



**Newsletter No. 43**

**May 2005**

I was recently given an old school textbook printed in 1882. It was Isaac Todhunter's *Plane Geometry*. Todhunter was a prodigious writer of texts for colleges and schools in all branches of mathematics, his titles include: *A Treatise on the Differential Calculus*, *An Elementary Treatise on Laplace's*, *Lamé's and Bessel's Functions* and *A History of the Mathematical Theory of Probability*.

What amazes me is the detail in his books. The one on plane geometry begins with a discussion of the derivation from the Greek of the words used, then goes on to describe how angles are measured (degrees, grades and radians). By page 15 he's covered all the trig ratios and by page 50 all the material included in our school curriculum. The book contains thousands of examples for exercise and many practical applications. It runs to 340 pages but is compact and hardbound. In one chapter Todhunter shows how performing a calculation in a particular way introduces an error in the 7<sup>th</sup> decimal place and explains how to improve the accuracy. In a chapter called 'Miscellaneous Propositions' he explores infinite series as they relate to the trig ratios,  $\pi$  and exponential functions. I remember books like these when I started secondary school. They had titles like *Elementary Geometry* that seemed anything but elementary and *A Shorter Algebra* - I hated to think what the longer version looked like.

Of course, these texts were aimed at the top echelon of students. I suspect that most of the contents would be completely beyond any students today unless they were specially prepared. What the consequences are, I've no idea. Maybe the level of trigonometry in Todhunter's book, solving those types of problems, is no longer necessary. Some of the trig identities we had to teach were always hard to justify. Maybe life goes on without them. After all, computers these days can iterate solutions far quicker than mathematicians can calculate them.

*We tend to think of multibase arithmetic as being a fairly recent topic in the curriculum but my father did it from Hall and Knight's Elementary Algebra in the 1920s and his father did it from Isaac Todhunter's Algebra in the 1880s!*

**R. A. Dear**

## INDEX

Diary dates  
Numeracy Reference Group Meeting  
Food for Thought  
Solution to April's problem  
Problem of the month  
Junior problem of the month  
Afterthought

### Diary Dates

The annual conference of the Mathematics Education Research Group of Australasia, MERGA, is scheduled for July 7-19, 2005 in Melbourne. Put your search engine onto MERGA 28 for further details.

Maths Week this year is on between 9 - 13 August. For more details see the NZ Association of Maths Teachers' site at [www.nzamt.org.nz](http://www.nzamt.org.nz). This site also has details from previous years. Apart from helping you to prepare for this year's Maths Week the site has material that you might find useful in your classroom at any time.

NZAMT Conference to be held at Christ's College, Christchurch from 27-30 September. 2005. For more details check out [www.nzamt9.org.nz](http://www.nzamt9.org.nz).

### Food for Thought?

What do we mean by  $2x$ ? Does it mean  $x + x$ ,  $2 \times x$ ,  $(x + x)$  or doesn't it matter?

Work through the following:

1. What does  $2x$  mean?
2. What is the value of  $2x$  when  $x = 3$  ?
3. What is the value of  $12 \times 2 \times 3$  ?
4. What is the value of  $12 \div 2 \times 3$  ?
5. What is the value of  $12 \div 2 \times x$  when  $x = 3$  ?
6. What is the value of  $12 \div 2x$  when  $x = 3$  ?

Did you get these answers?

1.  $2 \times x$
2. 6
3. 72
4. 18 (convention tells us to work from left to right)
5. 18
6. Most people give the answer 2, different to that of question 5. What does it tell us about  $2x$  and  $2 \times x$ ? Nothing is quite as simple as it first

appears, perhaps. Perhaps the moral of the story here is to use brackets everywhere where there might possibly be ambiguity.

## Numeracy Reference Group Meeting

We would be grateful for any comments on this meeting report.

**Introduction:** Probably most people know that the Numeracy Project is something that has been going for several years now. It started out as a pilot project using the Count Me In Too programme developed in New South Wales by Bob Wright. It then grew to a home-grown product that aimed to improve the numerical skills of primary students by helping teachers to better understand how children learned number. But it has now blossomed into a series of projects from primary school through intermediate to secondary school. These are the Early Numeracy Project for children in years 1 to 3; the Advanced Numeracy Project for years 4 to 6; the Intermediate Numeracy Project for years 7 and 8; the Secondary Numeracy Project for years 9 to 10; and Te Poutama Tau for children in years 1 to 10 of Maori medium schools.

A couple of times a year, the Numeracy Project's Reference Group meets and its first meeting of 2005 was held in Wellington on the 16<sup>th</sup> March. The Group's job is to review the Numeracy Project and to make recommendations for its future. The membership of the Group covers all aspects of education and consists of teachers and principals at all levels of schooling, people involved in teacher education, academics, and representatives of the ERO.

The agenda of last month's meeting consisted of reports from Jenny Young-Loveridge, Tony Trinnick, Gill Thomas, Joanna Higgins, and Kay Irwin on various research projects that they had been involved in relating to the Numeracy Project. The results of their work will appear in a publication from Learning Media later this year. (This will be announced in the July or August newsletter.) There was also an update on statistics relating to the Project by Malcolm Hyland and a report by Ro Parsons on its sustainability. But perhaps the most interesting aspect of the meeting was that the Associate Minister for Education, David Benson-Pope, attended for part of the day (more on that later).

**Statistics:** The Hyland statistics are fairly impressive and we list some of them below.

Teachers involved 2000 to 2004

Early Numeracy Project	7,800
Advanced Numeracy Project	5,750
Intermediate Numeracy project	475
Te Poutama Tau	335
Total	<hr/> 14,225

In the secondary arena, approximately 175 teachers completed exploratory studies in 2001 – 2003 and about 800 teachers participated in the 2004 “workshops for all”.

Teachers involved 2005

Early Numeracy Project	1,300
Advanced Numeracy Project	2,900
Intermediate Numeracy project	1,100
Te Poutama Tau	295
Total	<hr/> 5,595

About 400 teachers are taking part in the secondary pilot project this year.

In addition to the teachers involved, there are roughly 62 full time equivalent (FTE) facilitators and 3 FTE national and regional coordinators.

Overall this represents a budget of over \$6 million. It's nice to know that a significant amount of money is being spent on the development of mathematics in this country.

**Sustainability:** Ro Parsons of the Ministry presented a paper on the sustainability of the Project. As background she listed the strategic objectives of the Project. These are:

Improving the knowledge, skills and confidence of all primary teachers;  
Improving the achievement of all New Zealand students in mathematics;  
Improving the achievement of Maori and Pasifika students in mathematics; and  
Building the mathematics education community in New Zealand.

A good start seems to have been made in all of these and now it was necessary to ensure the sustainability of the Project and build on the gains that have been made. Ro then listed short-term and potential medium-term strategies. The short-term strategies are:

Reducing the facilitator/teacher ratio;  
Implementing a nationally coordinated approach to building the capability of facilitators;  
Developing the professional learning programme;  
Increasing the support of Numeracy lead teachers in schools; and  
Building the capability of teacher training institutions to respond to sustainability issues.

The medium-term issues are:

A coherent and connected approach to improving the quality of teaching and outcomes for all students in mathematics;  
A balance between decentralization and the maintenance of a national overview;  
The promotion of professional autonomy in mathematics education;  
Building professional leadership in mathematics; and  
Promoting the creation and dissemination of new knowledge in the area.

**Research findings:** We now look at some of the research findings. In due course the complete papers will be on this web site but you might be able to get copies of the work in progress from the authors.

Among other things, Jenny Young-Loveridge's work showed that the earlier students are involved in the Project the better they do. Hence it is important to start children as young as possible.

One of Tony Trinick's aims in his research was to examine the features of two schools involved in Te Poutama Tau who had made promising stage gains in the Number Framework. Features common to both kura were that they had experienced principals who participated in the professional development alongside their staff and the students were reasonably fluent in Te Reo.

One of the events that stimulated the implementation of the Numeracy Projects was the international TIMSS testing of 1995. The reaction to the apparently poor performance of New Zealand compared to other countries eventually led to the introduction of the Projects. So Gill Thomas devised a test using 24 TIMSS questions and gave it to 31 schools at Years 4, 5 and 8 who had participated in the Numeracy Project. Her data is presented below.

	Better	Similar	Worse
Year 4	16	6	2
Year 5	19	2	3
Year 8	6	15	3

Consequently these schools would have done much better on the TIMSS questions than we generally did in 1995.

In her research, Jo Higgins saw four orientations towards the use of equipment. These represent a progression from algorithmic, to external, conceptual, and finally to dialogical. Teachers are still shifting towards the conceptual and dialogical end of the scale. Equipment use is complex as it is dynamic in terms of the shifting balance of teacher and student responsibility for the task as well as its shifting place in the learning area.

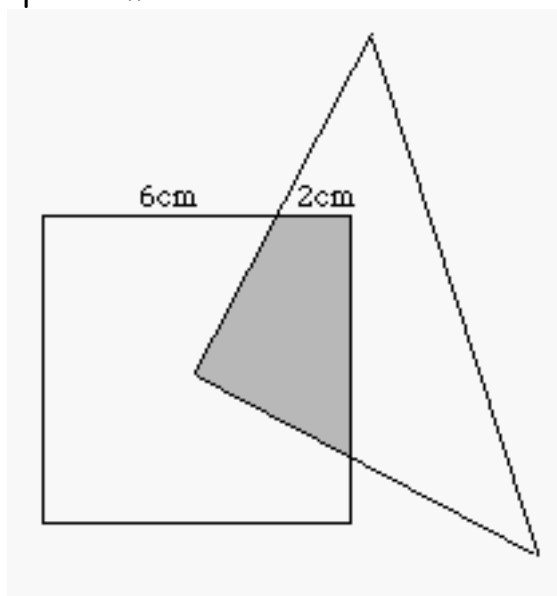
Kay Irwin spent some time with a teacher whose students were predominantly Pacific Island students. She found that the teacher's use of discourse was consistent with the pedagogy of the Project, that is that it emphasised the students' thinking rather than just working towards correct answers. There was evidence that the students used the same type of discourse between themselves when they were working on problems together. However, Kay found that there was less evidence that the Numeracy Project emphasised the use of correct mathematical terms and the presenting of complete evidence of the types that guide thinking in more advanced mathematics. She suggested that there be more emphasis on terms and logical explanations in the Project as she believes that this would increase the likelihood of success for all students.

**Associate Minister:** Finally and importantly there was the Minister. The morning of the Reference Group Meeting he had just got back from an OECD meeting in Italy. He went straight from the plane to parliament. Then he visited our meeting to make an announcement that the amount of money to be spent on the Numeracy Project at the secondary level was to be increased significantly. We were pleased that he was able to make time for us in his busy day. This was the first time in five years that a Minister had come to one of the Reference Group Meetings.

**Plenary session:** In the plenary session, the following recommendations (among others) were made.

That the October meeting of the Reference Group looks into the priorities for medium term strategies to progress and sustain the Project;  
That low decile schools continue to be supported;  
That reciprocal communication between teachers and students be emphasised;  
That publicity be given to the basic philosophy and theoretical underpinning of the Project and to what is effective teaching;  
That consideration be given to how the long-term sustainability of the Project can be effected in Maori medium schools;  
That professional development for teachers be ongoing;  
That a national picture of the performance of the Project over time continue to be taken;  
That the basic pedagogy of the Project be clearly enunciated.

**Solution to April's problem**



Last month we asked the following. The isosceles right-angled triangle shown has its shorter sides of length 10cm. The square has sides of length 8cm. If the vertex of the triangle is at the centre of the square, what is the shaded area (and why)?

If you rotate the triangle about the centre of the square, the overlapping area is unchanged. So rotate the triangle so that its arms are parallel to the sides of the square and the overlapping area is readily seen to be  $4 \times 4 = 16$  square centimetres.

What other shapes could we have used for this problem apart from a right angled triangle? How about a circle? And is what we did true for *all* right angled triangles? So should we have been careful to tell you more about the one in the diagram?

### This Month's Problem

The numbers  $a, b, c, d, \dots$  in the cells of square  $S_1$  are all either  $+1$  or  $-1$ . From  $S_1$  a new square  $S_2$  is formed (as shown) where each number in  $S_1$  is replaced by the product of the numbers in neighbouring cells (neighbouring cells have edges in common).

a	b	c
d	e	f
g	h	i

bd	ace	bf
aeg	bdfh	cei
dh	egi	fh

What happens if the process is repeated indefinitely?

We will give a book voucher to one of the correct entries. Please send your solutions to [derek@nzmaths.co.nz](mailto:derek@nzmaths.co.nz) and remember to include a postal address so we can send the voucher if you are the winner.

### This Month's Junior Problem

It has been suggested to us that we should run a monthly problem competition for students up to Year 8. So here is our first attempt. We'll give a \$20 book voucher to the first correct solution sent in to the problem below. And we mean solution; we won't accept just a 'yes' or 'no' or number answer. That is, we need some explanation of how the answer was obtained.

Amal has two bottles. One can hold 300 ml and the other 500 ml. Can she measure out exactly 100 ml? If not, why not? If she can, what is the smallest number of times she used the 300 ml bottle?

As with the regular Problem, please send your solutions to [derek@nzmaths.co.nz](mailto:derek@nzmaths.co.nz) and remember to include a postal address so we can send the voucher if you are the winner.

All students should also send parental approval for us to publish their names and school on the web site if they should win.

### Afterthought

You might have been wondering who on earth Laplace, Lamé and Bessel were and what they did. To find out you only have to go to the web site <http://turnbull.mcs.st-and.ac.uk/~history/>. This has a pretty good list of famous mathematicians and what they did – mathematically and otherwise. For instance, you will find that Laplace's father made his money in cider and that Laplace himself had to be a bit quick footed during the 1793 reign of Terror in Paris. Was the Reign of Terror why Lamé went to St Petersburg? Bessel wasn't French but what was his connection to the Königsberg Bridge Problem that Euler was fascinated by? And who got himself shot in a duel and why?

If the end of the world was to come on the first day of a new century, it couldn't be a Wednesday, Friday or Sunday. Isn't that a comfort!

What is not a comfort is that the triangle in April's problem might creep inside the square if it wasn't big enough to stay outside. Does that mean that the triangle had to have non-hypotenuse sides of length at least  $4\sqrt{2}$  or about 5.657 centimetres?