 2006**

There are at least four prestigious prizes that can be won by research mathematicians.

The Fields Medal, considered by some to be the greatest of these, was first awarded in 1936. It is given for outstanding mathematical achievement and is awarded to those under 40 as an incentive for younger researchers. New Zealander Vaughan Jones won a Fields Medal in 1990 for his work in knot theory.

Like the Fields Medal, the Rolf Nevanlinna Prize is awarded every four years. It was first awarded in 1986 and is presented for outstanding contributions to mathematical aspects of the Information Sciences.

The annual Abel Prize was first awarded in 2003 and recognises a lifetime's achievement in mathematics.

This year the Gauss Medal will be awarded for the first time. It is designed to honour scientists whose mathematical research has had an impact outside mathematics.

We've listed a great deal more information on these and some of the winners in the 'Afterthoughts' at the end of this newsletter.

With all those prizes to be won it might be time to get sharpening your pencil!

With me everything turns into mathematics.

Rene Descartes

You know, of course, that 53 is a prime number but did you know that Maria Callas, Rene Descartes, Hermann Goering, John Denver, Princess Grace of Monaco and Tchaikovsky all died aged 53? Incidently, the German submarine U53 sank 53 allied ships during World War II. How about that!

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What's new on nzmaths.co.nz

There are two new units of work in the Rauemi Reo Māori section of the site, Hangaia ki te māti and He Pānga Taurangi.

Keep a close eye on the site this month as there are several new developments arriving. Most excitingly, we will be adding a new piece to our jigsaw, linked to the learning objects in Te Pātaka Matihiko Our Digital Storehouse. While these learning objects are available to teachers in New Zealand schools already from http://www.tki.org.nz/r/digistore/ we have developed a more user friendly interface – this should be online in the first week of May.

We have also written a series of units which involve using the learning objects, these are currently being edited and the first units will be online in the first half of May.

Teachers in schools involved in the Secondary Numeracy Project will be pleased to hear that we have a collection of new resources almost ready to add to the SNP section of the site.

Diary Dates

July

8th Australasian Conference on Mathematics and Computers in Sport
http://www.anziam.org.au/

27 July

The Australian Mathematics Competition
http://www.westpac.co.nz/

14 -18th August

2006 Maths Week
http://ww.mathsweek.org.nz

26 - 29 September

Australian Mathematical Society Annual Conference
http://www.austms.org.au/People/Conf/

Numeracy Conference Plenary Sessions

In April's newsletter we presented a summary of the talk that Paul Cobb gave at the Numeracy Conference in Auckland in January. This month we do the same for Peter Hughes' plenary session. Once again we are thankful to Jenny Ward for organising this material for us.

The power point presentation, including full references, for this address is available online at http://www.nzmaths.co.nz/numeracy/References/keynotes06.aspx

Peter Hughes: Pedagogy in the New Zealand Numeracy Projects

The core of the Numeracy Project is derived from work on children's counting types, the Mathematics Recovery Programme and the Count Me in Too programme. The New Zealand Framework differs from these models in that it has renamed several of the stages at the lower end of the Framework, added an imaging phase to the advanced counting stage, and stages six to eight (Advanced Additive, Advanced Multiplicative and Advanced Proportional) have been added to the top end of the Framework.

The Number Framework provides a progression for teachers to follow as they guide students' thinking and in this way gives teachers something to work against. This is important as the teacher who understands where a child is on their conceptual development has a better chance of promoting reflective abstraction than a teacher who just follows the curriculum (Von Glasersfeld, 1996).

The teaching model is important as, in the progression of children's thinking, imaging is the link between materials and abstraction and as such it is vital. The use of concrete materials is also important, but rather than moving directly from physical representations to the manipulation of abstract symbols it is suggested that the emphasis be shifted to using visual imagery prior to the introduction of more formal procedures. This will support students to make the required abstractions. The teaching model is a tool for the teacher to make formative evaluations. Ideally, the teacher reacts to the needs of the students and alters the lesson in real time.

In addition to an understanding of the Number Framework and the teaching model, to be effective teachers also require a sound pedagogical content knowledge in mathematics. This includes an understanding of how particular topics, problems, or issues are organized, and adapted to the diverse interests and abilities of learners, and presented for instruction (Shulman, 1987).

Teaching is a complicated and challenging occupation and it can be difficult to identify quality teaching. Fraivillig (1999) identified a list of quality teaching actions and these include actions under the categories of eliciting, supporting and extending students. Further to this is the work done in the Manurewa Enhancement Initiative. In this work quality teaching actions include:

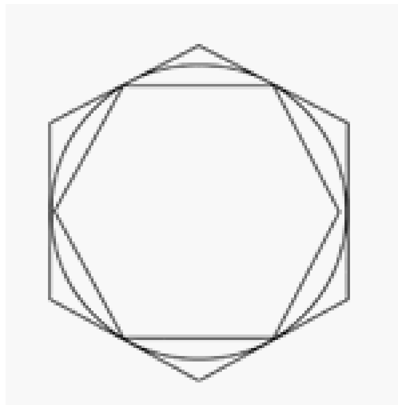
1. noticing student solution methods and listening without intervening;
2. understanding students' reasoning and the cause of any incorrect reasoning; and
3. taking appropriate teaching actions.

Successful professional development will result in sustained changes to practice. This is most likely to be achieved when teachers have a good understanding of the pedagogical content knowledge involved. Even expert teachers need ongoing support in this area.

Solution to April's problem

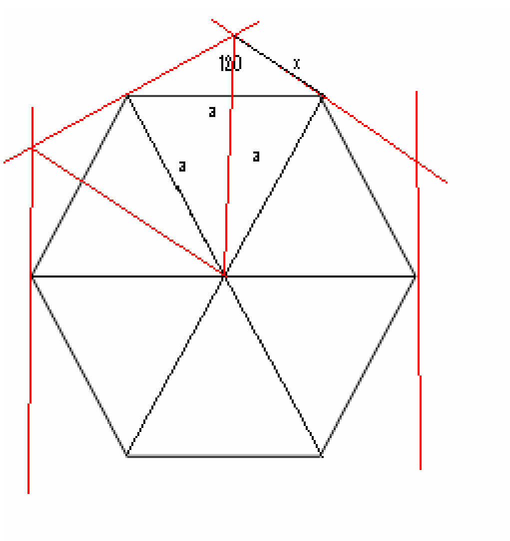
Last month's problem was as follows.

And while we're on the subject of ratio of areas, I wonder if you can tell us the ratio of the area of a regular inscribed hexagon to that of the regular circumscribed hexagon.



We had some good solutions from a number of people this month. The following two solutions came from Marnie Fornusek of Rotorua who gets this month's prize.

A circle with a radius of length a has a regular inscribed hexagon with an area of $\frac{3\sqrt{3}}{2}a^2$ (from previous problem solutions).



For the regular circumscribed hexagon (lines in red) with a side length of $2x$ we can find x in terms of a using the sine rule.

$$\frac{\sin 120°}{a} = \frac{\sin 30°}{x}$$

$$x = \frac{\sin 30°}{\sin 120°}a = \frac{1/2}{\sqrt{3}/2}a = \frac{a}{\sqrt{3}}$$

Therefore the regular circumscribed hexagon has a side length of $\frac{2a}{\sqrt{3}}$.

A regular hexagon of side length $x = \frac{2a}{\sqrt{3}}$ has an area of

$$6 \times \frac{1}{2} \times \frac{2}{\sqrt{3}}a \times \frac{2}{\sqrt{3}}a \sin 60^o = 3 \times \frac{4}{3} \times \frac{\sqrt{3}}{2}a^2 = 2\sqrt{3}a^2.$$

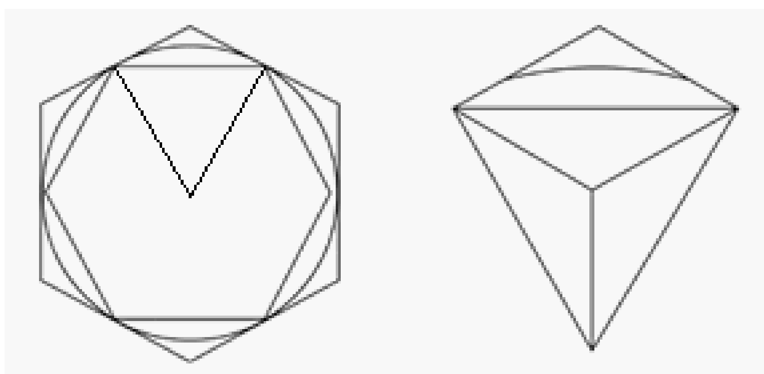
Therefore the regular circumscribed hexagon has an area $= 6 \times \frac{\sqrt{3}}{3} = 2\sqrt{3}$.

The ratio of area of regular circumscribed hexagon: area of regular inscribed hexagon is $2\sqrt{3} : \frac{3}{2}\sqrt{3} = 2 : \frac{3}{2}$ which simplifies to 4 : 3.

But Marnie adds: Another way of looking at it is that both hexagons are regular inscribed hexagons for circles - one of radius $\frac{2}{\sqrt{3}}a$ and the other of radius a. The ratio of the area of the circles should be the same ratio as the hexagons. So the ratio for the areas of the circles is

$$\pi\left(\frac{2a}{\sqrt{3}}\right)^2 : \pi(a)^2 = \frac{4}{3} : 1 \text{ or } 4:3.$$

Actually there is at least one other way to go about this. Maybe a couple of diagrams are a more elegant proof than words. If the inscribed hexagon is tessellated with triangles congruent to those between the two hexagons (one sector has been shown) it can be seen that the ratio of the inscribed hexagon to the circumscribed hexagon is three-quarters.



This Month's Problem

Deprimes, pronounced 'derprimes', are whole numbers whose digits when taken consecutively in pairs are all prime. Thus 137 is a deprime since 13 and 37 are prime. All two digit prime numbers, of course, are deprimes. You might like to explore the concept and list the deprimes between 100 and 200, there are ten of them (answers below).

This problem is not difficult, more one of search and find. The mathematical ideas involved with deprimes can be quite deep but all we want you to tell us is how many there are between 9,000 and 10,000 and list them. Are any of them prime, we wonder?

We will give a book voucher to one of the correct entries to the problem. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so we can send the voucher if you are the winner.

Solution to April's Junior Problem

First of all let's recall the problem.

Think of perfect squares: numbers like 4 = 2 x 2, 9 = 3 x 3, 100 = 10 x 10, 144 = 12 x 12. Right then, now think about the code GO TO IT. I'm going to tell you that

(i) each letter stands for a different number (1, 2, 3, 4, 5, ,6 ,7, 8, 9); and
(ii) each word stands for a perfect square.

What numbers do the letters G, I, O, and T stand for?

Well let's work it out. Maybe the simplest thing to do is to write down all possible 2-digit squares. These will be $4^2 = 16$, $5^2 = 25$, $6^2 = 36$, $7^2 = 49$, $8^2 = 64$ and $9^2 = 81$.

Now, two of the squares (GO and TO) end with the same number. This only happens with 16 and 36. So we've nailed down O to be 6.

Now if G = 1, T = 3 and we're looking for a square that ends in 3. There isn't one. However, there is a square that ends in 1. So T = 1 and G = 3.

The square that ends in 1 is 81, so I = 8.

So, G = 3, I = 8, O = 6, and T = 1.

I'm glad to say that we had several replies to this and they were all correct. That made it hard to choose a winner. So I decided that as there was nothing to choose between the answers, I'd give the prize to the first person who sent in their answer. So this month's winner is Henry Yuen from Remuera Intermediate in Auckland. Congratulations Henry.

But perhaps, if any of you students want to get an edge over your competitors for this month's prize, you can give me some reasons for your answers. All you have to do is to talk me through your solution, just as I did above.

This Month's Junior Problem

Anyway let us remind you that this section contains a monthly problem competition for students up to Year 8. While we're running hot on codes, let's try another one.

Each letter in the expression "GO AT IT" is a non-zero whole number. Each word is a perfect square. Now find ALL of the solutions for me and make sure you have told me why there aren't any more.

The usual $20 book voucher is available for the winner. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so we can send you the voucher if you are the winner.

Afterthoughts

… to the Introduction

Fields: J.C. Fields was the secretary of the 1924 International Congress of Mathematicians (ICM) in Toronto. It was there that a medal for mathematics was first proposed. Fields later donated funds to establish the medals.

The web site that will tell you about this medal, as well as to debunk a famous scandal about Mittag-Leffler that you must swear not to spread, is http://mathworld.wolfram.com/FieldsMedal.html.

Rolf Nevanlinna: R. Nevanlinna was active in the 1950s and had been Rector of the University of Helsinki and President of the International Mathematical Union (IMU). The IMU is responsible for the ICM.

In 1982, the University of Helsinki offered money to finance the prize that is awarded under the same financial conditions as the Fields Medal.

To find more details on the prize, try http://www.icm2002.org.cn/general/prize/nevanlinna.htm.

Vaughan Jones: If you'd like to know more about Vaughan Jones, you can look at the following web site: http://ifs.massey.ac.nz/mathnews/centrefolds/37/Aug1986.shtml. This is actually the Centrefold of one of the New Zealand Mathematical Society's Newsletters. Vaughan is shown there fully clothed you'll be glad to know. But if you want to find out more about local mathematicians, then search through these Centrefolds. A different mathematician appears every month.

Abel Prize: This year's winner is the Swede Lennart Carleson for ground-breaking work he carried out in the sixties related to Fourier analysis and work he did in the nineties on strange attractors. If you'd like to know more about Lennart Carleson and the Abel Prize peruse Professor Marcus du Sautoy's article *Life Begins at N = 40* in *New Scientist* for 1 April 2006 or check out the website http://www.physorg.com/news12076.html

Abel: Abel, after whom the prize is awarded, died at age 26 of tuberculosis, one of the most tragic early deaths in the history of mathematics. Like many great scientists, his ideas have permeated mathematics. In his short life, he revolutionized the theory of equations, complex analysis, number theory and algebraic geometry. One extraordinary result he proved was the impossibility of solving polynomial equations of degree greater than four, using surds. He is also responsible for some of the deepest work on algebraic integrals of the 19th century. Abel's Theorem is one of the basic results of algebraic geometry.

You can find more about Abel by going to http://www-groups.dcs.st-and.ac.uk/~history/Indexes/A.html and clicking on his name.

Galois: Perhaps up there in the race for the prize of tragic early deaths is Evariste Galois. He gets a whole site to himself at http://www.galois-group.net/. If you want to go through the whole gamut of political intrigue, love, duals, outstanding maths, an unnecessary death, and a sad ending, then Galois' story is for you.

Gauss: Gauss is the person who, in his teens, is famous for adding up the numbers from 1 to 100 in quick time and so upsetting his teacher. This was just the beginning of a life of overachievement in mathematics. You can find out more on him at http://www-groups.dcs.st-and.ac.uk/~history/Indexes/G.html by clicking on his name, and on both him and his medal, another IMU production, at http://www.mathunion.org/General/Prizes/Gauss/index.html.

… to This Month's Problem

The deprimes between 100 and 200 are 111, 113, 117, 119, 131, 137, 171, 173, 179 and 197. Six of them are prime.