Newsletter No. 48 **October 2005**

What sort of teenager is it that can not only make sense of but have a go at solving a problem like this? If a tap of a large cistern will empty it in 37 minutes, how many of such taps will empty it in $6^1/_2$ minutes? I'll leave you to work it out for now (answer below) but questions like this were just about standard in Victorian times. They certainly appear often in arithmetic books of the era. I guess they would be aimed mainly at what was called fifty years ago 'the grammar school' type of pupil - but not exclusively. Judging from my perusals of earlier textbooks, similar problems were being asked of secondary pupils in the early nineteen fifties but that was just about their swan song. Of course, in Victorian times there was not only the complication of fractions but peculiar, old-fashioned units like rods, poles, perches, ounces and acres. Take this problem for example: What length of cord will it be fair to tie to a cow's tail, the other end fixed in the ground, to let her have the liberty of eating an acre of grass and no more, supposing the cow and tail to be $5^1/_2$ yards? Express your answer in perches to three decimal places. The interesting part to this problem is how the compiler has mixed fractions and decimals, unlike the first problem where the pupil is expected to work in fractions throughout. You might like to explore, as a project, defunct units and have a go at the cow problem too (answer below).

Which half do you want, the bigger half or the smaller half?
Canteen lady

Did you know that the product of all the divisors of 48 is 48^5 and a hair's breadth is defined as one forty-eighth of an inch? Not only that but 48 is the smallest even number that can be expressed as a sum of two primes in five different ways (5 + 43, 7 + 41, 11 + 37, 17 + 31, 19 + 29). Well, this <u>is</u> the 48th issue of our newsletter. Incidentally, on a more sombre note, Enrico Caruso, Charles I of England, Nat King Cole, Freidrich Krupp, Anna Pavlova, and Cecil Rhodes all died at the age of 48.

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What's new on nzmaths.co.nz

Keep a close eye on the nzmaths site over the next few days. In the last six months we have spent a great deal of time working on converting the site to a more flexible format, and the fruits of this work are about to be revealed. Some time this week we will release a rebranded version of the site, with an improved menu structure, and a much more modern 'look'. We will be following this up in the next couple of weeks by adding a 'printable page' feature, so that you can print pages without the website menu structure.

For those schools with data in the Numeracy database, the reporting feature is now online, allowing you to compare a summary of your school's results with national data, or with previous results from your own school.

Numb3rs

Last month we talked about some films that had a mathematical flavour. This month we've come across a site that is an adjunct to a TV show. You've probably seen or heard of 'NUMB3RS'. It's on TV3 at 9.30pm on Wednesday nights. If you've seen it you'll know that the hero, or the hero's brother, is a mathematician who assists his FBI brother to solve crimes.

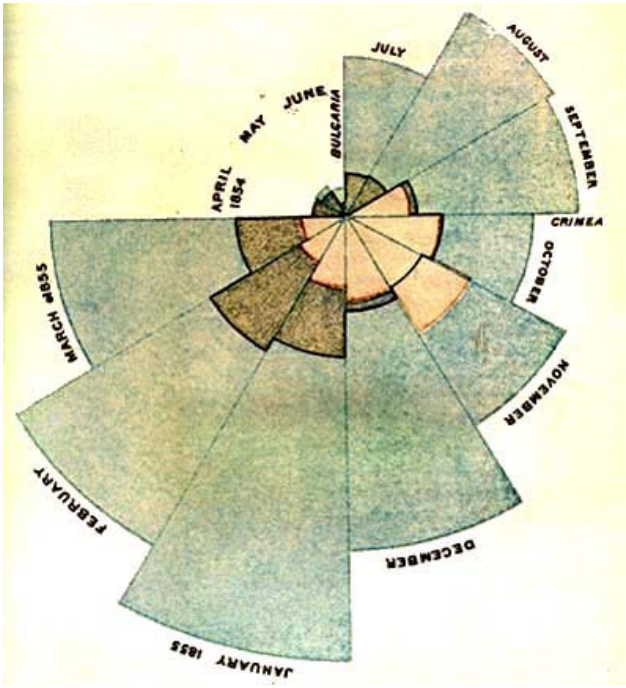
Well it turns out that CBS, (the channel where the programme is aired in the States), Texas Instruments, and the National Council of Teachers of Mathematics are teaming up to coordinate mathematics activities for the classroom around the hit show.

You'll get more details if you go to the web site:
http://www.cbs.com/primetime/numb3rs/ti/. There is a teacher's kit is available at the website. You can get direct access to these by clicking on the urls below.
Creating "Random" Numbers
Shifting Cells
Filtering Suspects and
How Tall is the criminal?

Of course is that the programmes in the States won't be synchronized with those shown here but you could record the programmes – or maybe it doesn't matter too much.

Weird object

So how did you go with this weird object from last month?



First of all look at it closely. You should see some valuable clues. For instance, the dates go from April 1854 to March 1855. What was happening on the world stage about then? Between September and November we can see 'Crimea'. And the date of the Crimean war was ...? 1854 to 1856.

So it is likely then that this object had something to do with the Crimean War. But what?

Here is another clue.

Do you recognize Florence Nightingale?

Nightingale used statistical data to create her Polar Area Diagram, or 'coxcombs' as she called them (for some reason she didn't call think to call them 'weird objects'.) These were used to give a graphical representation of the mortality figures during the Crimean War (1854 - 56).

The area of each coloured wedge, measured from the centre as a common point, is in proportion to the statistic it represents. The blue outer wedges represent the deaths from:-

... preventable or mitigable zymotic diseases

or in other words contagious diseases such as cholera and typhus. The central red wedges show the deaths from wounds. The black wedges in between represent deaths from all other causes. Deaths in the British field hospitals reached a peak during January 1855, when 2,761 soldiers died of contagious diseases, 83 from wounds and 324 from other causes making a total of 3,168. The army's average manpower for that month was 32,393. Using this information, Nightingale computed a mortality rate of 1,174 per 10,000 with 1,023 per 10,000 being from zymotic diseases. If this rate had continued, and troops had not been replaced frequently, then disease alone would have killed the entire British army in the Crimea. Nightingale's moves to introduce hygiene into war hospitals saved large numbers of lives.

To find out more about things of mathematical or statistical historical interest, have a look at the web site http://www-groups.dcs.st-and.ac.uk/~history/.

Number of the month

Because we've discovered the web site http://primes.utm.edu/curios/page.php, we decided to have a number of the month. This site is primarily about primes but there is quite a bit of other information about numbers that you might like to pour over or get your students to pour over.

This month we decided to find out a few things about 9 that you probably never knew and almost certainly didn't ever want to know. Here they are anyway.

Did you know that

(i) $10^9 + 9$ is prime;

(ii) two raised to the 9th power plus and minus 9 are primes! ($2^9 \pm 9$ are both primes.)

(iii) 19, 109, 1009 and 10009 are primes. No other digit can replace the 9 and yield four primes;

(iv) 2^^n-9 = 2^(2^(2^(....(2^2)...)))-9 is (for large enough n) always divisible by both 7 and 11. Note that 9 is midway between 7 and 11;

(v) there are exactly 3 = (sqrt(9)) pandigital improper fractions that reduce to 9 (provided each digit is used once);

(vi) $100000^9 - 9$ and $100000^9 + 9$ are primes. Note that 9 is the only known number with this property;

(vii) the 9th Fibonacci number plus 9 is prime. (Though it would seem that the 9th Fibonacci number minus 9 isn't.

We hope you feel a better person for that but what is a pandigital improper fraction?

Solution to September's problem

The surface area and volume of a cube have the same number of digits. Let the side of the cube be n. We were told that $n \geq 10$ and has all its digits the same. First of all here is our solution but you might like to look at the winner's, below, which is a lot shorter.

Let us first suppose the volume and surface area are a three digit number, then we need to solve the two equations:

$$100 \leq 6n^2 \leq 999 \text{ and } 100 \leq n^3 \leq 999.$$

These give the solutions: $2 \leq n \leq 12$ and $5 \leq n \leq 9$, respectively. Since both conditions must apply no solution to the arises (remember, n is an integer with at least two digits).

Let us now suppose that the volume and surface area are a four-digit number. We need to solve the two equations:

$$1000 \leq 6n^2 \leq 9999 \text{ and } 1000 \leq n^3 \leq 9999.$$

These give the solutions: $13 \leq n \leq 40$ and $10 \leq n \leq 21$, respectively. Since both conditions must apply no solution to the problem arises (remember, n has its digits all the same).

Let us now suppose that the volume and surface area are a five-digit number. We need to solve the two equations:

$$10000 \leq 6n^2 \leq 99999 \text{ and } 10000 \leq n^3 \leq 99999.$$

These give the solutions: $41 \leq n \leq 129$ and $22 \leq n \leq 46$, respectively. Since both conditions must apply one solution to the problem arises, namely $n = 44$.

Looking for further solutions. Let us suppose that the volume and surface area are a six-digit number. We need to solve the two equations:

$$100000 \leq 6n^2 \leq 999999 \text{ and } 100000 \leq n^3 \leq 999999.$$

These give the solutions: $130 \leq n \leq 408$ and $47 \leq n \leq 99$, respectively. Since both conditions must apply no solution to the problem arises, in fact the two solution sets don't even overlap. Clearly, for values of the surface area and volume greater than six digits the solution sets of values for n get wider apart (n^3 increases more rapidly than $6n^2$) and have no common value.

Therefore the unique solution to the problem is 44.

Now Cathy Walker sent in this.

Just read the latest newsletter and since I haven't sent in a solution to a problem for awhile and you want solutions I thought I would have a go at your problem. Using the fact that the volume of a cube is the length cubed and the surface area is 6 times the length squared I generated this data on a spread sheet.

11	1331	726
22	10648	2904
33	35937	6534
44	85184	11616
55	166375	18150
66	287496	26136
77	456533	35574
88	681472	46464
99	970299	58806
111	1367631	73926

So the length of the side of the cube must be 44units- a whole number with digits (at least 2) the same – two 4s – thus the volume of this cube is 85184 cubic units and surface area 11616 square units – both results having 5 digits

This Month's Problem

A regular polygon is a polygon that has its interior angles all the same and all its sides of equal length. This problem is two pronged. Given that a diagonal of a regular polygon is a line connecting two corners which is not a side, (a) how many diagonals does a 20-sided regular polygon have? (b) how about a 200-sided regular polygon? You might like to generalise the result.

We will give a book voucher to one of the correct entries to the problem. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so we can send the voucher if you are the winner.

Perhaps we should emphasise that we are giving book vouchers and not petrol vouchers. We have been having trouble getting petrol vouchers. I know at the moment petrol vouchers are worth more than their weight in petrol but that's it I'm afraid.

Solution to September's junior problem

Alice and Blair took two 1 litre containers and filled them with a mixture of water and orange juice. In the first container the ratio of water to juice was 2 to 1 and in the other container the ratio was 3 to 1. They then put the combined mixture into a 2 litre container. What was the ratio of water to juice in the mixture in the 2 litre container?

Our winner this month is Zheng-chao Liu from Papatoetoe Intermediate. Her teacher is Ms Mistry. The answer she gave is that the ratio of water to juice is 17:7.

There are several ways to get this. Here is one of them. In the first container 2/3 of the mixture was water and 1/3 was orange juice. So 2/3 of a litre was water and 1/3 of a litre was orange juice.

In the second container, ¾ of a litre was water and ¼ was orange juice.

Let's add them up. The water would amount to 2/3 + ¾ of a litre. But 2/3 + ¾ = 17/12. On the other hand, 1/3 + ¼ = 7/12 of a litre was orange juice.

So we have 17/12 of a litre of water and 7/12 of a litre of orange juice. The ratio of water to juice then is 17/12:7/12. But the ratio 17/12:7/12 is the same as 17/6:7/6 or 17/3:7/3 or 17/1:7/1. The simplest way to write it is 17:7.

We know that that is a bit complicated but the easy way to do the problem is wrong! That easy way is to say that 2:1 + 3:1 = 5:2. Unfortunately you just can't add ratios like that and have things work our correctly. It comes about for the same sort of reason that if the average speed of walking from home to school is 2 kph and the average speed of walking home is 4 kph, then average speed for the total journey is not 3 kph.

This month's junior problem

We certainly had a bumper entry last month. Keep it up this month.

Peri walks to school from home and back again along the same route. She averages 2 kph going to school and 4 kph on the way home. What is her average speed for the round trip?

Now you should be worried if your answer is 3 kph. Just to make sure that you've got things right, can you tell us how you got your answer?

To put a claim in for the $20 book voucher, send your solution to Derek at derek@nzmaths.co.nz. It would help to have your school address, and teacher's name added to the message.

Afterthoughts

The answers to the two Victorian problems above are five and nine-thirteenths and 6.136 perches respectively. The problems come from *Walkingame's Arithmetic*, the 1852 edition. I wonder what 'five and nine-thirteenths' taps look like?