

Newsletter No. 35

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I guess it's mainly shift-workers that watch T.V. in the wee small hours. A couple of months back, though, purely by chance I noticed that the B.B.C. had an early-morning programme in their *Genius* series devoted to Andrew Wiles and his solution of the 300 year enigma that was Fermat's Last Theorem. I videotaped it. I'm certainly glad that I did, the programme was absolutely inspirational.

Fermat's so called Last Theorem (it should have been called Fermat's Conjecture – it couldn't really become a theorem until it is proved) was the generalisation of Pythagoras' Theorem. With Pythagoras, $x^2 + y^2 = z^2$, where z is the hypotenuse of a right angled triangle and x and y are the other two sides. Of course thre are lots of numbers that satisfy this equation. But perhaps surprisingly, there are alos lots of whole numbers that satisfy it too. For instance, 3, 4, and 5, are such that $3^2 + 4^2 = 5^2$. Fermat wondered if it was possible that there were whole numbers x, y and z that satisfied $x^3 + y^3 = z^3$, or $x^4 + y^4 = z^4$, or $x^5 + y^5 = z^5$. Or in general, is there any whole number power n bigger than two, such that $x^n + y^n = z^n$ has whole number solutions for x, y, and z?

In 1993 Andrew Wiles made headlines when he announced a proof of the problem. He thought he could show that for any number n bigger than 2, there were no whole numbers x, y, and z for which $x^n + y^n = z^n$. In this country its first T.V. appearance was a short throw-away item on Channel 3 News. That was not the end of the story, however, as a slight error in the calculation jeopardised the proof. I think it took about a year to put that right.

There are a few interesting things about Wiles' solution. First, he essentially locked himself away in an attic and did almost no other research while he devoted himself to the project. While many people believe that this is the way all mathematicians still work this is far from being the case. Most maths is done in small groups these days.

Second, Wiles' first proof, that had an error, was given a great deal of scrutiny by the mathematical community to make sure that it was right. When an error was found Wiles went back to work, this time with a colleague. When the second proof was presented, that too underwent careful scrutiny till it was given the thumbs up.

Third, actually 'mathematical community' is slightly misleading. At the time proofs were completed, only about a dozen people in the world knew enough about the area to be able to check out the proof.

And fourth, this process is gone through by every new result no matter how important. Whenever a mathematician (or group of mathematicians) proves a result, they send it to a journal to try to get it published. The result is always subjected to refereeing by a couple of mathematical peers. No result is accepted until it has passed through such a process.

It turns out that Andrew Wiles had come across Fermat's problem as a ten year-old browsing through a maths book in his local library. The solution was to consume him for many years and he'll be forever famous for it. Fermat mentioned the theorem in the margin of a book he lent to a friend. He added that he had a proof of the conjecture but had no room to include it. It was this proof, or one like it, that became the Holy Grail for mathematicians and not-so-mathematicians for the next 300 years! Today it is felt that Fermat did not have a proof or if he did it was certainly not the one Andrew Wiles discovered which requires knowledge of mathematics that wasn't invented till long after Fermat's time.

Don't you wish you'd been Andrew's teacher?

If you'd like to know more about the problem and its solution you might try one of these books:

Fermat's Enigma: The Epic Quest to Solve the World's Greatest Mathematical Problem by Simon Singh Fermat's Last Theorem for Amateurs by Paulo Ribenboim Invitation to the Mathematics of Fermat-Wiles by Yves Hellegouarch

Or browse the website: <u>www.pbs.org/wgbh/nova/proof/</u>

The person who looks at a mathematical formula and complains of its abstractness, dryness and uselessness has failed to grasp its true value. **Morris Kline** (Mathematics in Western Culture)

What's new on the nzmaths site this month?

This month there is another new staff seminar available on the website. It is called Gauss' Trick and is about summing arithmetic progressions.

There are also two new geometry units, Robots (Level 2), and Gougu Rule (Level 5).

Diary Dates

ANZ Maths Week 04: 9 – 13 August 2004

Planning for this year's Maths Week is well underway. This extremely popular week is held early in Term 3. There will be free resources for teachers, prizes and competitions

for students. The Survivor Series, Daily Challenges, and ANZ Auction will be back, plus several new interactive games.

If you wish to receive email updates of what is planned check out the website <u>www.mathsweek.org.nz</u>. Alternatively, if you would like more information, email: <u>i.stevens@inspire.net.nz</u>.

Actually you've got plenty of time to plan so why not do something special for maths week? How about a poster or story about a child's favourite number?

Reflections on Maths Education in China

At the last Otago Maths Association meeting, Julie Anderson, Head of the Maths Department at Dunedin College of Education, entertained with pictures and stories from her recent teacher exchange in China. These notes were taken by Jan Saville and first appeared in the OMA newsletter.

The first thing you notice as you leave the Nanjing airport is the traffic lights. Chinese drivers don't sit waiting idly for the light to turn green; their traffic signals count-down second-by-second to the next change of lights. Pressing a pedestrian signal gives a child the opportunity to count backwards with the display as they wait. The largest pedestrian start number that Julie spotted was 92. Now that is a lot of backwards sequencing experience before the child even gets to school!

Classes in schools are larger than ours would be but the teaching appeared well managed. Specialist maths teachers are used in the primary schools. One lesson Julie showed pictures of had a single teacher working with 62 children, all seven- or eight-year-olds, for a period of one hour. During the time each child answered at least one question. There was opportunity for buzz groups, demonstrations to classmates, bookwork and no difficulties with any of the children misbehaving.

Technology was to the fore in all the schools that Julie visited (but they were the more affluent schools). A common occurrence was two students standing on opposite sides of the data display screen, each with their own section of whiteboard, demonstrating their idea of how a proof would go or the method they would use for solving the problem presented by the teacher. The different ways that the student solved the puzzle would be compared, discussed, and dissected by the teacher and the rest of the class. A lesson would often involve the teacher presenting a single problem or topic. While there was some of the traditional "show and tell", the topic tended to be initiated by the teacher and then be developed by the students. Focus was on the students suggesting multiple methods of solving that problem, with the teacher critiquing and guiding, rather than always starting a lesson by demonstrating "the way it is done".

A typical secondary school teacher would spend 10 - 12 hours a week teaching classes of 50 - 60 students. There is much more time spent preparing student work than we would expect in New Zealand, and additional hours are spent conferencing with individual students. Lessons are very polished. Teachers often get together to discuss how a particular lesson might best be taught. Criticism amongst the staff (and of the students doing a demonstration problem) was robust and to-the-point but neither staff nor the students appeared to be deeply affected by it. Any mistakes by the students when they were demonstrating a point would be acknowledged, discussed and shrugged off as they moved to more mathematics. This is quite different from our desire in New Zealand to ensure that no student suffers the humiliation of getting a question incorrect.

More on the coins from Cathy Walker

If you remember, a couple of newsletters ago Cathy made some comments about a coin problem where you had to turn a triangle of coins around by moving as few coins as possible. She has now got this far with the problem.

I have found that if you know how many coins there are in the triangle you just need to divide by 3 and round to the nearest whole number and that will tell you how many moves you will need to make to change the direction of the triangle. (Don't know why though.)

There's other patterns going on in here as well. I just noticed a regular increase in the difference of number of moves with respect to the previous layers. I don't know how long that continues but it looks pretty regular.

no. of layers	no. of circles	÷3	no. of moves	diff between no. of moves cf previous layer
1	1			
2	3	1	1	
3	6	2	2	1
4	10	3.333333	3	1
5	15	5	5	2
6	21	7	7	2
7	28	9.333333	9	2
8	36	12	12	3
9	45	15	15	3
10	55	18.33333	18	3
11	66	22	22	4
12	78	26	26	4
13	91	30.33333	30	4
14	105	35	35	5
15	120	40	40	5
16	136	45.33333	45	5
17	153	51	51	6

18	171	57	57	6
19	190	63.33333	63	6
20	210	70	70	7
21	231	77	77	7
22	253	84.33333	84	7
23	276	92	92	8
24	300	100	100	8

Does anyone have any comments on this? Is what Cathy says true? Can you justify it?

Solution to June's problem

It turns out that Cathy Walker also won the voucher this week. Her method of solution went this way.

For this months problem I used a table and some knowledge of quarters and thirds If there are 100 married couples there are 100 men. There are 4 categories the men could fit into

1	Taller + Heavier	2/3 of 1 and 2	36 48
2	Taller & Lighter	half of 1	18 24
3	Shorter & Heavier	a third of 1	12 16
4	Shorter & Lighter	12	12 12

As I figured if this was a real life problem a lot of men are going to be in no 1 and looking for factors of both 3 and 4 I tried 36 but there weren't quite enough so I tried 48 and that gave me the 100 men and 48 are taller and heavier.

Now many problems can be done in more than one way. You can also do this problem by algebra. Try this.

There were 100 couples and two categories. A set of men taller than their wives which we call, T, and those heavier than their wives, H.

Perhaps the best way to picture them is with a Venn diagram,



where a, b, c and d denote the number of men taller and lighter, taller and heavier, shorter and heavier, shorter and lighter than their wives respectively.

We know that altogether there are 100 men, so, a + b + c + d = 100 ------ (e1) We also know that two-thirds of the husbands who are taller than their wives are also heavier, i.e. $2^{2}/3(a + b) = b$ From which, 2a + 2b = 3b, i.e. b = 2a ------ (e2)

Three-quarters of the husbands who are heavier than their wives are also taller, so $\frac{3}{4}(b+c) = b$, from which b = 3c ------ (e3)

We also know that there are 12 wives who are taller and heavier than their husbands, i.e. there are 12 men shorter and lighter and than their wives, so that

d = 12 ----- (e4)

From the four equations (e1) to (e4) we can deduce that b, the number of husbands who are taller and heavier than their wives, is 48.

This Month's Problem

Models of polyhedra are often made by drawing nets on cardboard then cutting and folding them along the internal lines to form the required shapes. Regular tetrahedra, for example, can be formed from the two possible nets of equilateral triangles below.





Your question this month is to find how many different nets of six squares will make a cube. The usual rules for nets apply. The following is not a solution!



We will give a petrol voucher to one of the correct entries. Please send your solutions to <u>derek@nzmaths.co.nz</u> and remember to include a postal address so we can send the voucher if you are the winner.