



Newsletter No. 52

As we discussed last month, over the last few decades with computers and other high technology playing an ever-increasing role in our lives, the range of applications of mathematics has risen dramatically. Subjects as diverse as genetic counselling, aircraft design, drainage basin networks and sharemarket fluctuations are explored in terms of their mathematics. Wherever phenomena need to be analysed in mathematical terms or where past information is examined with a view to telling us something about the likelihood of future events, mathematical models are set up. It is not possible for many of these subjects to be investigated in schools because of the level of mathematical sophistication involved but it is important that students understand the ideas involved and have the opportunity to work through the modelling process in a range of practical and relatively straightforward applications.

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Of course, that all begs the question; what situations are appropriate for classroom discussion? A little research on the web or in the library will give plenty of examples but I've found the classic example of how our understanding of the workings of the solar system has changed over time a useful introduction to the modelling process. Working from early Greek Earth-centred descriptions like that of Thales up to Kepler's Suncentred model, the idea of the improving predictive power of a model can be demonstrated. (You might like to look at Morris Kline's "History of Western Culture" is also useful and J.L.E. Dreyer's "A History of Astronomy from Thales to Kepler". There's also a good selection of biographies of the world's astronomers at http://wspace.danask.com/.) A formula (model) for the time period of a pendulum can also be explored in the classroom. Students might first list the things they think affect the time period, then undertake an experiment to test their ideas. You might also check out what is already being done in other subjects, particularly geography and science, they might surprise you.

One way of considering the real world is to assume that it consists of phenomena that behave according to certain rules. The process of descriptive modelling seeks to obtain these rules and apply them back to the real world. A descriptive model though is not a fixed entity, it evolves with time as more accurate measuring devices are developed and insight to our world grows.

Reg Alteo

This is issue 52 of the newsletter and of course you knew that there are 52 weeks in a year and 52 cards in a pack, but wait there's more about 52 you might not have come across before The mathematician Erdos calls a number 'untouchable' if it is never the sum of the proper divisors of any other number. 52 is untouchable and there are

two smaller ones. What are they? (The answers are given at the end of the newsletter.) By the way, if you forgot, the proper divisors of a number are all that number's divisors except the number itself. Thus the proper divisors of 6 are 1, 2 and 3. Did you know also that the original 'Robison Crusoe' Alexander Selkirk survived 52 months on Juan Fernandez Island and on a more sombre note; Thomas a Becket, Christian Dior, Harry Houdini, Erwin Rommel, Frank Zappa, Roy Orbison and William Shakespeare all died aged 52.

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Diary Dates

5 April

The National Bank Junior Mathematics Competition http://www.maths.otago.ac.nz/nbjmc/

July

8th Australasian Conference on Mathematics and Computers in Sport <u>http://www.anziam.org.au/</u>

1 – 6 July

Mathematics Education Research Group of Australasia 29th Conference http://www.aamt.edu.au/merga29/index.html

27 July

The Australian Mathematics Competition http://www.westpac.co.nz/

14 – 18th August

2006 Maths Week

http://ww.mathsweek.org.nz

26 – 29 September

Australian Mathematical Society Annual Conference http://www.austms.org.au/People/Conf/

Numeracy Conference Plenary Sessions

For the last several years, a conference has been held in Auckland in February for the Numeracy Development Project's facilitators. This year Paul Cobb from Vanderbilt University, Nashville, Tennessee, Peter Hughes, from the University of Auckland and Marj Horne, of the Australian Catholic University where the three plenary speakers. We are grateful for Jenny Wade for putting together summary notes of their presentations. This month we will give the summary of Paul's talk. Peter's will be in the May newsletter and Marj's in the June newsletter.

Note that the full power point presentations for all of these addresses is available online at <u>http://www.nzmaths.co.nz/numeracy/References/keynotes06.aspx</u>.

Paul Cobb: Supporting Productive Whole Class Discussions

As students participate in classroom programmes they come to an understanding of what is expected of them within those programmes. Two areas of interest in terms of teacher expectations for mathematics classes are classroom social and sociomathematical norms. *Social norms* are obligations students have as part of the classroom in general. For example, students are required to explain and justify solutions, attempt to make sense of others' solutions, indicate whether they understand, and ask clarifying questions. *Sociomathematical norms* are mathematical obligations and include understandings of what constitutes an efficient mathematical solution or an acceptable mathematical explanation.

Teachers need to be supported to establish classroom norms and this is important as explicit negotiation of classroom norms will help ensure all students have access to important ideas.

In considering what constitutes a valid mathematical argument, two types of explanation can be considered.

A *calculational* explanation: explains the process of arriving at a result. A *conceptual* explanation: explains the reasons for using a particular calculation approach.

For example: A directory of 62 pages has 45 names per page. How many names are in the directory? A calculational explanation for this problem would entail a description of the process of multiplying 62 by 45. A conceptual explanation would relate this multiplication back to the original problem. For example, first I multiplied 60 by 40 to give 2400 and that's 40 of the names on 60 of the pages, then I multiplied 5 by 60 which is 300 and that's now all of the names on 60 pages. Then I added 2 more lots of 45 to account for the other 2 pages. So 2400 and 300 and 90 is 2790.

Conceptual explanations are important as they use situation specific imagery to facilitate a problem solving approach. They can change mathematical tasks from purely doing something with numbers to helpful ways to think about the world.

A general pattern in teaching is for teachers to present material to students, students to explore some problems, either in groups or individually and then for the class to meet back together to discuss results and findings. In this type of whole class discussion, teachers try to pull together what has happened, incorporating students' solutions and interpretations. The teacher's goal is to achieve a mathematical agenda by building on students' contributions. In such discussions both the students' views and the mathematical concepts being explored are important and both need to be considered by the teacher.

In whole class discussions, reflection can be viewed as a critical aspect of maths learning, with student actions and explanations becoming the explicit focus for discussion and analysis. For example: There are 5 monkeys playing in 2 trees. How many different ways could the monkeys be arranged in the 2 trees? In a classroom discussion of this task it is reasonable to anticipate the discussion would change in focus as the task progresses. Initially the talk would focus on the ways the monkeys could be arranged and recording these, maybe on a table. Once the table is complete the teacher might ask students to check the table to see if there are any combinations that have been missed, a reflective shift in discourse as the focus of the discussion on to ask "Is there a way we could be sure we've got all the combinations?" In this example students are involved in finding patterns, a task central to maths learning, and in finding out "how can we know", a precursor to the mathematical concept of proof.

It is the teacher's role to support students to reflect on both collective and individual activity. Discussions that focus on conceptual explanations are important as they give students access to each other's thinking and significant mathematical ideas become the focus. In this context students learn to ask mathematical questions and maths ideas are seen as tools. Specific maths ideas can be focused on to explain specific solutions, and discussion can also include debate about when particular maths ideas are useful.

{proof} the Movie

Proof is important, indeed fundamental, in mathematics. It's what makes it different from all other areas of study. Or maybe it's just the kind of proof. Anyway, proof is important in two ways in the film of that name.

There are essentially four characters: the sixty something mathematician (Anthony Hopkins), his younger daughter Catherine in her late twenties (Gwyneth Paltrow), the young male academic mathematician (Jake Gyllenhaal), and Catherine's older sister Claire (Hope Davis). The action takes place in Chicago.

Now the mathematician made some startling discoveries when he was only in his early twenties. Unfortunately he later became mentally unstable and the younger daughter looked after him at home for several years. One manifestation of this instability is shown when he is seen doing maths in the garden in the snow. (Now you may think that that isn't normal behaviour but I have known mathematicians who have worked in freezing studies but didn't realise that it was cold until they stopped working. Ah well.) Suitably vulnerable Catherine, who had started out as a maths major, was greatly stressed by looking after dad, as well as realising that he wassn't ever going to get the spark back again. The mathematician goes through 103 exercise books in his bid to get the next big theorem. But the "Let x = ..." quote that Catherine reads out from one of these books lets us know in no uncertain manner that dad isn't going to produce any more theorems ever again while producing one of at least two of the film's tear jerking moments.

Independently Hal is coming to the same conclusion. A young lecturer in dad's department he is looking through the books to see if there is anything of any value there. Hal is hung up by producing his own world shattering result. Having decided that at 27 he is too old to produce anything great, he's hoping to find a hint to greatness n what dad has produced in his books.

But I have to say at this point that we all know that big results do come to older people. The infamous Fermat's Last Theorem fell to Wiles in the 90's when he was in his 40's. In fact he was too old to get the mathematician's version of the Nobel Prize, The Fields Medal. For some strange reason you can get a Nobel Prize so long as you are still breathing but you can't get a Field's Medal if you are over 40! Do I want to go into why there is no Nobel Prize for Mathematics? Nobody really believes that Mittag-Leffler and Mrs Nobel ... But I digress.

I don't really know what an obsessive-compulsive is but Claire must be close to it. She drives me bats – and Catherine too. Claire needs to get Catherine under her protection and back to New York where she lives. If Catherine isn't stressed before Claire gets to Chicago, then she certainly is afterwards.

Oh but I didn't say why Claire is in Chicago or why Hal is fossicking among dad's books. Sorry but dad died. And then we get to the turning point of the film – the 104th book. That looks as if it does have a great result. Hal is impressed anyway and wants to check it out. But is it a proof and could dad have really proved it?

I'll leave it there so that there is something for you to discover, if not prove, for yourself. But I thought that I should pontificate on the mathematical rightness of the film. What did I like and what did I not like?

Well we have the expected 'mad' bearded maths professor. And, of course I've never met any of those. OK some are slightly weird sometimes but some are almost normal. And Hal is pretty sane and even I guess to a woman, even sexy. Catherine thinks so anyway. And I've already tried to prove that life for a mathematician is not over after 25.

But there my criticism ends and even those 'errors', because they are essential for the film, can be forgiven. I thought that the script and shots managed to avoid my biggest problem with maths oriented films: that of showing wrong maths. We were very carefully not told the details of the big result of book 104. And when maths was shown it was not sufficiently

specific to enable an expert to see that it wasn't appropriate. So mathematically I give the film high marks.

I didn't like the end though. What really happened after Hal and Catherine talked on the seat in the quad? I like to have these things spelled out for me.

Solution to March's problem

Last month you we asked the following question:

If the sum of the lengths of the edges of a cube is equal to the sum of the lengths of the edges of a (regular) tetrahedron, which has the largest surface area, the cube or the tetrahedron, and what is the ratio of these surface areas?

Wehad no correct answers this month so we present our own here.

To start with it's a good idea to let the length of side of the cube be x units and that of the tetrahedron y units. Now, since there are 12 edges to a cube and six to a tetrahedron, 12x = 6y and hence y = 2x.

Now the surface area of the cube is $6x^2$. The surface area of the tetrahedron consists of six faces, each face being an equilateral triangle of side y = 2x. So the problem really reduced to one of finding the area of an equilateral triangle. Whether you use 1/2absinC, 1/2base×height or Heron's formula, you'll end up with $\sqrt{3y^2}/4 = \sqrt{3x^2}$ for the area of each triangular face, so taking into account the fact that the tetrahedron has 4 faces the ratio of the surface areas, cube to tetrahedron, is,

$$6x^2: 4\sqrt{3}x^2 = 3: 2\sqrt{3}$$

Clearly the tetrahedron has the largest surface area. Did you expect that?

This Month's Problem

And while we're on the subject of ratio of areas, I wonder if you can tell us the ratio of the area of a regular inscribed hexagon to that of the regular circumscribed hexagon.

We will give a book voucher to one of the correct entries to the problem. Please send your solutions to <u>derek@nzmaths.co.nz</u> and remember to include a postal address so we can send the voucher if you are the winner.



Solution to March's Junior Problem

It's clear that averages are pretty difficult. We had no answers to the problem below.

During term 1, Alice averaged 80 over all of her tests. During term 2 she averaged 70 for all of her tests. Can you find a way that her overall average for the first two terms was 75? Is it possible that she could have averaged 90 for the first two terms?

What I had in mind was that Alice can get an average of 75 provided she does the same number of tests in both terms. Otherwise she's doomed to get something else.

On the other hand, there is no way that Alice can get an average bigger than 80 or smaller than 70. So she can never get an average of 90. I would have accepted several examples to show this because the only argument I can think of requires algebra. (Sorry about that.)

This Month's Junior Problem

OK so please send some answers to this.

Think of perfect squares: numbers like $4 = 2 \ge 2, 9 = 3 \ge 3, 100 = 10 \ge 10, 144 = 12 \ge 12$. Right then, now think about the code GO TO IT. I'm going to tell you that

- (i) each letter stands for a different number (1, 2, 3, 4, 5, 6, 7, 8, 9); and
- (ii) each word stands for a perfect square.

What numbers do the letters G, I, O, and T stand for?

The usual \$20 book voucher is available for the winner. Please send your solutions to <u>derek@nzmaths.co.nz</u> and remember to include a postal address so we can send you the voucher if you are the winner.

Afterthoughts

The two smallest untouchable numbers are 2 and 5.