

## Newsletter No. 34

June 2004
Some time back I set the members of my Year 10 class looking for possible relationships between the perimeter and area of rectangles. For some reason they felt that a particular perimeter would always hold the same area no matter how it was configured (the context was fencing rectangular paddocks). It didn't take the pupils long to realise that this was not the case but I let them continue experimenting to see if any other ideas arose. After a few minutes a pupil told us that she had found a rectangular shape with area and perimeter numerically equal. It was the four by four unit square. After a bit of class discussion I foolishly suggested to the class that this was the only example of its kind. Someone very quickly corrected me - they had found another rectangle with perimeter numerically equal to its area. In case you don't know it and would like to think about it I'll leave the solution until later.

I hadn't expected the second solution and later found that there were only the two solutions in whole numbers - an infinite number of them in rational numbers, of course.
Neither the pupils nor I had, in the first instance, even considered numbers other than whole numbers. That set me thinking and next lesson I offered the class a game to check out my suspicions (see below for that).

Here are a few of the answers I received when I asked pupils the question, 'What is a decimal?'

> A number less than one.
> A fraction of a number.
> A part of a whole number.
> It's less than zero and has a decimal point in front of it.
> It comes from fractions.
> A smaller number between two numbers.
> A number which is placed after the 'ones' column.
> A number with a remainder after it.
> A broken-down fraction.

Do the answers suggest children's poor understanding of decimals or a lack of experience in defining things? I'll leave you to ponder on that.

## What's new on the nzmaths site this month?

In the last month the Rauemi Reo Māori section of the site has been overhauled. This area of the site has grown in size dramatically in recent months and needed to be made more streamlined. If you teach in Māori medium have a look at the improvements. Two extra weeks of work have also been added. The Papa Pukeko unit has been extended to two weeks and a new unit, Heketea-a-Irä, has been added.

## Diary Dates

Maths week on 8-15 August is rapidly approaching! Initial information about Maths Week 2004 and resources from previous years are available on the NZAMT site:
http://www.nzamt.org.nz/
Applications for Teaching Fellowships close on the $16^{\text {th }}$ of July. For more information see http://www.rsnz.govt.nz/awards/teacher_fellowships/. We have provided a suggested topic last month, and another is described below:

At Tufts University, Boston, USA, as part of his work with the Centre for Engineering Education Outreach (CEEO), Professor Chris Rogers works with teachers, schools, industry, and government to make engineering an integral part of every child's education. With the support of National Instruments and LEGO, his work at CEEO has led to the development of ROBOLAB software (www.ceeo.tufts.edu/robolabatceeo/, www.lego.com/dacta): an educational toolkit for use with LEGO Mindstorms kits. It has been adopted in more than 4000 schools worldwide to teach a range of computing, physics, mathematics and engineering concepts via robotics. It has won the English BETT award for best educational software (www.bettawards.co.uk/aboutbettawards/) and has been translated into 12 languages. It is being used across the entire education age range from primary school through to university. The overall aim of using ROBOLAB and LEGO Mindstorms is to increase the interest and enthusiasm that school students have for science and engineering and thereby increase the participation level at tertiary level and beyond.

We wish to further develop this study at Lincoln University with the involvement of a school teacher for an extended period of time.

The work that would be undertaken would include:

- becoming familiar with RoboLab software
- becoming familiar with existing school projects that have been developed using RoboLab
- development of further projects that use RoboLab in the classroom
- contributing to further development of ROBOTABLE
- school visits to give children hands-on experience with the hardware and software (Lincoln University have sufficient number of LEGO Mindstorms kits to run school sessions for class sizes up to 30)

Interested parties can contact Keith Unsworth at unsworth@lincoln.ac.nz .

## Mathematical Game

As mentioned in the editorial above, responding to a problem in which neither pupils nor I first thought of non-whole solutions, I devised a classroom game to see how long it would take pupils to think of using fractional values, that's decimal fractions. It's in a question and answer format between teacher and class.

I begin by asking the class to give me a number between one and ten. Typical enactments of the game when played for the first time have been:
"Seven."
"Good. Now please give me another."
"Eight" was the response.
"I won't accept that," I replied, adding that we were playing a game and I was following some rule which they had to discover.
"Four."
"No, sorry, that's not acceptable either".
The game continued for a long time before I was able to accept an answer. Numbers were repeated and even solutions greater than ten offered. Eventually someone suggested four-and-a-half which I didn't accept but responded that they were on the right track. Was there another way they could say four-and-a-half? 4.5 was then given correctly.

The rule I was following and which took pupils many attempts to discover was that each number had one more decimal digit than the number before. Thus $6,7.2,3.34,5.555$, 6.2323 was an acceptable sequence of numbers. I had expected that the pupils would have some difficulty with the concept of an extra decimal digit required each time (in fact they didn't) but not that they would have a problem in thinking of a non-whole number in the first place (if you'll excuse the pun). For them, as it had been for me at first in the rectangle investigation, it was as though only whole numbers existed. Particular ones were offered again and again despite my negative responses. Pupils were even happy to break the rule that the number had to be between one and ten before trying a fractional value. There is clearly a resistance to the idea of decimal fractions as solutions to problems but I make no further comment.

## Book Review by Helen Goldblat $\dagger$

## Brain Benders: Mathematics Problems in Logic and Reasoning for $7 \mathbf{- 1 2}$ year olds

 by Barry Brocas and Brenda Bricknell, Kanuka Grove Press, 2004, \$29.95. ISBN 1-877249-27-0.Barry Brocas and Brenda Bricknell have written this book for primary school teachers and it is a welcome addition to any teacher's repertoire of maths resources. As a collection of 'problems' it will create opportunities for class and group discussion requiring students to use and further develop their skills of logical and systematic thinking.

All of the problems are presented clearly and attractively with one "brain bender" per page (with the solution available on the reverse). Furthermore they have been written in a language that the authors believe children will best understand.

The book lends itself to multiple uses. The one-page presentation allows for whole-class use (using overhead transparencies) enabling a teacher to demonstrate, model and to develop the mental skills useful for solving problems. The layout also allows for the problems to be used as "daily challenges" (by being photo-copied, with the problem on the facing page and the solution on the reverse). When attached to a wall, students could individually, in pairs, or in groups come up with possible solutions. The latter approaches will allow students to debate their answers before the solution is revealed. Alternatively, the pages could be reduced to 'card size' with the problems being presented as independent activities.

In developing this book the writers used the problems with a range of ages (from children through to adults) and the 100 problems tabled on the contents pages are divided into Junior, Middle and Senior levels of ability. Some of the problems are fairly straightforward, while others are challenging, providing suitable material for gifted and talented students.

As a guide, here are some samples of problems: one from each of the three groupings.

## Junior: Problem 12, Lining Up

Seven children are lining up in order of their heights. Put the children in order from the shortest to tallest from the following clues.

1. David is in the middle of the line.
2. Paul is taller than Sarah.
3. Cath is the shortest.
4. Mark stands between Anne and Jane.
5. Anne is not the tallest.

## Middle: Problem 55, Weighing the Marbles

Suppose you have eight marbles, one of which is slightly lighter than the other seven, which all weigh the same. Using a balance scale, explain how you can identify the lighter marble in only two weighings.

## Senior: Problem 88, Gordy and Gimp

Two tribes live on an island. Members of one tribe always tell the truth and members of the other tribe always lie.

I arrive on the island and meet two islanders Gordy and Gimp.
Gordy says: We are not both truth-tellers.

Which tribe is Gordy from?
Which tribe is Gimp from?
The solutions are in the book.
I can report that the book is both addictive and lots of fun, though I found that even with the older students it was best to begin with some junior problems first. Modelling and introducing them to the ways of thinking needed enabled them to more readily tackle the higher order types of questions.

The book can be purchased directly from Kanuka Grove Press (fax 06351 3324) or from one of the Education Resource Centres.
(Helen Goldblatt is the Numeracy and Gifted and Talented Adviser, School Support Services, Wellington College of Education.)

## Solution to May's problem

The largest product from two numbers is obtained by using two principles:
(1) The largest digits are on the left. For example to make the largest product from the digits $1,2,3$ and $4 ; 43 \times 21$ gives a bigger answer than $34 \times 12$ (although it is not the largest product that can be made from these four digits).
(2) The product of two numbers whose sum is constant is maximised by making their difference as small as possible. For example, knowing that two numbers sum to 20 their largest product is $10 \times 10$ rather than any other pairing like $11 \times 9$ or $13 \times 7$.

Using the two principles the solution to the problem is 96,420 and 87,531 .
We got several answers but only two were correct. Choosing between the two was difficult but we decided on this answer from Sasima Thammarucha, a Year 9 Dunedin student. She got there iteratively as follows.

My answer from using of the ten digits once, find the five digit numbers which have the largest possible product is $\mathbf{9 6 4 2 0} * \mathbf{8 7 5 3 1}=\mathbf{8 4 3 9 7 3 9 0 2 0}$

I solved the question by:
Firstly I make the 5 boxes time 5 boxes


B1
B2

Secondly I choose the two highest numbers of ten digits which is 9 and 8 . Thirdly I put 9 into the first box of B1 and 8 into the first box of B2


B1
B2
Fourthly I choose the nest two highest number of ten digits which is 7 and 6 Then I put 7 into the second box of B1 and 6 into the second box of B2


I choose the next two highest number of ten digits which is 5 and 4 Then I put 5 into the third box of B1 and 4 into the third box of B2


Then I choose the next two highest number of ten digits which is 3 and 2 Then I put 3 into the fourth box of B1 and 2 into the fourth box of B2


Then I choose the last 2 number which is 1 and 0 .
Then I put 1 into the fifth box of B1 and 0 into the fifth box of B2.


The answer is 8428629020 but after that I swapped the number from the second to fifth box of B1 to the second box of B2 and from the second box to the fifth box of B2 to B1 but I changed it in order e.g. I put 7 which is in the second box of B1 to the second box of B2.

My New Result is:

$=\mathbf{8 4 3 9 7 3 9 0 2 0}$
This is higher than the old result. I didn't change 9 and 8 into the other box because it is the highest number when it multiplied it will get the biggest product than other number. Also I swapped the number from the second to the fifth box of B1 and B2 because I thought that it might get the bigger result and it is!!!

## This Month's Problem

A social gym restricts its membership to 100 married couples. Two-thirds of the husbands who are taller than their wives are also heavier and three-quarters of the husbands who are heavier than their wives are also taller. If there are 12 wives who are taller and heavier than their husbands, how many husbands are taller and heavier than their wives?

We will give a petrol voucher to one of the correct entries. Please send your solutions to derek@.nzmaths.co.nz and remember to include a postal address so we can send the voucher if you are the winner.

## Afterthoughts

The rectangle with its area numerically equal to its perimeter, the sides being whole numbers, is six units by three.

For those of you who are algebraically minded here's a way to get all of the answers, whole numbers or not. Suppose that the sides of the rectangle are $x$ and $y$ as we have shown in the diagram.


Then the area is $x y$ and the perimeter is $2 x+2 y$. Since these are equal, $x y=2 x+2 y$.
Now you can find a lot of solutions to this by substituting for one of the values. Just let x be anything you like, say, 5 . Then we get an equation like $5 y=10+2 y$. So $y=10 / 3=3.33 \ldots$

But there is a sneaky bit of factorising that takes us there a bit quicker. The thing to notice is that
$x y=2 x+2 y$ becomes $\quad x y-2 x-2 y=0$.
And here's the sneaky bit:
$x y-2 x-2 y=(x-2)(y-2)-4$.
Since
$x y-2 x-2 y=0$, then $(x-2)(y-2)=4$.

Let's have a look at the advantages of that form of the equation. First consider the situation where x and y are whole numbers. Then $\mathrm{x}-2$ and $\mathrm{y}-2$ must both be factors of 4 . The only positive factors of 4 (remember we are talking side lengths so $x$ and $y$ have to be positive) are 1 and 4 or 2 and 2. In the first case $x=3$ and $y=6$ (or vice versa) and in the second case, $x=y=4$.


Then what about the situation when x and y are not whole numbers? Well again just put in values of $x$ and find the corresponding value of $y$. If $x=5$, say, then $y-2=4 / 3$, so $y=10 / 3$. Maybe this is easier than before, maybe not. But the interesting thing is that if you graph the equation $(x-2)(y-2)=4$, you get a nice curve (see above). Every point on that curves is a solution of the area/perimeter problem. The three whole number solutions are shown. (You might like to find all of the integer solutions.)

Do you get anything interesting if you replace rectangle by triangle in the original problem?

