Newsletter No. 32 **April 2004**

An exercise I sometimes give my students is to ask them to write short biographies of the whole numbers. Just a few words or facts about each, from one upwards explaining why they are 'useful' and why we couldn't do without them. For example, 6 is useful because it's half-a-dozen (eggs are still bought in that number). It is also the smallest perfect number (see March Newsletter) and since $6 = 1 + 2 + 3$, a triangular number. 8 is a cube and the only cube one less than a square. It's also the smallest whole number with its letters in alphabetical order.

You get the idea? My approach is to challenge students to find the first whole number that has no particular use. It's all a bit tongue-in-cheek but much enjoyed by students and encourages research into properties of numbers as well as giving divergent thinkers the opportunity to exercise their skills. You'll often find someone suggesting that the smallest 'useless' number they've found is just that and therefore useful! David Wells in his book 'The Dictionary of Curious and Interesting Numbers' (Penguin, 1986) has 43 as the smallest whole number with no entry. It is prime of course but apparently hasn't much else going for it.

> *Mathematics is simply numbers being rearranged again and again. As there are infinite numbers their combinations are endless and sometimes frustrating.*
>
> Year Nine Pupil.

What's new on the nzmaths site this month?

Seven of our units have recently had links made to versions designed to support students for whom English is an additional language.

- Foil Fun
- Making Benchmarks
- Money Matters
- Tony's Spelling Troubles
- Grandpa's Chocolates
- Make a 1000
- A Prime Search

There is also a new Staff Seminar on the subject of Mind Reading, which is well worth a look if you are into number tricks.

Mindreading: One thing that has fascinated many people over the years is the idea of mind reading. I can remember that my wife-to-be and I did a little mind reading act as our party trick. It was simple really. She went out of the room while I stayed while someone chose an object in the room. We used a lot of mumbo jumbo but basically I would ask her if it was a *** and she would answer yes or no. In fact, I always said the chosen object immediately after I had said a four-legged piece of furniture. So this way we could do the trick a couple of times before we said that she was too tired to carry on. We must have done the 'act' many times over the years but we have never been found out.

If you are into mind reading you might like to look at the web site http://digicc.com/fido/. What's going on here?

If you really can't work it out, then try our new staff seminar on the subject of mind reading. It's something that you could present at a staff meeting or give a nice lesson on.

Trumpet blowing: We just thought we'd like to brag a bit. It turns out that it's not just Kiwis who get to look at our site. This month we have had positive comments from the USA and Sweden.

From the US we have received an editor's choice award from a website called Bonus.com. They specialise in Java applications for children and particularly liked our Bright Sparks activities.

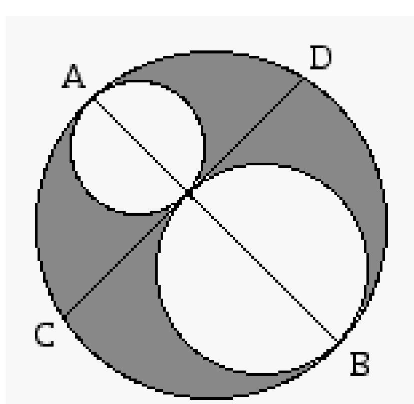
We are also glad to say that there is someone in Sweden who likes our site so much that he wants to translate parts of it into Swedish.

We're impressed by that. We hope that you are too.

One from Archimedes

This problem was posed, with solution, by Archimedes in his publication 'Works' written about 230 BCE. I'm not sure whether it was written on papyrus (probably) or tablets but either way I think it's O.K. to call it a publication.

In the diagram shown, the largest circle with diameter AB is of radius r, the radii of the smaller circles are x and y. The length of the chord CD is 2c.



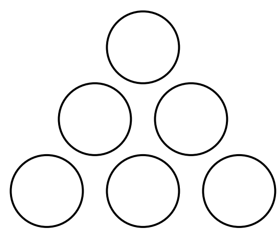
The problem is to find the shaded area. Some of the given information, i.e. x, y, r and c, is redundant. What is the minimum amount of information needed to solve the problem? [Answer below.]

Notice

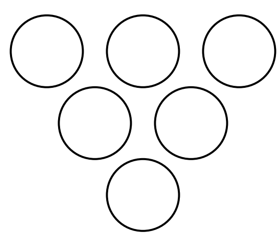
Those of you who are always on the lookout for new books should think about "Brain Benders". We hope to have a review of it in the next newsletter. In the meantime we're told that http://kanukagrove.massey.ac.nz have it as an online order at discount price right now.

A Coin Problem

In the diagram there are 6 coins arranged in the shape of a triangle.



It's not too difficult to show that, by moving three coins only the 6 coins can be made to point in the opposite direction, like the coins below.



The question is though what happens with 15 coins? If you put 15 coins in a triangular shape with 5 rows, what is the smallest number of coins that you can move that will point the triangle in the opposite direction?

We'll let you and your students work on that one for a month. We'll try to remember to give you the solution next month. If you solve it in the meantime, we'd be happy to print your answer. (There's no prize for this though.)

The Numeracy Project

Those of you who are interested in the Numeracy Project (at all levels) may like to know that last month the reference group for the project met in Wellington. As part of that day, the people who evaluated the various sections of the project reported on their findings. From that meeting it is clear that the Project is being well received by teachers and students alike. Students who are being taught this way are showing good progress and seem to be enjoying it more than by the more traditional route.

The findings are very similar to previous years. If you would like to read more, then you can find last year's reports on the web at http://www.nzmaths.co.nz/Numeracy/References/reports.htm. We'll let you know as this year's reports are published.

March's problem

You may remember that last month we went for something completely different. Back in the 1980s recreational mathematician Reg Alteo invented what he called 'Twoons', where the number 2 and various mathematical contrivances were used in the writing of song titles. For example:

On a bicycle made for 2^1 = 'On a bicycle made for two'
You were made 2^2 me = 'You were made for me'
$2^{1.585}$ (approximately) coins in a fountain = 'Three coins in a fountain'
Life begins at $2^2 \times 10$ = 'Life begins at forty'
When I'm 2^6 = 'When I'm Sixty-four'.

What we were looking for this month were more 'twoons', or 'throons' perhaps, even 'noons'. The idea was to encourage the more divergent thinkers among you.

The best solution that we got came from Deborah Lambie of Dunedin. Her answer is shown below.

The 36/3 Days of Christmas

On the 0.000001×10^6 day of Christmas
My true love sent 2^2-2^1 me
1^1 fraction, decimal and %

On the 1+1 day of Christmas
My true love sent 2^2-2^1 me
2^1 – numbers
1^1 fraction, decimal and %

On the $1^1 \times 2^1 + 1^1$ day of Christmas
My true love sent 2^2-2^1 me
333/111 equivalent fractions

2^1 – numbers
1^1 fraction, decimal and %

On the 2^1 x 2^1 day of Christmas
My true love sent 2^2-2^1 me
44/44*4 integers
333/111 equivalent fractions
2^1 – numbers
1^1 fraction, decimal and %

On the 25/5 day of Christmas
My true love sent 2^2-2^1 me
5^1 data displays
44/44*4 integers
333/111 equivalent fractions
2^1 – numbers
1^1 fraction, decimal and %

On the 6 (6+6) /12 day of Christmas
My true love sent 2^2-2^1 me
123-117 regular polygons
5^1 data displays
44/44*4 integers
333/111 equivalent fractions
2^1 – numbers
1^1 fraction, decimal and %

On the 0.000007 x 10^6 day of Christmas
My true love sent 2^2-2^1 me
49/7 problem solving questions
123-117 regular polygons
5^1 data displays
44/44*4 integers
333/111 equivalent fractions
2^1 – numbers
1^1 fraction, decimal and %

On the 1*2*4 day of Christmas

My true love sent 2^2-2^1 me
4096/64/8 statistical investigations
49/7 problem solving questions
123-117 regular polygons
5^1 data displays
4*4/4 integers
333/111 equivalent fractions
2^1 – numbers
1^1 fraction, decimal and %

On the 3^2day of Christmas
My true love sent 2^2-2^1 me
3^3-18 locus problems
4096/64/8 statistical investigations
49/7 problem solving questions
123-117 regular polygons
5^1 data displays
4*4/4 integers
333/111 equivalent fractions
2^1 – numbers
1^1 fraction, decimal and %

On the 100/10 day of Christmas
My true love sent 2^2-2^1 me
40/4 powers of numbers
3^3-18 locus problems
4096/64/8 statistical investigations
49/7 problem solving questions
123-117 regular polygons
5^1 data displays
4*4/4 integers
333/111 equivalent fractions
2^1 – numbers
1^1 fraction, decimal and %

On the 2*2*2+3 day of Christmas
My true love sent 2^2-2^1 me
15-4 square roots

40/4 powers of numbers
3^3-18 locus problems
4096/64/8 statistical investigations
49/7 problem solving questions
123-117 regular polygons
5^1 data displays
4*4/4 integers
333/111 equivalent fractions
2^1 – numbers
1^1 fraction, decimal and %

On the 144/12 day of Christmas
My true love sent 2^2-2^1 me
1+2+3+4+2 quadrilaterals
15-4 square roots
40/4 powers of numbers
3^3-18 locus problems
4096/64/8 statistical investigations
49/7 problem solving questions
123-117 regular polygons
5^1 data displays
4*4/4 integers
333/111 equivalent fractions
2^1 – numbers
1^1 fraction, decimal and %

This Month's Problem

A country estate keeps 1000 doves. The birds are free to fly during the day but always return to roost at night in a dovecote. The dovecote is a grand wooden structure consisting of a number of pigeonholes within which the doves rest at night or during bad weather.

Without considering the problem of why doves would stoop so low as to reside in pigeonholes, can you find the largest number of pigeonholes that could be occupied by the 1000 doves if each hole is occupied but no two contain the same number of birds?

We will give a petrol voucher to one of the correct entries. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so we can send the voucher if you are the winner.

Archimedes' Answer

The shaded area is $S = \pi r^2 - \pi(x^2 + y^2)$.

Also we know that $x + y = r$ and, from the intersecting chord theorem, $c^2 = 4xy$ or $\frac{1}{2} c^2 = 2xy$.

Now, $(x + y)^2 = r^2$
Also, $(x + y)^2 - 2xy = x^2 + y^2$
Hence, $r^2 - \frac{1}{2} c^2 = x^2 + y^2$
So that, $S = \pi r^2 - \pi(r^2 - \frac{1}{2} c^2) = \frac{1}{2} \pi c^2$

Perhaps surprisingly, all of x, y and r are redundant!

Afterthoughts

Overheard from a year 12 pupil:

Which way's horizontal, up or down?

Brickbat

In our November issue we posed an ambiguous problem about 12 grandchildren which, not surprisingly, elicited puzzled responses. The word 'twin' in the original should have read 'sister'. I'm still trying to work out how this typographical error occurred! All other aspects of the problem and its answer were correct. Our apologies if you were unduly ruffled.