

Newsletter No. 40

February 2005

Nga mihi nui o te tau hou. Welcome to 2005. We hope the year brings you your dearest wishes and, of course, many successful new teaching ideas. It's our plan to include lots of the latter on the NZ Maths website.

In the February 2003 issue of this newsletter we began by mentioning the four 4s problem. You know the one where you use four 4s and a prescribed set of mathematical symbols to express as many whole numbers as possible from one upwards. It's an excellent way to revise order of operations and so on and popular with pupils.

For example, $1 = 4 - 4 + 4 \div 4$ $2 = 4 \times 4 \div (4 + 4)$ $3 = (4 + 4 + 4) \div 4$ and so on.

These days a more common version of the problem is to use the digits of the current year, in order, instead of the four 4s. How successful you are at the puzzle, of course, depends on the operations allowed and your own persistence. The operations are usually taken as $+, -, \times, \div$, indices, and brackets. For higher numbers concatenation (placing digits side-by-side, like 20) is allowed and occasionally factorial notation. I wonder if 2005 will present much of a challenge. Let's make a start ...

 $1 = 2^{0} + 0 \times 5$ $2 = 2 + 0 + 0 \times 5$ $3 = 2 + 0! + 0 \times 5$ $4 = (20 + 0) \div 5$ $5 = 2 \times 0 \times 0 + 5$ (by definition and for consistency 0! = 1)

How far can you get?

An alternative version of the puzzle is included as this month's problem below.

'The moving power of mathematical invention is not reasoning but imagination.'

Augustus De Morgan

What's new on nzmaths.co.nz

We have been very busy over the summer holidays adding plenty of new content to the site. In the Numeracy section of the site we have added several more equipment animations in both English and Māori, as well as adding new Material Masters and Numeracy Project booklets. We have also added links to most of the units in the Algebra section of the site, describing how you can emphasise the Numeracy concepts within them.

There have been many new units added to the site. In particular, look in the Measurement and Geometry sections of the site to find new material. There will also be several new units added in the first week of February, so keep checking back!

Brilliant Bryson

I have to say that, despite the fact that we live in Dunedin, none of our grandchildren have the same advantage: two are in Christchurch and one is in Sydney. Now my wife hasn't yet managed to work out how to get to Sydney several times a year but we do find ourselves travelling the 360 km between D and C at least once a month. Of course it's good getting there but the driving can be a bit of a bore. Now it's not that we don't have much to say to each other – we clearly don't, but it is nice to have something to while away the 5 hour trip. The 'something' that we have cottoned on to is talking books. Stick them into the machine on the dashboard and the passenger can sit back and relax till we hit a fast food factory (we want to minimise the time on the road and maximise the time with the children), eat and change drivers.

The last pilgrimage was only a few days ago and the talking book was Bill Bryson's "A Short History of Nearly Everything". Now I'm not sure whether or not I am allowed to advertise on this site but I've done it anyway. The amazing trick that he has pulled off in this book is to make all that science stuff that is so hard, and at school and university even dry, interesting and alive. My wife is not a scientist and I'm only a mathematician, and we certainly didn't understand everything that he was talking about but we were taken in from beginning to end.

Among other things (actually is there anything else?), the book covers the Universe and the Big Bang, the Solar system, the Earth and geological forces, the development of Homo Sapiens, Genetics and the cell, and the structure of the atom. Bill takes us on a leisurely history of it all showing how our knowledge in each area developed and who the prime movers were along the way. On the road we find out tangential things such as the fact that Newton's hair has been analysed and was found to contain 40 times the normal amount of Mercury. Did you know that two famous women scientists died of cancer? One was Mme Curie who did much of the early work on X-rays. It's not surprising given the amount of radioactivity that she was exposed to. You can get some idea of how much when you hear that her notebooks are still putting out radiation and cannot be touched. As for the second woman, she died of ovarian cancer. I'll let you find out who this was and what she was doing to contract this disease. Probably you knew

that Einstein worked as a second class patent clerk in Switzerland but did you know that he had trouble being promoted? I guess the five ground breaking papers he had published in one year weren't of much interest to his bosses. However, they did send physics racing off in new directions and the one on the photoelectric effect earned him a Nobel Prize and made television inevitable. (I assume that was a good thing – the TV not the Nobel Prize.)

Now I'm not that ancient but I remember in first year geology being told that there was this crazy idea around that all the continents had been joined at some time. It's amazing now to think that it was only in the 60's that the lads really started to get that idea on a sound footing. The interesting thing about that is that it took the old geological guard some time to accept it. As a result of the theory of tectonic plates we now know what caused the horrendous 'Christmas' tsunami even if we have no way of stopping it. Just to cheer you up even more, of course that's not the only thing that is hard to stop. If a lump of space junk decided to head this way we might find ourselves in the same pickle as the dinosaurs. A collision with Earth may not necessarily break up the planet but the fall out from the explosion would make it hard to see the sun for a while and that might make it hard to grow the odd sheep or a few vegies. And you can't take any comfort from the fact that New Zealand was the least affected country during the cataclysmic dinosaur episode. We were just lucky that the impact site was in the Northern Hemisphere. Oh and don't think that we will all be saved by some heroic astronaut and a dose of nuclear bombing. We don't have the hardware. Even if we had sufficient warning to intercept the impending debris (a week or so would be far more than we could expect), we don't have any spacecraft that could get us up there and far enough away from the Earth. The last one vehicle retired with the Moon programme. And while we are on the subject of disasters, did you know that it was the same person who contributed both lead to our petrol and CFCs to our cans?

I'm afraid that there's not much maths that sneaks in though. Sure there's a passing reference to the maths of radioactive decay related to the age of the Earth but little more. The big problems of maths don't get a mention. But maybe maths is harder to make interesting than physics (did I say that?).

I can't think of the last book that I 'read' twice, back to back. But I'm about to start off again on Bryson's Short History again. There is bound to be something that I missed when I was concentrating on overtaking. I'd thoroughly recommend it to you as highly entertaining and informative too. My first year maths class is going to see it on their reference list AND THEY WILL ENJOY IT!

Mindreading

There are several sites on the internet that claim to be able to read your mind. We've listed two of them in one of our staff seminars

(<u>http://www.nzmaths.co.nz/HelpCentre/Seminars/mindreading.htm</u>). We've shown there why one of them isn't mind reading at all but a very nice trick involving the properties of 9. Suppose we want to know if 38910348476 is divisible by 9. All we have to do is to

add up all the digits of the number. If the sum is divisible by 9, then so is the original number.

Let's see this in operation with the number above. 3 + 8 + 9 + 1 + 0 + 3 + 4 + 8 + 4 + 7 + 6 = 53. Now 9 doesn't go into 53 so 9 doesn't go into 38910348476.

But the interesting thing now is that when you divide 53 by 9 you get 5 and 8 left over. Now try dividing 38910348476 by 9. You'll find that the remainder is 8. To see more about this have a look at

<u>http://www.nzmaths.co.nz/Number/Operating%20Units/guzzinta.htm</u>. It should tell you all you want to know about divisibility by 9.

And the same thing holds for divisibility by 3. Given a number if you add up all the digits and divide that sum by 3, whatever the remainder is it will be the same remainder as in the original number.

Now the trick goes even further. If you are working in base 8, you can see if any number is divisible by 7 (one less than 8) by adding up all the digits in that base and dividing that sum by 7. And for base 21, the same test works for 20, and so on.

In the Mind Reading staff seminar we've shown how the first mind reading trick works. Can you see what's going on with the second one at <u>www.albinoblacksheep.com/flash/mind.php</u>? We hope that we'll be able to use someone's explanation in the next newsletter. Send it in to <u>derek@nzmaths.co.nz</u>.

All About Pi

This month's problem is about pi (or π). Just in case you feel that you want to learn a little more about that famous number – the ratio of the circumference of any circle to its diameter – then you could do worse than read "A Biography of the World's Most Mysterious Number" by Alfred S. Posamentier and Ingmar Lehmann (published by Prometheus books, Amherst, New York).

This book would certainly be good reading for a bright young student. It covers just about all you can think you might want to know about π including my favourite quick method for justifying that the area of a circle is πr^2 . To do this divide a circle up into a large number of equal sectors and then lay them out as in the diagram. This diagram clearly has the same area as the original circle.

If there are a large number of sectors, then they roughly form a rectangle with the long side of length a half of the circumference and the short side of length r, the radius of the circle. So we find that the area of the circle = $\pi r.r = \pi r^2$.

You might be worried about the little inaccuracies there but as the number of sectors increases, the shape of the diagram gets closer and closer to a rectangle and the whole thing works well in the limit.

As for the rest of the book here are some questions that it will answer:

- How close did Archimedes come to a value of π and how? (Not far and by approximations.)
- How many places did Zu Chongzhi get? (Quite a few but we don't know how.)
- Who gave the letter π to this ratio? (Confine your attention to the 17th Century.)
- Which State tried to legislate the value of π and why? (We're at the end of the 19th Century now.)
- What has π got to do with needles or should it be what have needles got to do with π? (Think Buffon.)
- How can Pythagoras' Theorem be extended to semicircles? (Try it and see.)
- When is π -day? And what time on that day is most auspicious? (You need to be an American to work this out.)
- What's so interesting about the 360th digit of π ? (You need to know the relation between π and 360 degrees.)
- Put a rope around the equator. How much longer will be another rope that is a metre all round above the equator? (Just a quick calculation plus a surprise.]

Well apart from this, the authors have listed π to 1.24 trillion places! It only took a Japanese team about 600 hours. You might ask why and how. Well this is a good exercise to test out the speed of a new computer. And, as for 'how' we'll let you read the book to find out. Suffice to say that there is more than one way.

Solution to November's problem

Our usual plan at this point is to go carefully through the solution of the problem from last month and announce the winners of the prize. Unfortunately there were no solutions, correct or incorrect, sent to us in the three months since the November newsletter was put on the web. We can only put this down to the fact that there was too much pre-Santa excitement in the air. Anyway, as a result we have decided to give you all one more month to solve the problem. So we will have two prizes on offer this month. Of course, through the haze of Christmas you may have forgotten what the problem was. So we repeat it below.

You were asked to find the 465th term in the sequence given by the arrangements of the digits 1 to 6 when they were put in numerical order from least to highest.

The smallest is 123456 and the next 123465. Continuing we have 123546, 123564 and so on. The first 120 numbers in the sequence begin with 1 followed by the 5! = 120 arrangements of 23456. So what is the 465^{th} term?

This Month's Problem

Moving on from the four 4s and the 2005 problems mentioned above we set you the following challenge. Using the digits 2, 0, 0, 5 *in that order* and any of the operations $+, -, \times, \div, \sqrt{}$, indices, and factorials, with brackets and concatenation also allowed, how close can you get to the value of π ?

For example, $(20 + 0) \div 5 = 4$ which is a start but a poor one. Somewhat better is $\sqrt{(20 + 0 - 5)} = 3.87$ (2 d.p.) which is approximately 23.3% in error. Can we get any closer?

Well,	$\sqrt[3]{(200 - 5!)} = 2.99$	(% error 4.8)
and closer still,	$\sqrt{(\sqrt{200} - 5)} = 3.02$	(% error 3.9)

I'm sure you can do better.

[For those who have forgotten or not come across them, factorials arise from the mathematics of arrangements. The factorial symbol '!' denotes the product of a number and all the whole numbers below it down to one. Thus $3! = 3 \times 2 \times 1 = 6$ and $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$. They arise when numbers of arrangements are needed. For example, in how many ways can the letters of the word CAT be arranged? Listing them gives $6 = 3 \times 2 \times 1 = 3!$. This is because there are three choices for the first letter, two for the second and then there is only one left for the third. How about the number of arrangements of the letters of the word MATHS? Listing and counting them is time-consuming but there are 120. A quicker method is to note that there are five letters for the first choice. When that has been taken there remain four choices for the second, giving 5×4 choices for the two letters. There are three choices for the third letter, two for the fourth and one for the fifth giving $5 \times 4 \times 3 \times 2 \times 1 = 3!$ altogether. Remember, by definition and for consistency 0! = 1.]

We will give a book voucher to the answer closest to π under the given conditions. If there are two equally close we will choose the one that is the simplest, i.e. that uses the

fewest mathematical symbols in an elegant way. Please send your solutions to <u>derek@nzmaths.co.nz</u> and remember to include a postal address so we can send the voucher if you are the winner.