Newsletter No. 50 **February 2006**

Welcome to a new year. We hope you are refreshed, happy to be back in the swing of things and keen to try new ideas with your pupils. It's what keeps us young, they say!

I don't know if you watched much TV over the Christmas period but TVNZ took a bit of a chance, I thought, when they screened *The Story of One* at peak viewing time on Boxing Day evening. The programme was an hour and then-some look at how numbers developed alongside human civilisation. It showed how recording and writing gave us some control over our environment; from scratches on bones to inscriptions on clay tablets, from the inefficiency of Roman numerals to India's greatest invention - place value, from bead frame reckoning to pencil and paper calculation and the computerisation of everything. There were vignettes of mathematicians including the importance and death of Archimedes, the world view of Pythagoras, algebra and Alkarismi, Fibonacci's popularisation of the Arabic numerical system and Leibnitz's development of binary numbers (I hadn't known that). It was all something different anyway - educational television for people on holiday - bizarre.

On another track entirely, it's a fact that the opossum has 50 teeth and that swimming records can only be set in pools of length 50 metres. On a more sombre note Captain Cook, Errol Flynn, Lady Hamilton, Tsar Nicholas II, Michael Clark and Virgil were all 50 years old when they died. Incidently, 50 is the smallest number that can be expressed as the sum of two squares in two different ways: $50 = 5^2 + 5^2 = 7^2 + 1^2$. It is also expressible as the sum of primes in two distinct ways such that all the primes involved are consecutive (beginning with 2): $50 = 2 + 5 + 7 + 17 + 19 = 3 + 11 + 13 + 23$.

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What's new on nzmaths

Hopefully you have managed to have a break over the Christmas period. We here at the nzmaths website have been very busy. There are a host of new resources which have been made available since our last newsletter:

- In the Numeracy Project Section of the site we have added a Representational Framework, illustrating students' responses to problems at different stages of the Number Framework. We have also added 10 new equipment animations. These are all available in both English and Māori versions. Have a look around this section of the site, because there are also a few other new additions, including planning sheets, a workshop on quality teaching, and content tutorials for teachers.
- We have also added 20 new units of work to the site, plus a further 11 units to the Māori section of the site.

Hopefully these new resources will be useful to you in your teaching; as always we appreciate your feedback!

Some More on 50

From the intro I started to think about adding numbers up to get 50. I discovered that you can add two lots of consecutive numbers together to get 50. The sums are

$$8 + 9 + 10 + 11 + 12 \quad \text{and} \quad 11 + 12 + 13 + 14.$$

And that got me thinking about (i) what other numbers are the sum of two strings of consecutive numbers and did they have anything in common with 50; (ii) what numbers, if any, were the sum of only **one** string of consecutive numbers; and (iii) what numbers, if any, were the sum of **no** strings of consecutive numbers?

The only way I could get started was to try to find all the ways of adding strings of consecutive numbers to get the numbers 1 to 30. This took a lot of work but then it seemed clear what the conjecture for (iii) ought to be and maybe there was a conjecture there too for (ii). I think that only 9, 18, and 25 gave me two sums. Is that right?

I'd be interested to hear how you got on. Please send your work and conjectures to derek@nzmaths.co.nz. I'll take up the ideas here in March.

Cricket and Averages

Just the other day, I think it was about 1958, I came across this problem. It appears that a lad, Allan, and his mate Marty, were in the same cricket team. Now Allan had the better batting average for the first half of the season and he had the better batting average for the second half of the season. So when it came to the end of season awards, Allan was a sure fire bet to get the batting trophy because there was no one else anywhere near Allan or Marty.

But when the club President was dishing out the trophies he called Marty up to get the batting trophy.

Well, as you can imagine, there was a great fuss. Not so much from Allan we're glad to say. No he was a right gentleman. But his mates cried enough foul for him and a whole army of cricketers.

The President calmed the 'gentlemen' down by saying that he'd deal with it after the presentation, which he duly did.

"OK, so here are the figures. For the first half of the season Allan had an average of 70 and for the second half he had an average of 30." The President read out the numbers. "As for Marty, his averages were 50 and 15."

"Well that proves it then", one of Allan's friends yelled out. "Allan's better by 35! Marty give Allan the trophy!"

At this point the team's scorer got in a word. "But Allan got his first average by scoring 280 runs for 4 times out and his second average for 600 runs at 20 times out. That's an average for the season of …"

"70 + 30 divide by 2 is 100/2 = 50 and that's much more than Marty can average with a 50 and a 15!!"

"But", the scorer said, "It doesn't work like that. To get Allan's average for the season you have to add all his runs together and divide by the number of times he was out. So that's 280 + 600 = 880, divided by 4 + 20 = 24. And 880/24 = 36.67."

"Surely that's still enough though?"

"I'm afraid not. Marty scored 600 runs for 12 outs and then 30 runs for 2 outs to give an average of 630/14 = 45."

Allan's supporters were gob smacked. And after that, the club didn't ever again announce the two half season averages.

* * * *

What's really going on here? Well first it's clear that the overall average isn't the average of the averages, if you know what I mean. In Allan's case 36.67 isn't 50. You might like to play around with the numbers to get a feel for things. Just tinker at first with the 30 average – that could be obtained by 600 for 20 outs; 570 for 19; …; 30 for 1. Find Allan's overall average for several of these. Do any of them give an overall average bigger than Marty's 45? It seems that the number of outs somehow weights the total average. And what's really strange is that sometimes an average of 70 and 30 will give an overall average that is bigger than the overall average you get from a 50 and a 15.

This should remind you a bit of the October Junior Problem. There we looked at two average speeds for a trip to school and home again. There the overall average speed was also ***not*** the average of the two averages. You have to be careful with averages.

Golf and Designs

Just as we were about to put this newsletter to bed we had an unexpected problem turn up on our email.

"My problem is that 20 of us are going to have a 5 day golfing holiday and I need to try and work out a formula so that (i) we all play with someone different each day; and (ii) we don't play with anyone more than once. I know we won't play with everyone but that doesn't matter as long as we play with different people each time."

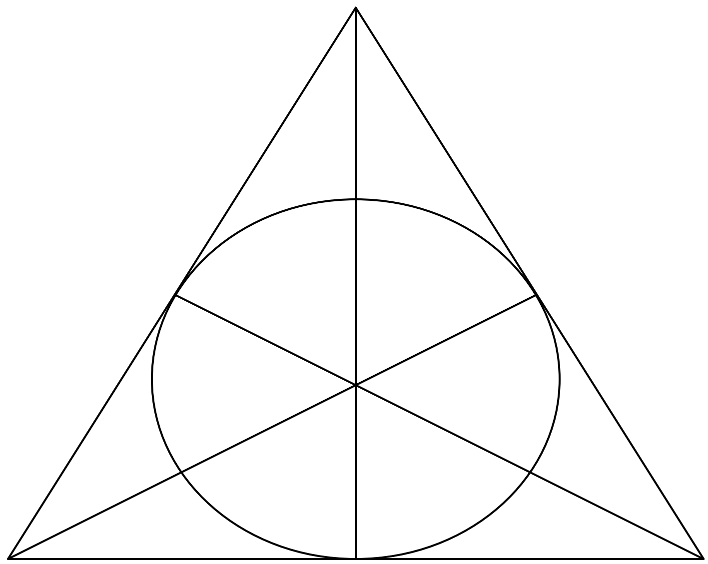
There was one extra piece of information: they were going to play foursomes.

I drew the short straw to look into it so I thought that, having solved it, I could make use of it here. First though I lucked into a solution. I was actually thinking that I would need a block design and I haven't worked with them for ages. My books on the subject were all at work and I was at home and not anxious to drive down to work to sort things out.

Block designs aren't things that you usually come across in school – they're certainly not in the Burgundy Bible. They have an interesting history and have grown up through application, attempting to produce geometries with a finite number of points, and from pure fun problems.

Statisticians got into them trying to analyse agricultural experiments. The normal way to test out new treatments (fertiliser, say), was to divide the land up into plots and apply the treatments. The problem with this was that some of the plots were naturally more fertile than others. They might have been closer to water, say, so it was necessary to allow for variations within the plots themselves. To get some balance in this, the plants were arranged in groups with certain properties. The whole 'experiment' then was called a Block design.

Thinking of geometries, you at least want two lines to meet in a single point and two points to define a line. Here then is a finite geometry with those properties. You should be able to see that there are 7 points where the lines cross, and 7 lines, although one of the lines looks more like an ellipse. This is commonly called a ***Fano plane***.



This ‘plane’ has 3 lines on every point and 3 points on every line. It also satisfies the axioms of a block design (which I have carefully avoided telling you).

And then there was Kirkman. He had a schoolgirl problem. I don’t wish to impune his name. I don’t mean that he had a problem with schoolgirls but rather:

“Each day for a week, fifteen young ladies of a school walk back to their dormitory in groups of three abreast. Can they be arranged so that in a week, no two will walk in a group of three more than once?”

This is a block design problem too. If Kirkman had thought of 7 young ladies instead of 15, the Fano plane would have told him how to arrange the girls. But now you have to try to find the block design that gets the girls back to their dorm.

You know I always feel a bit sorry for the Rev. Kirkman. He was a vicar in England in the 19th century. He got mathematically wronged on two occasions. The first time it was to do with the problem above. After the problem had been posed, a German named Steiner came up with the same idea, quite independently, and the generalization that grew out of the problem is now called ***Steiner Triple Systems***.

On the second occasion he lost precedence at the hand of an Irishman. Kirkman played around with an idea in Graph Theory (not the $y = x^2$ type of graph). Then came along William Rowan Hamilton, who was famous for a lot of other things in maths, used the same idea in a game, and got the credit for what are now known as ***Hamiltonian Cycles***.

Hamilton has an important concept in applied maths named after him – ***Hamiltonians***. And he also invented things called ***Quaternions***. Actually he was out walking with his wife on the banks of a river when the quaternion idea came to him. He was so excited by the discovery that he carved the defining relations on the first stone bridge that he came to.

Anyway the reason I haven't told you what a block design is is because (i) it's complicated (but you can find it in http://www.york.cuny.edu/~malk/tidbits/blockdesign-tidbit.html); and (ii) the golf problem isn't solved by a block design anyway.

To solve the golf problem I first called the players 1, 2, 3, …, 20. Being pretty smart, I decided that the first day they should play in the following five foursomes:

1., 2, 3, 4;   5, 6, 7, 8;   9, 10, 11, 12;   13, 14, 15, 16;   17, 18, 19, 20.

That, of course, took years of experience to find and I doubt that any of you readers would have come up with that.

But getting the second day wasn't quite so easy. I played around and discarded a lot of ideas including block designs. Then I thought to rewrite the foursomes vertically rather than horizontally.

1	5	9	13	17
2	6	10	14	18
3	7	11	15	19
4	8	12	16	20

Now I wanted to use symmetry. That way it would be easier to generate days 3, 4 and 5. So I went down the diagonals. That gave me day 2:

1, 6, 11, 16;   5, 10, 15, 20;   9, 14, 19, 4;   13, 18, 3, 8;   17, 2, 7, 12.

Then it's all down the diagonals from there so I'm sure that you don't need any more hints. But it is fascinating to note that on the first day $4 - 3 = 3 - 2 = 2 - 1 = 1$, etc. And on the second day $16 - 11 = 11 - 6 = 6 - 1 = 5$. (Though you might not see how $3 - 18 = 5$ unless you changed 3 to 23 but that is getting stupid.) On the third day you'd have to have the differences being 9 and on the fourth day 13 and on the fifth day 17.

Ah, I've said too much.

Solution to November's problem

Now last year we posed:

20 posts, equally spaced d metres apart, are in a straight line. 10 birds randomly alight on the posts (one per post). Find the maximum average distance between adjacent birds. You might like to generalise this problem for m posts and n birds.

Investigating with various positions of the birds will show that the maximum average occurs when the end posts are occupied. The arrangement of the other birds is irrelevant.

The average distance between adjacent birds is the quotient given by the total distance between the birds divided by the number of distances between the birds. The total distance between the birds is simply the distance between the extreme birds. It is maximum when the end posts are occupied.

Looking at the general case, the distance between the end posts which are each d metres apart is $(m - 1)d$. The number of adjacent distances between n birds is $n - 1$. Hence the solution to the problem is $[(m - 1)d]/(n - 1)$.

This month's winner is Marnie Fornusek of Rotorua. He came up with this solution.

The maximum distance is when the first bird lands on the first post and the 10^{th} on the post #20. The distance from the first to 20^{th} post is 19d. If there are 10 birds then the number of gaps between them is 9. Therefore the maximum average distance between adjacent birds = 19d /9.

In terms of m posts and n birds this is $(m-1)d /(n-1)$

This Month's Problem

An equilateral triangle and a regular hexagon have equal length perimeters. What is the ratio of their areas?

We will give a petrol voucher to one of the correct entries to the problem. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so we can send the voucher if you are the winner.

Solution to November's Junior Problem

Look at the game in the More on Games section. Now consider the '102' game where each player may say ***one, two, three, four or five*** consecutive numbers. Will Alice or Blair win this game? Give your reasons for your answer.

We have two joint winners from Remuera Intermediate. These are Jonathan Bacon and James Koo. Hopefully you'll still be at the school. If not perhaps your teacher can send it on to you. Their solution was:

Blair will always win since Alice always has to start. This is the reason. There are magic numbers in this game. These magic numbers are multiples of 6. Because whatever number Alice says Blair can always say the multiple of six. And since 102 is a multiple of six, Blair will win if he just says the magic numbers.

(Just a reminder: the game is played with the first 102 numbers. Each player takes a turn saying one, two, three, four or five consecutive numbers. The player who says 102 is the winner. Whatever Alice says Blair can say the numbers up to the next multiple of 6. In that way, the multiples of 6 are magic.)

This Month's Junior Problem

Suppose in the cricket problem described earlier in this newsletter, Allan had averages before and after Christmas of 70 and 30, while Marty had averages of 50 and 20. Is it possible that they could end up with the same overall average for the season? If so, how? If not, why not?

To put a claim in for the $20 book voucher, send your solution to Derek at derek@nzmaths.co.nz. It would help to have your school address, and teacher's name added to the message. Make sure that your teacher or parents know that you have sent in a solution.