

Newsletter No. 42
I don't know about you but with all the great weather we've had over the summer my concentration span has only been working in short bursts. A lot of spare sunlit hours have been spent messing about at the beach or reading in the garden. Fortunately relatives had bought me a supply of anthologies for Christmas which I still hadn't read and which didn't require much sustained concentration. You know, one's full of obscure facts like how Aeschylus died apparently he was hit on the head by a tortoise dropped by a passing eagle - or that copper bowls are, for some reason, best for whisking eggs. Two of the books had information about dice, some of which are worth passing on. Did you know, for example, that there is a standard die? Most of us know that opposite numbers on a die sum to seven but there is more to it than that. The standard die has the numbers 1, 2, 3 anti-clockwise around a vertex. See below for some activities related to dice, standard and otherwise.

Writing about dice reminds me of something I once overheard in a classroom (my own!).

Teacher: "If you threw a die 60 times which number would occur the most?"
Student: The number 3 because it's nearer the middle.

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## What's new on nzmaths

There are 8 new units of work in the Measurement and Geometry sections of the site this month:

- The Gingerbread Man, Measurement, Level 1
- Weighing Stations, Measurement, Level 2
- Calibrating Clocks, Measurement, Level 2
- Fold and Cut 2, Geometry, Level 2
- Now you see me, now you don't, Geometry, Level 2
- Quadrilaterals, Geometry, Level 4
- Cos rule, Geometry, Level 6
- I'm so sorry I ate chocolate, Geometry, Level 6

We are currently working on translating the Copymasters for the Problem Solving activities into Māori. All level 1-3 problems now have both English and Māori copymasters.

The Numeracy Project Material Masters have also been translated into Māori, and are available from the Numeracy Project section of the site.

## Diary Dates

As we mentioned last month, the annual conference of the Mathematics Education Research Group of Australasia, MERGA, is scheduled for July 7-19, 2005 in Melbourne. Put your search engine onto MERGA 28 for further details.

27-30 September. 2005: NZAMT Conference to be held at Christ's College, Christchurch. For more details check the website www.nzamt9.org.nz/ or look at last November's issue (no. 39) of this newsletter.

## Dicey Doings I: How Many Dice?

There's a lot that can be said about standard die. Perhaps first of all you might want to know that standard die have been standard for centuries. But even before that die were used way back, certainly by the Romans. They would square off the sides of a bone so that it was a relatively long thin 'box' shape. Initially this only gave them die that had four possibilities (try rolling one of these onto its small end!). And, of course they had to have the end shape square to give all the numbers a fair chance.

But then some genius came up with a cube shaped gambling instrument. And that caused a few problems. If you were going to put a different number on each face couldn't you have rather a lot of dice? In fact you can have 30 of them. To see this, first of all make a blank cube. Now when you start off it doesn't matter where you put the 1 because all faces are equal. But we can put 5 different numbers on the side opposite to the 1 . So we have 5 potentially different dice already.

Again because of symmetry, the next number we add to the cube can be put anywhere just hold the cube between your finger (where the 1 is) and your thumb (where the second number went) and rotate it. Now you can bring any face towards you. So the third number can be put on any face you like. Put on the third number. Now the next, fourth, number can be put on any of the 3 empty faces. So far then we've made $5 \times 3$ choices. There are 2 choices left for the fifth number, no matter where the fourth number went. That gives us $5 \times 3 \times 2$ choices so far. But since there is only one face and number left, that number goes where it has to go.

Right, so there are $5 \times 3 \times 2=30$ possible dice. You might like to get your class to make them all.

## Dicey Doings II - How Many Proper Dice?

But we don't like most of those 30 dice we've produced above because opposite sides don't add up to 7. Why do we want the opposite sides of dice to add up to 7? Probably because we want dice to be balanced. If you put all of the big numbers on one 'side' of the die, then, especially if you are gouging out holes in a bone die, you might find that the die is unbalanced and falls onto one side more often than not. Surprisingly, or maybe not surprisingly, a very short time after die went cubic, they also went for the opposite sides to add to 7 .

So that then means we have to ask how many dice have opposite sides adding to 7? If you have now got a class set of 30 different dice you can quickly count the ones that have this property. If you don't then you'll have to work it out - but it's not too hard. You've been through it all before.

The 1 goes anywhere you like because every face is the same at the start. This forces the 6 to go opposite the 1's face.

Where does the 2 go? Holding the cube between your thumb (the 1 face) and forefinger (the 6 face), you can see as you did in Dicey Doings I above, that all the faces are now the same again, so we can put 2 on any face we like. So 5 goes on the face opposite to that.

Note that so far there have been no differences - you have been forced to do what you have done. But now we get a choice for the first time. Holding the cube in your thumb and forefinger as before, turn the 2 face towards you. Where can the 3 go? Well you now have a choice. You could put it to the left of the 2 or to the right of the 2 .

OK then there are two possible dice with opposite faces adding to 7 . One is the standard die and the other isn't.

But we have to be careful about this 'standard' business. The die that we've called 'standard' is only standard in the West. Once again, for centuries the East and the West have chosen different ways to make their standard dice. So if you have a die that was
made in the East it almost certainly isn't the die that we've called 'standard' above. Because more dice are made in the East (probably because they are cheaper), then you may find more of your dice are not standard. (But how does that explain the comment in Question 1 below? Perhaps I've largely been given expensive games!)

## Dicey Doings III - A Quiz

1 Can you say which of these two dice is the standard one?


I checked with my own dice and found that only about three-quarters of them were standard!

2 Seven is the most common total uppermost when two dice are thrown. Which is the better bet? That you will score seven twice with two rolls of two dice or that you will score totals of six and eight?

3 When two dice are thrown which is more likely; that a one or six will come up or that neither will come up?

4 Here are two unusually numbered dice:

$$
\begin{array}{ll}
\text { Dice 1: } & 1,2,2,3,3,4 \\
\text { Dice 2: } & 1,3,4,5,6,8
\end{array}
$$

You want to play a game that requires the usual totals in the same proportions when two dice are thrown. Can you use these dice?

5 How can two cubes be labelled, with each side bearing a number from one to six or left blank, to make a pair of dice that will throw a sum total of one through twelve with equal probability?

Answers in Afterthoughts below.

## Dicey Doings IV - A Strange Die (or two)

The last question causes me to think. Can you produce two dice whose sums are 1, 2, 3, $4,5,6,7,8,9,10,11$, and 12 ? What's more can these sums occur equally often? And can we use numbers other than 1 to 6 ?

If this has to happen, then there are a few things that are implied. You can read about these in Afterthoughts.

Checking out probabilities like this can be best done by using a table. Put the numbers on the faces of the first die across the top and the numbers on the second die down the side. In the body of the table go the sums of the respective numbers. You can see what happens for a normal pair of die in the next table.

|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 2 | 3 | 4 | 5 | 6 | 7 |
| $\mathbf{2}$ | 3 | 4 | 5 | 6 | 7 | 8 |
| $\mathbf{3}$ | 4 | 5 | 6 | 7 | 8 | 9 |
| $\mathbf{4}$ | 5 | 6 | 7 | 8 | 9 | 10 |
| $\mathbf{5}$ | 6 | 7 | 8 | 9 | 10 | 11 |
| $\mathbf{6}$ | 7 | 8 | 9 | 10 | 11 | 12 |

You might like to finish off the peculiar dice problem by yourself. For some hints you could try the web site Problem Solving, Level 5, Lara's Equiprobable Dice I.

By the way, you might like to know that $0,0,0,3,3,3$ is one of a pair.

## More Mind Reading

We left you to look into the mind reading problem from http://digicc.com/fido/. This problem gets you to play around with 3 or four digit numbers. It then asks you to leave out a number and it tells you what the number is.

If you haven't tried it give it a go now. If you have, then go to the seminars section of the website. You can get there via Info Centre and then Seminars. From there the Mind reading seminar will tell you all you want to know.

## Solution to March's problem

This was last month's problem. While Jenny was in the shop she exchanged a one dollar coin for six other coins. On the way home she lost one of them down the back seat of the car. What is the probability that the lost coin was a 10 cent piece?

There are three ways Jenny could change her dollar coin for six other coins:
$650 \phi, 20 \phi, 10 \phi, 10 \phi, 5 \phi, 5 \phi$
$750 \phi, 10 \phi, 10 \phi, 10 \phi, 10 \phi, 10 \phi$
8 20ф, 20ф, 20ф, 20ф, 10ф, 10ф

Without any further information we will assume these are equi-likely, i.e. each of these outcomes has probability $\frac{1}{3}$. Drawing up a probability tree to show the possible outcomes when a $10 \phi$ coin is chosen (in this case lost) we have,

giving the probability that Jenny lost a $10 \notin$ coin as,

$$
1 / 3 \times 2 / 6+1 / 3 \times 5 / 6+1 / 3 \times 2 / 6=9 / 18=1 / 2 .
$$

This was clearly much too hard as we had no solutions sent in. Maybe we should give more questions on probability to save our voucher money.

## April's Problem

The isosceles right-angled triangle shown has its shorter sides of length 10 cm . The square has sides of length 8 cm . If the vertex of the triangle is at the centre of the square, what is the shaded area (and why)?


## Afterthoughts: <br> Revisiting February's problem

Last month we included some solutions to February's problem. The original problem was:
Using the digits $2,0,0,5$ in that order and any of the operations,,$+- \times, \div, \sqrt{ }$, indices and factorials, with brackets and concatenation also allowed, how close can you get to the value of $\pi$ ?

Some examples are: $2+0!+0^{5}=3$. Well, it's a start! It represents an error of approximately $4.51 \%$. $\sqrt{ }(\sqrt{ } 200-5)=3.0236$ ( $4 \mathrm{~d} . \mathrm{p}$.) is better.

The only solutions received were sent in by Derek Smith of Lower Hutt. He actually gave four but although they were close to the value of $\pi$ they did not conform to the conditions of the problem. Still, as they were the only entries we gave him the prize anyway. His 'attempts' used, for example, the Integer Value Function, Int, which keeps only the integer value of a number. It removes the decimal part of the real number, e.g. $\operatorname{Int}(5.64)=5$. Elsewhere he used the determinant of a matrix, the exponential function $\mathrm{e}^{\mathrm{x}}$ and the inverse tan function $\tan ^{-1}$.

Perhaps you'd like to have another look at the problem and find a value as close as you can to $\pi$ using only the conditions stated above.

## Answers to Dicey Doings III

1. The $1,2,3$ corner on a standard die (and the $4,5,6$ corner) is numbered anticlockwise. So it's the die on the right.
2. Oddly enough, 6 and 8 is more likely because it can be scored in two different ways - the 6 first then the 8 and vice versa.
3. From the table above you'll see that 1 and 6 is more likely.
4. These dice will do the job. They produce sums from 2 to 12 in the same proportions as standard dice. As far as I know, they are the only other dice to have this property. Can this be proved?

|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 2 | 3 | 3 | 4 | 4 | 5 |
| $\mathbf{3}$ | 4 | 5 | 5 | 6 | 6 | 7 |
| $\mathbf{4}$ | 5 | 6 | 6 | 7 | 7 | 8 |
| $\mathbf{5}$ | 6 | 7 | 7 | 8 | 8 | 9 |
| $\mathbf{6}$ | 7 | 8 | 8 | 9 | 9 | 10 |
| $\mathbf{8}$ | 9 | 10 | 10 | 11 | 11 | 12 |

5. There are 36 outcomes from two six-sided dice, so if sums from 1 through 12 are to be made with equal probability each must be made in three ways. The only way a sum of 12 can be made three ways is with a six on one die and three sixes on the other. The only way a sum of 1 can be made three ways is with a one on one die and three zeros on the other. Thus there is only one solution to the problem - one of the dice is standard and the other has three zeros and three sixes.
