Newsletter No. 38 **October 2004**

I was enjoying a bit of winter sunshine recently, lounging on the verandah looking at the latticework squares of the woodwork and noticing how they were distorted by perspective to rectangles and maybe trapeziums, when a question came to mind. Can everything be mathematicalised?

In the physical world we certainly feel that is so. Some phenomena may be extremely complicated like the renowned problems of modelling turbulence across an aerofoil or accurately forecasting weather patterns but we have no doubt that scientists will solve them eventually even though it may require totally new developments in mathematics.

The physical world though is not quite the same as the world of physics. There are the arts, for example - painting, sculpture, literature, music and so on. While mathematics can describe some of their properties, paintings and sculptures can be reduced to finite sets of points and literature to sets of words after all, there are aspects that lie outside the physical world. I'm thinking of such things as emotions, ambitions and beliefs. These properties, of what we might call inner human life, provide much more of a challenge to mathematicians. It is true that statisticians have looked and in some cases found patterns in these phenomena, applying significance tests and so on to describe human group behaviours, but most of us feel that it is not the same thing. Deterministic modelling seems more authentic somehow than its stochastic relative - but that's another story.

> *Mathematics and civilisation have always gone together but in this age of increasingly sophisticated science and engineering mathematics has assumed even greater relevance.*
>
> **Prince Philip**

What's new on the nzmaths site this month?

In the last month we have updated the national data in the reporting feature of the numeracy database. The national data provided is now based on 2003 results. Also, we have added the option to compare data by demographic sub category. So if you want to compare just one gender or ethnicity of your students, or if you want to compare your school to other schools of similar decile, you can do so.

Also in the Numeracy Project area of the site, we have added a set of Frequently Asked Questions about the project, with their answers.

While there are no new units available on the site this month, watch this space, as we have had writers working on units, and these will be added to the site very soon.

Diary Dates

The NZAMT 9 conference, 27th to 30th September 2005, Christchurch.

Curriculum Project

You may not realize that there is a major curriculum review on at the moment that is looking at all of the subjects of the curriculum. A Mathematics group has met on five occasions already and we thought that it would be useful for you to see the ways that we are moving. You might also like to have some input into the process. To get a better idea than the intermittent news from this newsletter, you should access the Teach2Learn website. This can be done by linking to the curriculum project website in the curriculum project kete on tki. … http://www.tki.org.nz/r/nzcurriculum/cp_online_e.php

As far as Mathematics goes, there has been thought given to the name of the subject. Given that there is a reasonable amount of Statistics in the current curriculum, it has been suggested that Mathematics and Statistics a might be a better name than just Mathematics. How do you feel about this? It does not mean that there will be more Statistics in the curriculum it just reflects the place of Statistics overall.

It is worth pointing out at this stage that there is no thought of changing the content very much at all. The layout may well change a lot. This is because the curriculum project plan is to have one document that will reflect all of the essential learning areas and their main points. The achievement objectives for all subjects will be included in the one document. This clearly means that there will be fewer, broader objectives in all curriculum areas.

As a result, there will also be separate explanatory documents for all subjects that will amplify the slimmed down version in the collective document. So far the Mathematics group has only just begun to think about what this might look like.

Much thought has also been given to the strands of the new curriculum. When you look at the development of Number it is clear that it moves smoothly into Algebra. The foundation of Algebra is certainly Number and, in some sense, Algebra is a generalisation of Number. Consequently, it is currently being suggested that Number and Algebra should be merged into one strand.

Statistics is clearly another strand that stands some way apart from Number and Algebra and has its own way of thinking and operating. That leaves Geometry and Measurement. As much, but not all of Measurement is geometrical, Geometry and Measurement fit well together too. Hence it is likely that they will form the third strand of the curriculum. This will then make three relatively large strands.

All subjects will have an essence statement that reflects the unique aspects of each one. Considerable work has been done to arrive at an essence statement for Mathematics and Statistics. Your comments on this draft would be welcome:

> "Mathematics and Statistics are human creations. They have distinct, yet connected, bodies of knowledge and methods of enquiry. Mathematics and Statistics have contributed to the cultures in Aotearoa, New Zealand, and throughout the world. Our ancestors in first travelling to the land and thriving in its environment used mathematics to solve problems, from navigation to food gathering, design, and construction. Our descendants will need to use mathematics and statistics competently to be critically thinking citizens in the global community of the future.
>
> Mathematics is the exploration and use of patterns and relationships in quantities, space and time. It has its own ways of classifying and describing these patterns and relationships using related ideas like number, shape and space, and measurement. Mathematics also has unique methods of reasoning, generalising, and decision making.
>
> Statistics is the exploration and use of patterns and relationships in data. It is primarily abut looking for and explaining similarities and differences in data, variability within data, and how much confidence can be placed on any conclusions drawn from it.
>
> Both Mathematics and Statistics involve solving problems that help us to explain, interpret, and investigate the world in which we live. Through this process the body of knowledge is increased. Patterns and relationships are represented and communicated through displays and symbols that facilitate the creation of new ideas.
>
> Mathematics and Statistics are important parts of the New Zealand Curriculum Framework. They have a broad range of practical applications to everyday life, from financial literacy to recreation and leisure. Many apparently simple systems in real life are based on complex mathematical models.
>
> Mathematics and Statistics are useful ways of thinking that serve the other Essential Learning Areas. They provide students with powerful ways of thinking, and are important requirements for many vocations. Through studying Mathematics and Statistics students develop the ability to think creatively, critically and logically, to structure and organise, to process information, and to enjoy intellectual challenge."

If you have anything to communicate to the Mathematics group you can do so via the Teach2Learn site, by contacting one of the group's members, or by emailing either Derek Holton (at derek@nzmaths.co.nz) or Gill Thomas (gill@nzmaths.co.nz).

NZAMT9

In every odd year the New Zealand Association of Mathematics Teachers holds a conference. From 27th to 30th September 2005 it is going to be held in Christchurch. If you never go to conferences make a New Year's resolution next year to attend NZAMT 9. By the look of the conference's invited speakers this is a conference not to be missed. You can look up their credentials for yourself (go to www.nzamt9.org.nz) but the collection of Ian Stewart, Doug Clarke, Claudi Alsina, John Mason, Clio Cresswell, Gillian Heald, and Don Fraser, is one of the best that ***any*** conference in New Zealand, mathematical or otherwise, will have had for many a long year.

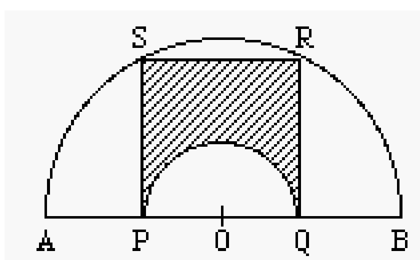
Apart from plenary speakers there will be workshops, talks, displays, and many graphics calculators will be given away. If you have a neat way to teach some aspect of the maths curriculum, then why not think about giving a talk or workshop. There'll certainly be a lot to see and do and it will also give you a chance to meet old friends and to make new ones. By the way, this conference is not just for secondary teachers. All teachers have a place there.

It's going to be good. Plan to be there.

And one from South America!

A friend recently came across this problem in a Chilean magazine. It doesn't seem to matter that it's in Spanish, one can still make sense of it.

En la figura AB y PQ son semicircunferencias de centro O y PQRS es un cuadrado. Si AB = 2r, entonces el área de la superficie sombreada está representada por:



It was a multichoice question and there were then five answers to choose from (see 'Afterthoughts' below). You might like to have a go.

Paul Erdös

Unfortunately, Paul died in 1996. He was undoubtedly the most prolific mathematician of the 20th Century having produced over 1500 publications. When most people think of research mathematicians they think of a bearded, bespectacled, be-tied loner who hides away somewhere and produces whatever mathematicians produce (and most people are not really sure what that is). However, Paul was a relatively small man who did wear glasses but had no beard and never seemed to wear a tie. Instead of hiding away from the world, he spent most of his adult life travelling from country to country, visiting university after university, and working with any mathematician who would care to work with him (and there were many).

But hosting Paul was not without its hazards. He seemed to manage on about 3 hours sleep a night and that gave him a few extra hours more than his host in which to think mathematics. And if he was making progress on a problem he would have the need to share his ideas. One mathematician recalls being wakened in the early hours of the morning in the marital bedroom with "now about that conjecture, I think …"

Paul's peripatetic life style left no time for wife or children. However, for a long time his mother travelled with him and he certainly showed a great interest in children. He called them 'epsilons'. This was just one of his specially coined words. The Greek letter epsilon is used in some areas of mathematics to denote a small quantity and so for a mathematician it was natural to refer to children as epsilons. His various prejudices might be seen in the fact that to him women were the 'boss' class and men were 'slaves'. Paul called God the 'Supreme Fascist'.

Along these lines there was The BOOK. You may not know that research mathematicians have an aesthetic feeling when it comes to their work. They refer to 'nice' and 'elegant' proofs. These are ones that have an unexpected twist or a surprising simplicity. Paul imagined that all the nicest proofs could be found in The BOOK, which was presumably kept from the sight of mere mortals by the Supreme Fascist.

One of his early results was in the area of what is known as Ramsey Theory after the person who introduced it. The basic problem can be expressed in terms of people at parties. It can be shown that there have to be 6 people at a party before there are 3 people who mutually know each other or 3 who mutually do not know each other. In general we might ask how many people do there have to be at a party in order that some number mutually know each other and some number mutually don't; finding that 'number at the party' turns out to be very difficult. Even after well over 50 years of trying, only relatively small such numbers have been found. In most cases we only have bounds on what the numbers can be. The early result mentioned above produced bounds which are still pretty good even today.

There is still an aura around Paul. In his native Hungary he was treated like a prince of mathematicians. It was certainly an honour to work with him. This led to the development of what is known as Erdös numbers. If you have written a paper with him your Erdös number is 1. If you have written a paper with someone who has Erdös number 1, your Erdös number is 2, and so on. Since Erdös' death, of course, if you don't have an Erdös number of 1, then the best you can do is 2. So you need to choose your research partners wisely.

Paul never seemed to have any significant amounts of money although he had won one or two valuable prizes. He moved from place to place living simply and usually being funded by his next host. But he did offer a variety of monetary prizes for problems that he thought should be solved. If he offered $50 then he thought that it was relatively easy. $5000 showed that the problem was pretty hard. Many mathematicians have benefited from this process. In the early days the recipient usually cashed the cheques that appeared. However, as people realised his importance, the prize cheques were prized and often framed and never presented to the bank. Then someone realised that you could cash the cheque and have it returned to you by the bank. You could cash your cheque and frame it.

(For a biography and more details on Paul Erdös, see www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Erdos.html. This site has a great set of biographies of mathematicians. You might like to use it for background material for class or to send your students there for historical information. For a very easy to read and entertaining book about Paul Erdös you might want to look for *The Man Who Loved Only Numbers: The Story of Paul Erdos and the Search for Mathematical Truth* by Paul Hoffman.)

Solution to September's problem

In the prime factorisation of a square number, each factor has an exponent divisible by two. For example, $5184 = 72^2 = 2^6 \times 3^4$. Similarly, in the prime factorisation of a cube each factor has an exponent divisible by three. Generalising, in the prime factorisation of a power of n each factor has an exponent divisible by n.

Consequently, if a number is a square, cube and fifth power every prime factor has an exponent divisible by two, three and five and the number is thus a perfect 30^{th} power. The next smallest such number, after one, is hence 2^{30}.

Derek Smith of Lower Hutt put it this way and so wins this month's prize.

Consider a number a such that:
$a^2 = b \rightarrow a = \sqrt{b}$
$a^3 = c \rightarrow a = \sqrt[3]{c}$
$a^5 = d \rightarrow a = \sqrt[5]{c}$ where a, b, c and d are non zero whole numbers.

Equating these equations yields:
$a^{30 =} b^{15} = c^{10} = d^6$, thus as a = 1 is given, is the next smallest 2?

Equating the powers as factors gives 15 = 3 x 5 10 = 2 x 5 6 = 2 x 3

Since the powers do not have a common factor the next smallest common factor for 15, 10 and 6 is 30.

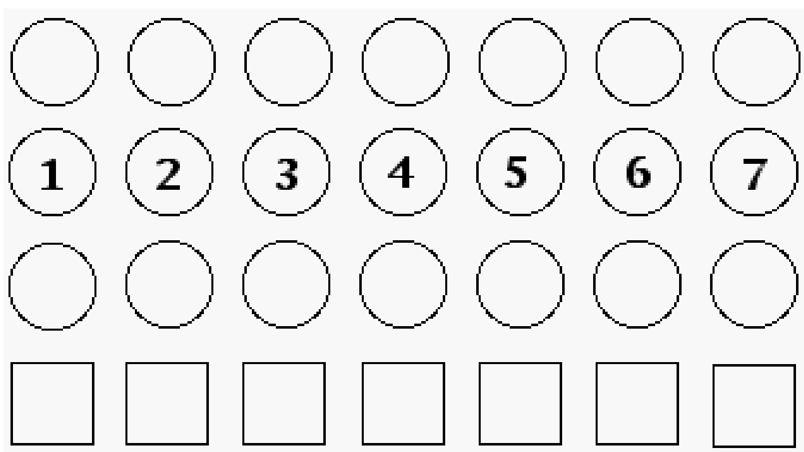
number	sqrt	cube rt	fifth root
1073741824	32768	1024	64

$64^5 = 1073741824$
$1024^3 = 1073741824$
$32768^2 = 1073741824$

Therefore, $2^{30} = 1073741824$ is the next number that is simultaneously a perfect square, cube and fifth power.

This Month's Problem

Your task, should you accept it, is to find a permutation of the numbers one to seven with the property that when placed in both the first and third rows, the seven column totals are all perfect square numbers.



We will give a petrol voucher to one of the correct entries. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so we can send the voucher if you are the winner.

Afterthoughts

The given five multi-choice answers were,

A) $^4/_5\, r^2 (\pi + 1)$

B) $^4/_5\, r^2 (8 + \pi)$

C) $^1/_{10}\, r^2 (5\pi - 8)$

D) $^1/_{10}\, r^2 (8 - \pi)$

E) $^1/_{10}\, r^2 (5\pi - 1)$

Which one is correct, do you think? And why?