Newsletter No.4 **June 2001**

This month we get a continuation of the decimal discussion started in our last newsletter and get some thoughts on patterns. Our contributor here is Kaye Stacey who is Foundation Professor of Mathematics Education at the University of Melbourne. She and her students have been involved in research into students' understanding of decimals for some while now. There is a great deal of useful information on their website http://online.edfac.unimelb.edu.au/485129/DecProj/index.htm.

Bruce Moody has provided us with another contribution this month. This time it is about patterns. Many thanks to Kaye and Bruce for taking the time to produce something for this newsletter.

In addition to the above, we have the usual regular features. And we'd be glad to hear from you on anything that has been raised or on anything that you would like to raise.

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What's new on the nzmaths site this month?

On the site this month we have added a number of problem solving lessons for levels 4, 5 and 6.

Web Statistics: In each newsletter we include some statistics from the previous month. In May:

There were 17, 843 distinct users. A distinct site is either a single computer or a school connecting to the website. A user is only counted once per month, regardless of how many times they come back to the site.

Traffic from the USA now accounts for 16% of the usage of the NZMaths site.

There were 127,175 pages of information downloaded. This is 2,581 megabytes of information.

What’s the point • ? continued by Kaye Stacey

In the last newsletter, Bruce Moody has pointed out that there is a strong link between the way that students read decimal numbers and the way that they think about them. For instance, they read numbers such as 0.75 as "oh point seventy five" and, as a result, believe that 0.75 is larger than 0.8 because they compare the whole number 75 with the whole number 8. They conclude that 0.75 > 0.8 because 75 > 8.

Research in many countries finds that many children, especially in upper primary school, think like this. Like Bruce, in our Australian research, we have also found that children who say "oh point seventy five" rather than the correct "oh point seven five" are especially likely to have this misconception and we advise our teachers to use this as a warning sign to investigate how a student is thinking about decimals.

However, there is no necessary connection between pronouncing 0.75 as "oh point seventy five" and having misconceptions about decimal numeration. We know this because in some languages, this is the correct way to pronounce decimal numbers. In Norwegian, for example, the correct way to pronounce 0.75 is "oh point seventy five" and the correct way to pronounce 0.1234 is "oh point one thousand two hundred and thirty four" , although in Norwegian, of course. But Norwegian children are no more likely to misunderstand decimals than Australian children are.

In Australia, about 30% of children aged about 10 - 12 interpret the numbers after a decimal point as another set of whole numbers and so conclude that 0.75 is greater than 0.8 etc. This is certainly partly due to using money to teach about decimal numbers. When we think about money, we think about two sets of whole numbers: the dollars and the cents. Understanding that the cents are part of a dollar is not the foremost idea, yet this is what is needed for learning about decimals. The important idea is to establish the "one" or the "unit" very strongly in students' minds and to ensure that the decimal parts are interpreted from the place value. Bruce's suggestion to use measurement is a good one, but remember always to first establish strongly what the unit is. For concrete materials, we use LAB, where the unit is a very long piece of pipe - it makes a big impression on students.

Another difficulty with using money too much is that students mix up the tens and the tenths: so, for example in $6.95 the 9 is 90 (9 tens) cents but 9 tenths of a dollar.
Another consequence of too much association of decimals with money is that some students (even adults) cannot tell whether 2.54367 is greater or smaller than 2.54 because they believe they are both really the same. They would represent the same money. These students do not have ANY understanding of place value: they believe that the decimal numbers beyond the second place have no meaning and are just errors. Poor teaching about rounding can exacerbate this.

We noted above how many students think longer decimals are larger numbers. You may be more surprised that we have found that about 10% of students of all ages throughout secondary school actually think longer decimals are smaller numbers. So, in Bruce's test above, these students could conclude that 0.75 is less than 0.8, but they would also conclude that it less than 0.4. Sometimes this is because they know decimals are another way of writing fractions and so they think 0.75 < 0.4 because 1/75 < 1/4. Sometimes, they associate decimals and negatives so they think 0.75 < 0.4 because -75 < -4. Sometimes, they use a little place value knowledge and so they think 0.75 < 0.4 because 0.75 is 75 hundredths, 0.4 is 4 tenths and hundredths are less than tenths. Look for these students in your classes by asking them to compare numbers like 0.75 and 0.4 or 2.578 and 2.3. And test out 0.30 and 0.3 as well.

There are many activities that teachers can use to focus students' attention on the meaning of decimal numbers. Analysing the numbers in terms of place value is fundamental and integrating knowledge of fractions, decimals, whole numbers and negatives is important. Don't always teach them in isolation. There are many teaching ideas on our website http://online.edfac.unimelb.edu.au/485129/DecProj/index.htm. This is still under construction, so we are keen to be sent corrections and suggestions.

Books on decimals

You might be interested in the two books relating to decimals that are available from: Teaching Decimals Project, Department of Science and Mathematics Education, University of Melbourne, Parkville, VICTORIA 3010, AUSTRALIA. These books are:

Lesson Ideas and Activities for Teaching Decimals by Caroline Condon and Shona Archer
Linear Arithmetic Blocks: A concrete model for teaching decimals by Shona Archer and Caroline Condon.

The books contain short lesson plans and photocopy masters. The material is suitable for ages 10 - 14. The cost of the two books, along with packing and postage costs to New Zealand, is NZ $40. (The lessons can also be downloaded from the website, but the books are a more convenient form in which to have the material.)

Diary Dates

3 July to 6 July: Wellington 2001: A Maths Odyssey (NZ Association of Mathematics Teachers Conference No. 7)
Information can be found on www.nzamt.org.nz .

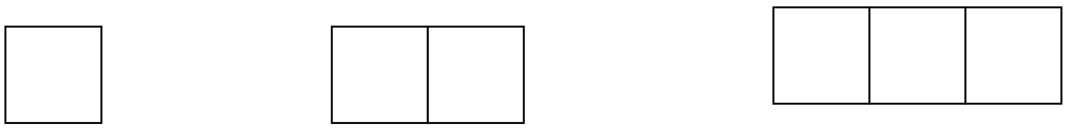
27 July: Closing date for Applications for the New Zealand Science, Mathematics and Technology Teacher Fellowships.
Information, can be found on http://www.rsnz.govt.nz/awards/teacher_fellowships.

13 August to 17 August: Mathematics Week
Information can be found on www.nzamt.org.nz .

25 December: Christmas Day
We believe this is a holiday. We hope to have more information soon.

Match the Patterns by Bruce Moody

One common form of representing patterns involves using matches to make up a repeating unit, and then asking children to work with their observations regarding the number of matches used. The example given below comes from MINZC p144:



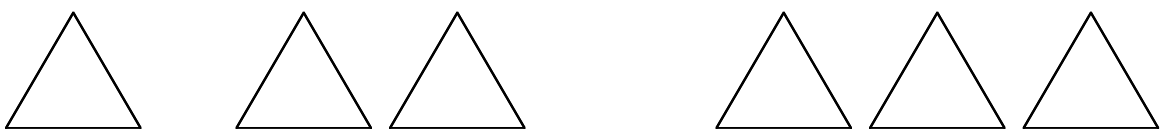
4 **7** **10**

"*Predict the number of lines needed for 20 squares, and then graph the sequence .*"

The difficulty for students is seeing the constant. The first match is an integral part of the first square, and is hard to see that it is different from the other sides. Visually, the left edge of every other square is identical, but in terms of the pattern they are not treated the same.

A modified approach takes this kind of work in two stages.

Stage 1 Working with Discrete Shapes



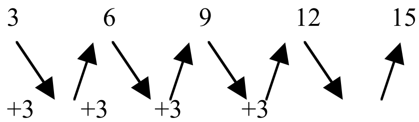
3 **6** **9**

3, 6, 9, … What is the pattern? *Going up in 3s.*
Why does it go up in threes? *We use 3 sticks for each triangle.*
How many will we use for 4 triangles? For 7?
What will the 15^{th} number be? Get them to verbalise the fact that they could perform a multiplication to obtain the answer.

At this stage we should also be introducing the way to represent this information on graphs, and using those graphs to make predictions about how many matches are needed for a given number of triangles. We can also start informally solving equations; how many triangles would there be if you used 45 matches? This can be solved both by using the graphical display and by linking in with division as sets of three.

Having the one context for patterning, graphing and equations helps reinforce the concept that the sections of algebra are related.

Get the students to look to see how the numbers in the pattern are related. Students are often confused between what they see as *adding* three and the teacher's insistence that this means that they must *multiply* by three. What we would like students to do is to make the connection between going up in 3s and the 3 times table. Students usually learn multiplication as an extension of skip counting, but once the tables are mastered many seem to lose that link.

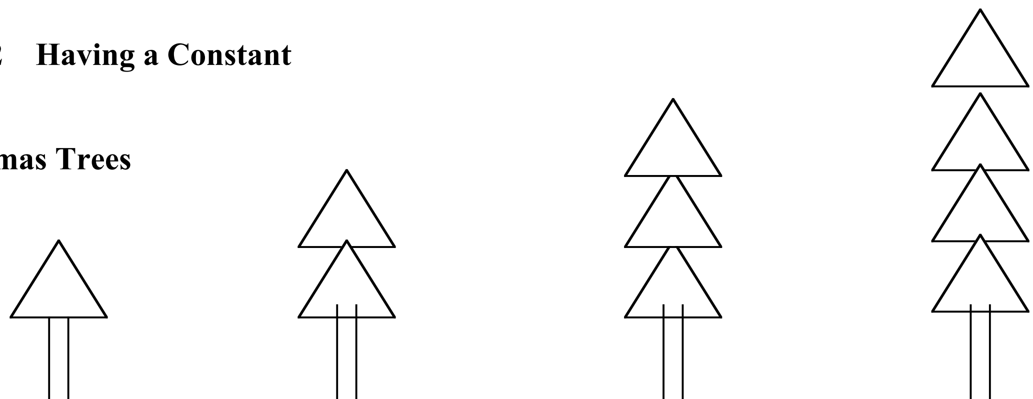


The repeated addition of 3 gives us the pattern of the 3 times table.
It is this linkage that is often omitted in the difference method of writing rules and thus the chance to build upon prior knowledge is missed.

Repeat the concept with squares, and other discrete shapes, until students can tell you what is happening each time.

Stage 2 Having a Constant

Christmas Trees



Sets of Branches	1	2	3	4
Matches Used	5	8	11	14

Here the constant is clearly visible and identifiable as the unchanging part of the pattern.

What changes each time? Why does it change? How is it changing?
What stays the same? How does it fit into the total number of matches used?
What we are looking to establish is that we are adding triangles as previously, and so 3 more matches are required each time. This needs to be linked with the 3 times table. The trunk remains the same each time and requires 2 matches and this 2 is included in each total. That is why the pattern has the same properties as the 3 times table, but each number is 2 more than its equivalent in the 3 times table.

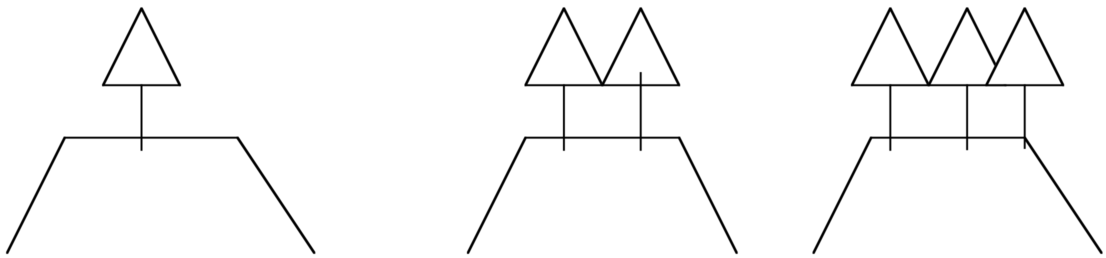
Predictions concerning what would be needed for 5 or more sets of branches should be made. Students should be encouraged to write their rule in words, speak it out, and test it.

Graphing and solving equations should also be incorporated with these patterns.

From the question "if I used 32 matches, how many sets of branches are in my tree?" it is a logical step that the two matches used for the trunk should be removed before tackling the number of triangles. *Get rid of 2 'cause that's the trunk; then find how many 3s there are in 30 and that gives you the number of branches.*

I think that students will make more sense of the general procedures for solving equations if their first experiences of handling rules like “remove the constant before dividing” are ones they create with their own language.

Here is another example: **Maungakiekie**



One tree hill — 7 lines

Two tree hill — 11 lines

Three tree hill — 15 lines

Once students have worked through the pattern, focus their attention on how examples like this are made up; i.e. they have a repeating section and a fixed section.
Get students to create their own problems and get them written on cards.
i.e. they have a pattern with several examples and then ask questions like “how many lines/sticks would you need for the next drawing/model?”, “how many would you need for the 6th one”, “can you write the rule?” The creator's answers could be written on the back of the card for checking. These cards can be used for other students to work on in the next lesson, and creating such questions gives a good indication of whether understanding of the principles has occurred.

While not essential to the level of algebra I would aim this work at, it is interesting to compare the treatment of the constant when these patterns are graphed. With the MINZC example of joined squares, we are placed in the unfortunate position of suggesting that 0 squares use 1 match! With the discrete shapes, 0 triangles require 0 matches. A Christmas tree with no branches still has a trunk of 2 matches. A zero-tree hill still needs 3 lines to represent the hill.

The contributor: This article was contributed by Bruce Moody. Bruce has been a Secondary Teacher before working as the Secondary Maths Adviser in the Waikato/Waiariki region. He is currently employed by the Ministry of Education for a special project aimed at improving numeracy in the Rotorua area.

We welcome contributions like this.

The May Solution

If you remember, last month we posed the following problem.

Four prisoners are placed so that they can only look straight ahead of them in the direction of the wall. (We show the way things are in the diagram.) They are all waiting to be executed (or forced to cook tea or something equally gruesome). They know that, between them, they are wearing two black hats and two white hats. If any one of them can say what colour hat they are wearing they will all be saved (or given takeaways or something equally happy). Which one shouts out and why?

A B C D

We have had two entries at last! What's more they where correct ones. To celebrate we are giving petrol vouchers to both entrants.

Congratulations and the petrol vouchers to Cathy and Faye. Cathy's solution is as follows:

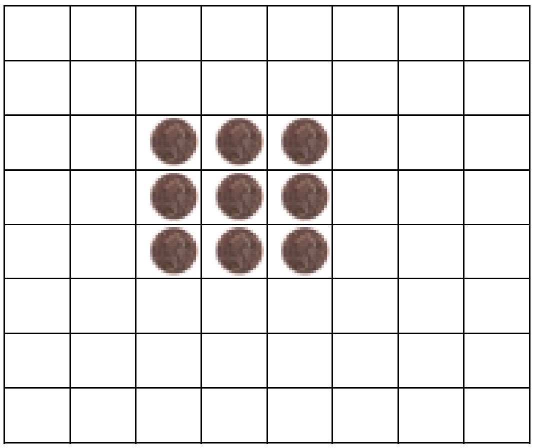
> "From memory of what the picture looked like, well the guy who is second in line on the right is the one who gets to live or have a hot dinner or whatever. This is because he knows the one in front of him has a white hat on and he knows that if he had a white hat on the guy behind him wouldyell out black. Since the guy behind him doesn't call out that he is wearing black then he knows he must be wearing black."

As a footnote to this, B then works out that his hat must be white. This is because he makes the same deduction as C and knows that he and C have different colour hats. Since C has now declared his hat to be black, B can deduce that his hat has to be white.

A and D can never reach any conclusions about their hat colour.

Problem of the Month

Place 9 coins in a 3 by 3 array somewhere in the middle of a chessboard. A coin can ‘take’ another coin by jumping over that coin and landing on an empty square. Show that in a sequence of 8 ‘takes’, all the coins except one can be removed. Is it possible for the last coin to end up on its original square? (More than one coin needs to be moved.)



Once again we will give a $50 petrol voucher to (i) anyone who sends us a solution to this month’s problem (we’ll choose one at random if we have a deluge) or (ii) anyone who sends us a problem that we can use here next month.

Please send your solutions to derek@nzmaths.co.nz

All the best for your teaching.

Gill, Derek and Joe.