Newsletter No. 36 **August 2004**

We've mentioned in earlier newsletters about some of the long-standing maths problems awaiting solutions (Issue no. 17, Sep. 2002). One problem, Fermat's Last Theorem, which had defied solution for 300 years and was solved ten years ago by Andrew Wiles was discussed in our last issue. Another, Hardy's Prime Number problem, was picked off this year.

G.H. Hardy is probably most well known for his 1940 memoir *A Mathematician's Apology*. Number theory fascinated Hardy, particularly the distribution of prime numbers. Not only do they seem to occur at random but paradoxically they show patterns. For example, there appear to be rather a lot of what are called prime pairs, i.e. prime numbers like 3 and 5 that differ by two. 29, 31 and 41, 43 come to mind, I'm sure you could think of many others. And although there is no polynomial that generates all the primes, $n^2 + n + 17$ seems to generate a lot of them while $n^2 + n + 41$ generates even more.

Let's have a deeper look at $N = n^2 + n + 17$. With a bit of computing we can produce the table below.

n	1	2	3	4	5	6	7	8	9
N	19	23	29	37	47	59	73	89	107

There are all sorts of patterns there but N is surely always prime, at least in the table it is. So is N always prime no matter what value n has? Well, no. If you think about it for a while you may be able to see one value of n for which N isn't a prime. In fact there is an infinite number of values of n for which N ***isn't*** a prime. (Think about it for a while and then check the footnote to this newsletter.)

The interesting thing here is that there is no quadratic Q in n (that is, nothing of the form $Q = an^2 + bn + c$, where here we restrict a, b and c to be whole numbers) for which Q is prime for all whole number values of n. (You can see why in the footnote too.) Nor is there any cubic ($an^3 + bn^2 + cn + d$) or quartic ($an^4 + bn^3 + cn^2 + dn + e$) or indeed any polynomial in n (where the powers of n go up as high as you like) which produces primes for every n.

If you want to stick to just sums of powers (or maybe products of powers of different letters), you can get a polynomial with 26 variables that produces a prime for every value of each variable. But this is too horrendous to contemplate.

Now we were talking about Hardy, the Cambridge don who considered himself to be a PURE mathematician. By that we mean that he boasted that nothing he did would ever be useful in the real world. It's interesting that the number theory that he loved should turn out to be the basis of one of the most used codes of the last ten years. The RSA code is almost certainly being used by banks and for national security. It relies on the fact that big numbers are hard to factorise in a reasonable time.

But we've strayed from the prime pair idea. An extension of that is that of primes being in arithmetical progression. Choose a prime and increase it by constant jumps. For example, beginning with five and increasing in jumps of six gives 5, 11, 17, 23, 29, five prime numbers in arithmetic progression before the pattern breaks down. Hardy wondered how many consecutive primes an arithmetic progression could have and conjectured that there was no upper limit.

In 1939 it was proved that there were infinitely many arithmetic progressions which contain three primes, the so-called 3-term prime progression. This year Ben Green of the University of British Columbia and Terence Tao of the University of California, having set out to prove that there are infinitely many four-term prime progressions, discovered a proof of Hardy's conjecture - and there it was solved!

It's worth noting here that Terence Tao is not a wizened old mathematician. In fact he's now only in his late 20s. Terence is an Australian who was a gifted mathematician from a very young age. When he was only 10 he won a Bronze medal for Australia's Maths Olympiad (see the IMO article later) team. The next year he got a Silver and in 1988 he won a Gold. He is still the youngest person ever to win a Gold medal! This led on to a PhD at 17, the youngest ever at Flinders University in South Australia. His main base these days appears to be the States but he regularly returns to Australia because of an arrangement he has with the Australian National University in Canberra.

The nzmaths website is all about problem solving so why not encourage your pupils to exercise their problem-solving skills. You never know, one of them may turn out to be someone who solves one of the big ones.

> *Remember, problem solving is not a spectator sport, it requires practice, practice, practice.*
>
> **Anon**

What's new on the nzmaths site this month?

There are two new units on the site this month, both in level 5 Geometry (Shape):

- Ruler and Compass Constructions; and
- Dizzy Heights.

Diary Dates

If you've forgotten Maths week it's still not too late!

ANZ Maths Week 04
9 – 13 August 2004

You can still check out the website www.mathsweek.org.nz
or alternatively, if you would like more information, email: i.stevens@inspire.net.nz
but be quick!

The IMO

The International Maths Olympiad (IMO) is an annual international maths competition for secondary students most of whom are 17 or so. This year it took place in Athens. New Zealand has sent a team of 6 students since 1988, the date that is now famous for Terence Tao's Gold and some anniversary or other in Australia.

The problems at the IMO are very difficult and require not just knowledge of a range of areas of mathematics but being able to fit ideas together in an original way. It took our teams several years before anyone got a Gold, though we did get a Silver in 1988. This year two of our students won a Bronze each and we were 53rd among over 80 countries.

For more details on the Olympiad, see the schools section of www.maths.otago.ac.nz.

Staff seminars

If you go back to the nzmaths site, on the same page that you get access to the newsletters you can gain access to a series of staff seminars. These are things that you might like to organise for your staff at school.

At the moment there are 9 seminars available, though some of them could take up a lot more than one session. The seminars are entitled Angle (Geometry); V-Numbers (Problem Solving); Frogs (Bright Sparks); Snakes and Ladders, Monopoly, and Lotto (Probability); and Four Problems and a Funeral, Mind Reading, and Gauss' Trick (Number).

You might be interested in Lotto because it shows you how to work out your chances of winning Lotto. (Maybe that's too depressing!) Then Frogs gives some background for the Bright Sparks piece of that name. Or maybe you'd like to catch up on a bit of mind reading that's linked to number. Anyway, have a look at them and see what you think. We'd also like to hear if you would have some ideas that we can use or would like some help some topic that you've always wanted top know about.

And even if you don't want to give a seminar, there are some useful mathematical ideas there that you might be able to use in class.

Looting and Pillaging in Denmark

This year's International Congress on Mathematical Education (ICME) took place in Copenhagen, though if you know the region at all well it was actually held at the National Technical University which is in the suburb of Lyngby (which seems to be pronounced more like Loongboo.) Pronunciation was one of the things that I guess I should have been prepared for but wasn't. On the face of it, the Nordic languages are very different from those of English and French but in Copenhagen language was, in fact, never a problem. As I found to my embarrassment, almost everyone speaks English and speaks it well. (My embarrassment is caused because of my concern about the English language 'taking over' the world.)

Anyway, this was the 10th ICME, and they are held in every Olympic year. ICMEs are designed to cover all aspects of mathematics education from pre-school to university. Though the majority of participating people are from a university or teachers' college, there are many sessions that are attractive to teachers. Consequently, ICME attracted 2,300 conferees and the organisers seemed to be disappointed that they hadn't reached the 3,000 mark.

This ICME was broken up into a number of different types of session. These were (i) plenary sessions, where an invited speaker or panel addressed the whole Congress (with TVs in adjacent rooms to the main hall for the overflow); (ii) regular lectures, where selected people talked about their research or about teaching ideas; (iii) topic study groups, where particular issues were discussed; (iv) discussion groups that seemed to be a little less formal than the topic study groups; (v) small group activities, where there were workgroups and people sharing their experiences; (vi) affiliated study groups, where organisations such as the International Organisation of Women and Mathematics Education met; (vii) posters and round table discussions; and (viii) national presentations. If you would like to find out more about any of these, you should get onto www.icme-10.dk. In many instances there are outlines available there of talks that were presented at ICME.

But the above does not cover all of the action. In addition to this there were hectares of publishers and technology companies, a mathematical circus would you believe, and naturally Happy Hour! If you were interested in calculators, there was intense activity centred around Casio and Texas Instruments. There seemed to be almost continual graphics calculator demonstrations. If you haven't seen one of these instruments before, they are calculators with small screens. The machines are as powerful as a computer was only a few years ago. They can be valuably used for teaching and learning especially in the secondary school.

But the best part of any conference is the Happy Hour simply because that is where you can catch up with old friends and meet new ones. And the same goes for the conference excursion. Naturally there were a large number of excursions but I managed to get on one that went to Roskilde where they had found several Viking boats in the mud. They had been buried in a stream there as part of the defences of the local people. There is now a museum built around the remains of these boats. They are really beautifully shaped, as you can tell by the boats in the picture. Near the museum building, they are currently making a replica Viking warship that will have 60 oarsmen and carry 90 warriors.

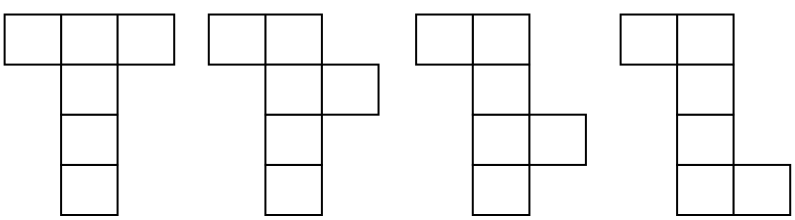
But there is more to Roskilde than museums. Once there we had the chance to sail a Viking boat. This involved learning how to row in time with 11 other people (6 on each side of the boat) using rope rowlocks. Not easy! Once we had mastered this enough to get out of the little harbour there (see picture to the right), we then learnt how to raise and lower a square sail. I was fortunate enough to have tiller duty. This wasn't done the way I was used to with a rod directly attached to a rudder, where moving the tiller from left to right sends the boat off in a new direction. Apparently Viking boats had push-me-pull-me tillers. A hinged arrangement meant that a forward or backward movement on the tiller caused the rudder to turn. It was a bit strange at first but I soon got used to it and it was a lot easier than rowing.

I think that everyone in our boat was disappointed to have to leave the boat. And then, of course it was off for looting and pillaging lessons!

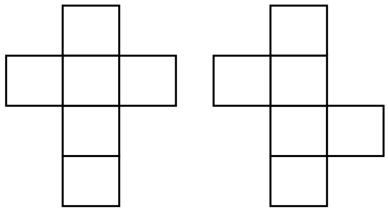
Solution to July's problem

You were asked to find the number of nets of six squares that could be folded to make cubes. It pays to be systematic in your approach and soon becomes clear that if five of the squares are in a row (one was shown last month) the net can't be folded to make a cube.

First look for all the nets with 4 squares in a row. Starting from placing the 5th square adjacent to the top right square look for all possible placements of the 6th square. There are 4.

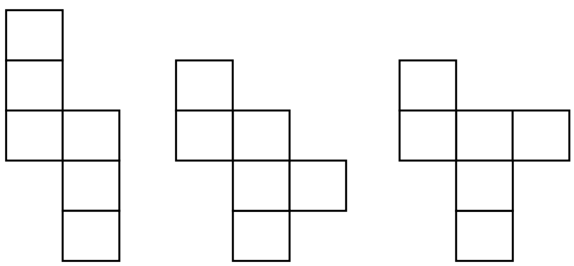


Then place the 5th square one space lower and look for all possible placements of the 6th square. There are 2.

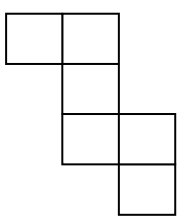


If you place the 5th square adjacent to the 3rd square then you are repeating earlier nets (only rotated) so this must be all the nets with 4 squares in a row.

Next, start with 3 squares in a row. It is more difficult to be systematic from this point as there are 3 squares remaining to be added. First add the 4th square next to the top square again. Then look for places where another square could be attached to this one. The only place a 5th square could be attached to the 4th one is above it. If this is done, there are three places a 6th square can be added.

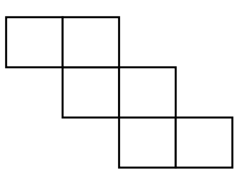


The only other places the 5th square could have been added (other than above the 4th) are diagonally opposite it, in which case there is only one net that can be formed,



or beside the middle of the original three in which case the only net is a repetition of one above.

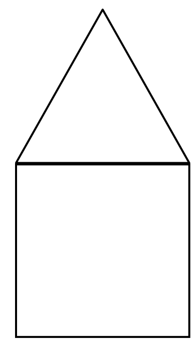
There is only one net that can be made with no more than two squares in a row.



So, in total there are eleven distinctly different nets.

I’m afraid that we didn’t have any winners this month despite there being several entries. All the people that entered made small errors.

This Month's Problem



On a farm in South Otago there is an unusual storage shed. It is 20 metre wide and shaped like a cube with a pyramidal roof. Each face of the pyramid is an equilateral triangle. What is the area of the roof?

Remember we will give a petrol voucher to one of the correct entries.

Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so we can send the voucher if you are the winner.

Footnote

If n = 17, then N isn’t a prime because it has a factor of 17. To see that, just notice that for $n = 17$, $n^2 + n + 17 = 17^2 + 17 + 17 = 17(17 + 1 + 1) = 17 \times 19$. OK so it is the product of two primes but that doesn’t qualify it for primehood.

Is this the only value of n for which N is not a prime? We did suggest that there were an infinite number of values of n for which $n^2 + n + 17$ is composite. Can you see what they might be?

So that takes us on to $M = n^2 + n + 41$. By now we know that you have drawn up a table and used Excel or some other means to generate large numbers of values of M. Is it true that for $n < 41$ M is a prime? So 41 must be the downfall of primeness for M. It’s the same argument as for N when n = 17. (No need to do the calculation – just do a factorisation like the one we did above.)

Alright but how do we find an infinite number of numbers that make M composite? What happens when $n = 2 \times 41$? Then $M = (2 \times 41)^2 + (2 \times 41) + 41$. Now take out the factors of 41. So $M = 41(2 \times 41 \times 2 + 2 + 1) = 41(4 \times 41 + 2 + 1)$ and 41 is obviously a factor while the bracket is not 1. So here M isn’t prime.

And the same thing happens with $n = 3 \times 41$. In this case, $M = 41(9 \times 41 + 3 + 1)$ – composite. Or $n = 4 \times 41$, where $M = 41(16 \times 41 + 4 + 1)$. And it’s at this point that we can use the power of algebra. Take $n = k \times 41$, where k is any whole number you like to think of. In this case $M = 41(k^2 \times 41 + k + 1)$ and that is composite for any whole number value of k. So we have discovered an infinite number of values of n for which M is not prime. (And, of course, the same thing goes for N.)

It should be easy now to see what goes wrong with any quadratic or indeed any polynomial. If we take $Q = an^2 + bn + c$, then if n is any multiple of c, Q will be composite. Now take any cubic $C = an^3 + bn^2 + cn + d$. Any multiple of d will give C composite. So for any polynomial in n with constant term q, if n is a multiple of q, the polynomial will not have a prime value.

Going back to $n^2 + n + 41$, the big question now is, if n isn't equal to $k \times 41$, is M necessarily prime? We'd like to hear from you on this one. Let Derek know what you've found on derek@nzmaths.co.nz.