Newsletter No. 49 **November 2005**

Symbols may not present many difficulties to mathematicians but to the lay person and not a few scientists the story is different. Trying to express statements like $3x + 4 = 7$, $\{\exists x\colon x \in D_6\}$ or $\int f(x)dx$ in words can be extremely difficult, even near impossible. To the trained mind the symbols take on a life of their own. Just as words can be manipulated to give different meanings, symbols, taking the place of words, can do much the same thing but at a deeper level. The inventors of the Calculus both devised notation, symbols, to shortcut their mathematical manipulations. Leibniz seems to have won that battle as his notation is the one most often used by higher mathematicians today. Using symbols can bring out differences or even similarities between different systems. The symbols $\times$ and $+$ when used in their most familiar way lead to statements of algebra like $x \times x = x^2$ and $x + x = 2x$. When used, as they sometimes are, in Boolean algebra for intersection $\cap$ and union $\cup$ respectively, statements like $x \times x = x$ and $x + x = x$ result - an entirely different algebra altogether. Mathematicians can explore the differences or similarities between the two algebras so much more easily when using symbolism. Their use accelerates the thought processes and hence saves mental energy. A symbol is not just a sign. We see signs wherever we turn; telling us to slow down at corners, keep left, leave our library books here, don't use in a microwave and so on. As E. H. Gombrich wrote, ‘the hallmark of a symbol is the feeling that it is much more than a sign.’

As this is the last newsletter of the year we take the opportunity of wishing you all a wonderful summer recess. May it provide all you need to refresh and refurbish. In this issue, as we did last year, we have included a few Christmas Crackers for your entertainment.

To a mathematician, symbols are catalysts for thought.

Michael Holt

Did you know that there are 49 letters in Sanskrit and the speed limit on Japanese motorways is only 49 miles per hour? And the digits of 49 cubed end in 49. I wonder how common that is. I also have it on good authority that the digits of 1/49 are the powers of 2 in sequence, eventually overlapping so that the pattern, although still there, cannot be seen. On a more sombre note (we had one of those last issue) Thomas Aquinas, Robert Clive, Davy Crocket, Isadora Duncan, Gene Clark (of the Byrds) and Marty Feldman all died aged 49.

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What's new on nzmaths.co.nz

The new branding of the website was 'launched' early last month, with the vast majority of feedback that we have received thus far indicating that the new 'look' is very popular; with the dropdown menus making navigating the site easier, and the new style of banners and new colour scheme looking much more modern than the old site.

We have now made it so that the site branding (banners and menus) does not appear when you print pages from the site. This will save on ink and paper for users who print off our units.

Book 5 of the Te Poutama Tau material is now available on line.

Teachers in Numeracy Project schools looking for a different way to test their students on the knowledge domains may be interested in the IKAN forms, now online at: http://www.nzmaths.co.nz/Numeracy/Other%20material/IKAN.pdf

Number of the Month

We think that we've got the number of the month correct this month. In some versions of the last newsletter we made an error with October. Anyway, 11 is what we are about to explore. If you want to see all about 11 and more then you should have a look at the site: http://primes.utm.edu/curios/page.php.

Fine, so here is all that you wanted to know about the number 11 and more.

First it is always easy to see if a number is divisible by 11. All you do is to first add together the digits in the odd positions (starting from the right these are the units, 100s, 10000s, and so on), Say this gives you A. Then add together all of the digits in the even positions (starting from the right this covers the 10s, 1000s and so on). Say this gives you B. Then if A – B (you might want to take B – A if A – B is negative and you don't like dividing into negative numbers) is divisible by 11 then so is the original number.

As the result of this, every palindromic number with an even number of digits is divisible by 11. So this means that the only palindrome with an even number of digits that is prime is 11.

But then there are the ***really*** useful facts about 11. Rather surprisingly there are precisely 11 of these!

(i)The Colonel's famous secret formula for Kentucky Fried Chicken includes 11 herbs and spices.

(ii) Bar codes consist of 11 numbers.

(iii) The Apollo 11 spacecraft reached Earth's parking orbit 11 minutes after takeoff. This was just 66 (6 x 11) years after Orville Wright flew two feet off the ground for over 11 seconds.

(iv) There are 11 stars in Vincent van Gogh's *Starry Night*.

(v) The No. 11 London Bus runs from Fulham Broadway to Liverpool Street Station.

(vi) According to the FBI, a burglary occurs once every 11 seconds in the United States.

(vii) No book in the Bible has exactly 11 chapters.

(viii) In any given 12 hour period, on an analog clock, the minute hand resides in the same position as the hour hand exactly eleven times.

(ix) World War I ended at 11AM on the 11th day of the 11th month of the year.

(x) The name for the planet Pluto was proposed by 11-year-old Venetia Burney of Oxford, England. For her it was the name of a favorite Disney character.

(xi)A hendecagon is an eleven-sided polygon.

More on Games

Working with a bright young mathematician a little while ago one of us came across a combinatorial game that we hadn't met before. The game is called '22'. Each player takes it in turn to say one two or three numbers in sequence starting from 1. The player who says 22 is the winner.

So for instance, the game might go like this:

Alice:	1, 2
Blair:	3, 4, 5
Alice:	6
Blair:	7, 8
Alice:	9
Blair:	10, 11, 12
Alice:	13
Blair:	14, 15
Alice:	16, 17
Blair:	18
Alice:	19
Blair:	20, 21, 22

Now it may seem rather grand to call this game a combinatorial game but that is what mathematicians call them. They are games for two players where everyone has complete information all the time. Nothing is hidden. We analysed the combinatorial game Noughts and Crosses in September. Now let's analyse '22'. In the game above, Blair won. But will he always win or can Alice win sometimes? Or is it all preordained? If Alice knows what she's doing, can she always win? By the way, I'm assuming on the principle of Political Correctness that Alice always goes first (even though there may be a law against this soon).

The young mathematician quickly said that you wouldn't want to say 19, 20 or 21 because then your opponent would always win. So 19, 20 and 21 are ***bad numbers***. These numbers are clearly to be avoided. So are there ***good numbers***, numbers that you would want to say so that you could be sure of winning (or maybe of at least not losing)?

Well obviously 22 is good. Both Alice and Blair want to say 22. Now maybe 18 is good too. If you say 18, then your opponent has to say either 19; 19, 20; or 19, 20, 21. In each case you can get to 22 from these bad numbers so 18 is good too.

What are the bad numbers before 18? Well you can get to 18 from 15, 16 or 17. So 15, 16, 17 must be bad numbers. And that must make 14 good.

Now you can keep working like that or you can look for a pattern. The first way is best because it justifies the next good number whereas if you guess a pattern you might not guess right. But whatever way you go you have to think that 10 is the next good number. And then 6 and then 2.

OK but what does this tell Alice and Blair? Now Alice always goes first. And she can always get to 2. So her best strategy is to say "1, 2". No matter what Blair responds with, Alice can then make the next good number 6. (If he says "3" she should say "4, 5, 6". If he says "3, 4," she should say "5, 6". And if he says "3, 4, 5" she should simply say

"6".) After that she can always say 10 and then 14 and then 18 and then 22. So, even if Blair knows the trick, Alice will always win if she plays with the good numbers in mind.

That's fine but is there a game that Blair can win? Would he do better with 23 or 24 or …?

If you analyse the '23' game the same way we did the '22' game you'll quickly see that the good numbers are 23, 19, 15, 11, 7, and 3. If Alice says "1, 2, 3" she must win (assuming she remembers the right strategy and keeps aiming at the good numbers.

If you analyse the '24' game the same way we did the '22' game you'll quickly see that the good numbers are 24, 20, 16, 12, 8, and 4. If Alice says "1, 2, 3, 4" she must win. Oh but that's not allowed! She can say at most three numbers. So Blair can win the '24'game.

But what about the '1198' game? Who will win that? Is that good for Blair or good for Alice? If you think about what we have done so far, the numbers that are multiples of 4 are good for Blair. The rest are good for Alice. Does 4 go into 1198? Unfortunately for Blair, no.

So we now know (i) how to play the 'n' game for any number n; and (ii) whether we want to start the game or whether we want the other player to go first.

But as with all good mathematical problems this is only the beginning. How does our strategy change if we allow either player to say up to four consecutive numbers? What are the good numbers then? Which games should Alice win? Which games should Blair win?

Or maybe you should allow up to 15 consecutive numbers to be said by either player. What are the good numbers then? Which games should Alice win? Which games should Blair win?

But then how much more difficult is it to analyse the three-player version of the '22' game? Can Alice, Blair or Chris force a win now? And then there's always the four-player game.

First Christmas Cracker

What does the word OZ─OZ spell?

Second Christmas Cracker

One year we included a mathematical word finder as a Christmas Cracker. This year we have hidden the names of 40 mathematicians in a letter from Russell to Reg. Some of the names may be more familiar for their contributions to science (answers below).

12 Cote Street,
Napier

Hi Reg,

Well, I drove the Merc at Orawia races - number four. I erred with the car, Dan's car not mine. I rolled it and lamed an ankle in the crash. Driving pell mell into a wall is why I lie in the general hospital! Adam scored a record, he romped home. Your hairy friend Jacob is hooked and will be, we hope, another legend really soon.

We had a ball at the O'Nallys' party. Mabel fell in your son's lap, laced his drink with her own and collapsed! It's all new to Neil and he's in a daze no doubt - the young don't deserve parents. There'll be no alcohol tonight!

By the way, he can't organise his course for next year. We offer Maths, Bio, T.D. and Physics.

Au revoir mon général, be a survivor, yea?

Russell

P.S. The Soviet attaché was there.

Solution to October's Problem

There are a number of ways of attacking the problem of finding the number of diagonals of a regular polygon. For the 20-sided polygon you could try a solution by drawing but I think it would be difficult. You could try solving the problem for smaller numbers of sides, say, 3, 4, 5, and so on, and spot the pattern. Alternatively you could consider the general case at the outset and solve both the 20-sided and 200-sided cases simultaneously.

n points equally spaced around the circumference of a circle can be joined to each of n - 1 others. However, two of these connections, those to the adjacent points, form the sides of the polygon under consideration. This leaves n-3 possible diagonals from each of the n points. These are, however, each counted twice, so there are $^1/_2 n(n-3)$ diagonals. Substituting n = 20 and n = 200 solves the two particular cases.

This month we deicded to split the prize between our two most regular solution contributors: Cathy Walker and Derek Smith. We've also given their solutions as they give you some idea of the thinking that went to producing the answers.

Cathy's solution:
I had a bit of fun working out the solution to your problem this month. As I drew out the shapes and drew in the diagonals I recorded the info in a spreadsheet and as I expected patterns emerged.

no. of sides	diagonals drawn from corners						no of diagonals
3	0						0
4	1	1					2
5	2	2	1				5
6	3	3	2	1			9
7	4	4	3	2	1		14
8	5	5	4	3	2	1	20

From the first corner I was able to draw 3 less diagonals than the number of corners (a line to each corner apart from the one I was drawing from and the 2 adjacent ones). This was also the same for the second corner (I chose an adjacent one). For each remaining corner (using next adjacent corner) there was one less diagonal I could draw so I figured I had to find out the sum of all these different numbers.

I decided to call the number of corners I had c – then I had to find the sum of c-3 + c-3 + c-4 + c-5 + c-6 and so on. This looked quite tricky so I looked at the numbers of diagonals from each corner for a 9 sided figure.
So letting c-3 = 6
6 + 6 + 5 + 4 + 3 + 2 + 1 – I had 2(c-3) then if I added the other numbers together to make 6 eg 5+1, 2+4 then I had 2 and one half of these or 5 lots of 3. I knew both of these numbers would change as the numbers of corners changed so I tried to express them in terms of the numbers of corners the 5 = c-4, the 3 = ½(c-3). Then I checked to see if this worked for other numbers of corners.

10 sided figure
7+7+ 6 + 5 + 4 + 3 + 2 + 1 =
7+ 7 = 2(10 – 3)
6 + 5 + 4 + 3 + 2 + 1 = ½ (10 – 3)(10 – 4) – were there 7/2 lots of 6 (not immediately obvious!!) but there were 6/2 (3) lots of 7.

So odd and even numbers appeared to work a little differently but the 'formula' I had come up with appeared to be working for both.It appears when c – 3 is even, there are ½(c-3) in each group and c-4 of them. Eg c=13 so c-3 = 10.
Numbers to add are – 10, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1.
The 2 10's are 2(c-3).
The other numbers make groups of 10 – 4 ½ of them or 9 groups of 5 or c-4 groups of ½ (c-3).

It appears when c – 3 is odd, there are ½ (c-3) in each group and c-4 of them. Eg c=14 so c-3=11.
Numbers to add are – 11, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1.
The 2 11's are 2 (c-3).
The other numbers make 5 groups of 11 – ½ (c-4) groups of c-3

The commutativity of multiplication definitely helps here

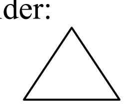
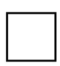
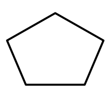
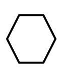
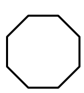
So therefore there should be 2(c-3) + ½ (c-3)(c-4) or ½ c(c-3) – simplified - diagonals for polygons where c is the number of corners.
So a polygon with 20 would have 10x17=**170 diagonals**.
A 200 cornered polygon would have 100x197=**1970 diagonals**.

Derek's solution:
A regular polygon is a polygon that has its interior angles all the same and all its sides of equal length. This problem is two pronged. Given that a diagonal of a regular polygon is a line connecting two corners which is not a side,

(a) how many diagonals does a 20-sided regular polygon have?
(b) how about a 200-sided regular polygon?

You might like to generalise the result.

Consider: Pattern spotting!

	Diagonals = 0		(3-3)x3/2
	Diagonals = 2	1 + 1	(4-3)x4/2
	Diagonals = 5	2 + 2 + 1	(5-3)x5/2
	Diagonals = 9	3 + 3 + 2 + 1	(6-3)x6/2
7-gon	Diagonals = 14	4 + 4 + 3 + 2 + 1	(7-3)x7/2
	Diagonals = 20	5 + 5 + 4 + 3 + 2 + 1	(8-3)x8/2
N–gon	Diagonals = 20	(n-3) + (n-3) + (n-4) + … + 3 + 2 + 1	(n-3)xn/2

Possible rule: Let n be the number of sides on the N-gon then $(n-3) + \sum_{n=1}^{n-3} n$

OR ${}^{n(n-3)}/_{2}$

(a) how many diagonals does a 20-sided regular polygon have?

$$(20-3) + \sum_{n=1}^{20-3} n = 17 + 17 + 16 + 15 + \ldots + 3 + 2 + 1 = 170$$

OR $^{20(20-3)}/_{2} = 170$ **170 diagonals for a 20-sided regular polygon**

(b) how about a 200-sided regular polygon?

$$(200-3) + \sum_{n=1}^{200-3} n = 197 + 197 + 196 + 195 + \ldots + 3 + 2 + 1 = 19700$$

OR $^{200(200-3)}/_{2} = 1970$ **1970 diagonals for a 20-sided regular polygon**

Any N-sided polygon can have (n-1) lines extending from each vertex (counting the sides of the N-gon). Removing the sides gives (n-3) diagonals and from each vertex the number if diagonals is the same BUT diagonals are counted twice, as diagonal AB is the same diagonal as BA hence $^{n(n-3)}/_{2}$.

Third Christmas Cracker

There are plenty of examples of the type $(5 + 1 + 2)^3 = 512$. That is where the sum of a series of digits to some power is equal to the concantenation of those digits. Here's a couple more:

$(2 + 4 + 0 + 1)^4 = 2{,}401$

and $(1 + 7 + 2 + 1 + 0 + 3 + 6 + 8)^5 = 17{,}210{,}368$

Your question is, excluding trivial solutions like $1^0 = 1$ and $2^1 = 2$, what is the smallest number of this type?

This Month's Problem

20 posts, equally spaced d metres apart, are in a straight line. 10 birds randomly alight on the posts (one per post). Find the maximum average distance between adjacent birds. You might like to generalise this problem for m posts and n birds.

We will give a book voucher to one of the correct entries to the problem. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so we can send the voucher if you are the winner.

Solution to October's Junior Problem

The problem here was: Peri walks to school from home and back again along the same route. She averages 2kph going to school and 4kph on the way home. What is her average speed for the round trip?

We had two good solutions here so we are giving a book prize to each of Michael Rooke of Botany Downs Primary School and Athene Laws of Remuera Intermediate School.

Michael’s solution:
Answer: 2 2/3 or 2.66666 etc. kph

Working: if the distance was 2km she'd take 1 hour there at 2kph and 1/2hr home at 4kph. Thus the round trip of 4k at 1 1/2 kph. 4k divided by 1 1/2 = 2 2/3 kph average speed.

Athene’s solution:
I read this month’s junior problem and came up with the answer 2.6kph.
I worked this out by using the example of 8 km each way. If you are going 2 kph then it will take you 4 hours and if you are going 4 kph then it will take you 2 hours making it 6 hours in total. So if it takes you 6 hours for 16 km then the average kph will be 2.666 because 16/6 equals 2.66666666666666667.

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We should point out that they have assumed different distances to work with. In actual fact it doesn’t matter what distance you assume you will get the same answer. Let us show this by using the distance d each way.

If you are going at 2 kph for a distance of d km it will take you d/2 hours and at 4 kph it will take you d/4 hours. So the whole trip of 2d km takes 3d/4 hours. This means that the average speed is 2d ÷ 3d/4 = 2d/(3d/4) = 8d/3d = 8/3 = 2 2/3 = 2.66666666666666667 kph.

We should also add that you can’t just average the two speeds. This would give you 3 kph which is obviously not true.

This Month’s Junior Problem

Look at the game in the More on Games section. Now consider the ‘102’ game where each player may say ***one, two, three, four or five*** consecutive numbers. Will Alice or Blair win this game? Give your reasons for your answer.

To put a claim in for the $20 book voucher, send your solution to Derek at derek@nzmaths.co.nz. It would help to have your school address, and teacher’s name added to the message. Make sure that your teacher or parents know that you have sent in a solution.

Afterthoughts

Answers to Christmas Crackers:

1. Rotate the whole word 90° clockwise and read downwards.
2. Cotes, Napier, Hire, Mercator, Fourier, Cardan, Carnot, Rolle, Lame, Klein, Pell, Wallis, Lie, l'Hospital, Adams, Record, Hero, Airy, Jacobi, Hooke, Peano, Legendre, Ball, Theon, Abel, Laplace, Colla, Newton, Neil, Zeno, Young, Parent, Holton, Cantor, Fermat, Biot, Monge, Ivory, Russell, Vieta
3. $(8 + 1)^2 = 81$

Many Thanks

The nzmaths newsletter would like to thank the following people who all made a contribution to this year's newsletters: Brian Bolt, Russ Dear, Derek Holton, John Stillwell, Lynn Tozer, Ian Stevens and Andrew Tagg (and anyone else we might have missed!).

We would also like to thank all of you who sent in answers to problems.

See you next year!