Newsletter No. 28 **October 2003**

If you were asked for the names of the most prestigious New Zealand mathematicians, those born in this country, I wonder who would be on your list. A little research, I think, would quickly suggest two people - one alive and one dead, one from the North Island, one from the South and both Fellows of the Royal Society.

Vaughan Jones was born in 1952 in Gisborne and attended Auckland Grammar School and the University of Auckland. He then immediately set off on an academic career picking up numerous scholarships on the way and finally achieving his science doctorate in Geneva in 1979. He moved to America in 1980 and became Assistant Professor of Mathematics, then Associate Professor, at the University of California at Los Angeles before being appointed as Professor of Mathematics at the University of California at Berkeley in 1985.

Alexander Aitken was born in Dunedin in 1895 and attended Otago Boys' High School and the University of Otago. After interrupted study due to war service (he saw action and was wounded on the Somme) Aitken followed his original intention and became a schoolteacher at his old school. With encouragement from the Professor of mathematics at the University of Otago, he soon moved to Scotland where he studied for a Ph.D. at Edinburgh University. His Ph.D. thesis was considered so outstanding that he was awarded a D.Sc. for it. He spent the rest of his career at the University being appointed Professor of Mathematics in 1946.

Aitken's mathematical work was in statistics, numerical analysis and algebra. Jones also worked in a branch of algebra but his most remarkable contribution was his discovery of 'a new polynomial invariant for knots' which led to surprising connections between apparently quite different areas of mathematics. Jones was awarded the prestigious Fields medal in 1990.

If you'd like to know more about these two New Zealand mathematicians check out http://www-gap.dcs.st-and.ac.uk/~history/BirthplaceMaps/Countries/New_Zealand.html

You can claim to be a mathematician if, and only if, you feel that you will be able to solve a puzzle that neither you, nor anyone else, has studied before.

W.W. Sawyer

Fancy a bet?

Not that we want to encourage you to spend the family jewels on the TAB but we have something straight from the horse's mouth that might interest you. And it's why you might want to know about systematic lists – if you are a serious punter, that is.

At first sight there seems to be little money to be made on quadrella betting, unless you are very lucky. As you probably know, to win a quadrella you need to select the four winners in four specified races. So the odds against you are pretty high. After all, if there are 10 horses in each of the selected races, there are 10 possible horses that could win the first race, 10 the second, 10 the third, and 10 the fourth. This means that there are 10 x 10 x 10 x 10 possible outcomes. Choosing one set of four horses at random gives a probability of 1/10000 of winning the quadrella. Not worth the effort, huh?

Well suppose that you could find a way of reducing the potential winners to 2 horses per race. You would then only have to worry about 2 x 2 x 2 x 2 = 16 possibilities. That makes the whole thing more manageable. Given that the odds are going to be good it's almost worth just betting an equal amount of money on each of the 16 possibilities. But suppose you felt that in the first race Golden Ace was more likely to win than Fleetfoot, then you might decide to take out more units on the first horse. So let's make a systematic list of all of the possibilities. In the table, the gaps under a horse mean that the horse should be there but we were too lazy to put them in. So the sixth row indicates a win by Golden Ace in Race 1; Old Gorey in Race 2; Rum Red in Race 3; and Longshot in Race 4.

Race 1	Race 2	Race 3	Race 4	
Golden Ace (2)	Pepper Stake (1)	Rum Red (3)	ChardonnA (2)	12
			Longshot (3)	18
		Fire Chariot (1)	ChardonnA (2)	4
			Longshot (3)	6
	Old Gorey (1)	Rum Red (3)	ChardonnA (2)	12
			Longshot (3)	18
		Fire Chariot (1)	ChardonnA (2)	4
			Longshot (3)	6
Fleetfoot (1)	Pepper Stake (1)	Rum Red (3)	ChardonnA (2)	6
			Longshot (3)	9
		Fire Chariot (1)	ChardonnA (2)	2
			Longshot (3)	3
	Old Gorey (1)	Rum Red (3)	ChardonnA (2)	6
			Longshot (3)	9
		Fire Chariot (1)	ChardonnA (2)	2
			Longshot (3)	3
				120

But we can do better than just assume that each of the outcomes is equally likely. In brackets we've made a judgement of the relative merits of the horses. For various reasons we think that Golden Ace is twice as likely as Fleetfoot to win Race 1; that Pepper Stake and Old Gorey are equally likely to win Race 2; that Rum Red is three times as likely as Fire Chariot to win the third race; and that Longshot will beat ChardonnA 3 times out of 5.

If you now look in the last column of the table you'll see how many units we want to take out on each combination of horses. The number in each row is the product of the numbers in brackets in each row and tells how many times out of 120 possible outcomes, we'd expect that combination to win. And that's also the dollar amount that we are going to bet.

That's all very nice but how do you reduce the number of horses from 10 to 2? In practice it's never ever likely to be that easy. In all likelihood there are going to be several horses in each race that stand a reasonable chance. But how do you get 10 horses down to even 5? Well that's the really hard bit. That's where you pore over the form guides or look at the videos for hours or go to the track where the horses are exercising or spend ages in front of crystal balls or get it straight from the horse's mouth.

(The method above is used by a regular punter that we know of. His real skill is not so much mathematical but rather getting the 'really hard bit' right.)

Of Historical Interest

When I was a school pupil, in about Form Four if I remember correctly, our enthusiastic maths teacher showed us how to extract the square roots of whole numbers 'by hand'. We didn't have calculators anyway in those days and tables only worked to four figures. The method was capitalised upon when mechanical, hand-cranked calculators first hit the classroom, I remember. What I had forgotten, until recently, was the method we used, then I came across it.

In 'A Treatise of Arthmetic, in Theory and Practice' written by John Gough and published in Philadelphia in 1788 is material on how to extract square roots. What makes it interesting is that it is written in rhyme. Here it is, see if you can decipher the method …

First to prepare the square, this do
Point off the figures two by two:
Beneath the last the square next less
Put; and its root i'th' quotient place:
From the last period take the square,
Then the next lower period there
To the remainder must be brought:
Be this a dividend: The quote
Doubled must the divisor be

To all but units place; then see
How oft the greater holds the less,
The figure must the quote express,
And the divisor units too,
Then as plain Division do.
Thus every period one by one
We manage, and the work is done.

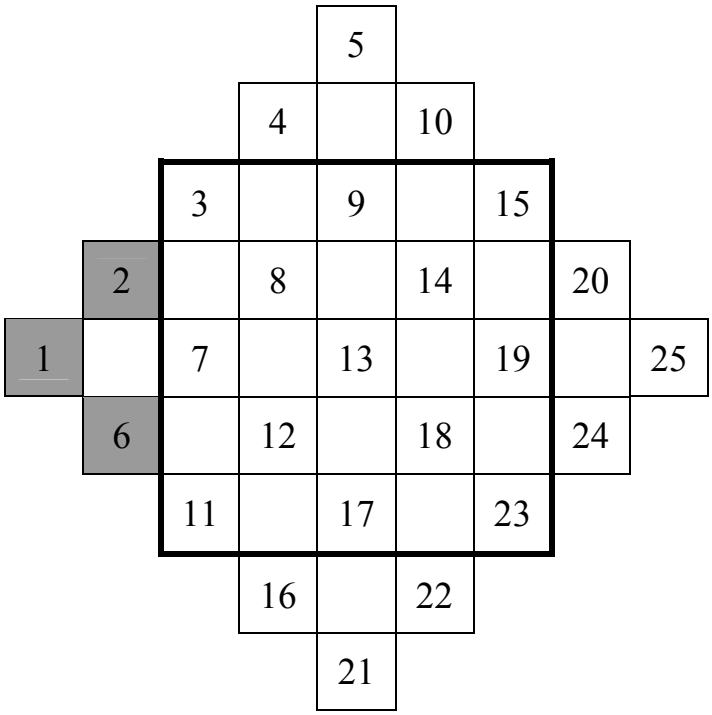
Gough also supplies a rhyme for extracting cube roots but it is fiendishly complicated!

5x5 Magic Squares by Brian Bolt

(You may remember that last month Brian wrote about 3 x 3 magic squares. Here he takes it up a notch.)

Over the years people have found neat ways of constructing odd order magic squares. The following way due to Bachet de Méziriac is particularly attractive as it can be used for any odd order square including 3x3.

3	16	9	22	15
20	8	21	14	2
7	25	13	1	19
24	12	5	18	6
11	4	17	10	23



First border a 5x5 square as shown, then number the diagonals from far left to top right using the numbers 1 to 25 in sequence. Next imagine sliding the numbers outside the original square into the spaces on the opposite side of the square without changing their arrangement. The result is a magic square.

		1		
	5			
4				
				3
			2	

(i)

		1	8	15
	5	7	14	
4	6	13		
10	12			3
11			2	9

(ii)

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

(iii)

A second method, which I always refer to as the North East rule, begins by putting a 1 in the middle of the top row, then moving diagonally one square up and to the right. If a move takes you off the edge then imagine that opposite edges are joined and start on into the opposite edge. See figure (i). This is fine for filling in 1 to 5 but then the way ahead is blocked as the square is already occupied. When this happens continue by dropping down one square and continue along the diagonal until it happens again. See figure (ii). In this example the 'drop move' is required after 5, 10, 15 and 20, the multiples of 5. The completed square, see figure (iii) is different from that of Bachet de Méziriac, but in each case the number in the centre is 13 and the magic total 5x13 = 65.

We now have two different solutions and this raises the question as to how many others there might be. It was many years before I found a way of producing further solutions using a variation on the NE rule based on an investigation I had developed in a different context with the Saturday morning master classes I lead for 13 year olds. Instead of moving one square diagonally I decided to try a knight's move, 2 squares to the left and 1 up.

					2		9									
			1		8						24	15	1	17	8	
	5		7			5					5	16	7	23	14	
	6			4							6	22	13	4	20	
		3		10			3				12	3	19	10	21	
		9		11	2						18	9	25	11	2	

nothing special about a knight's move, the approach can be generalised by using any vector step in a consistent way to produce yet more solutions.

In each of these solutions 13 always ends up in the centre. Is this inevitable? By comparison with the 3x3 square it seemed likely and I assumed that as all my solutions to date supported that fact I took it for granted. Imagine my surprise when I found a new method of constructing 5x5 magic squares using 1 to 25 which allowed me to have any of these numbers in the middle.

Take any five numbers a, b, c, d and e and put them in the first row of a 5x5 array. Now shift them 3 places to the left to form the second row and repeat the process for succeeding rows. The result is a magic square as each row, column and diagonal sum to a+b+c+d+e. Similar arrays constructed by shifts of 2 places are also magic.

a	*b*	*c*	*d*	*e*
d	*e*	*a*	*b*	*c*
b	*c*	*d*	*e*	*a*
e	*a*	*b*	*c*	*d*
c	*d*	*e*	*a*	*b*

Now consider the two magic squares below formed with 1 to 5 and a shift of 3 places to the left together with multiples of 5 from 0 to 20 using a shift of 2 places to the left.
Adding these together forms a 5x5 magic square See (iii).

1	2	3	4	5
4	5	1	2	3
2	3	4	5	1
5	1	2	3	4
3	4	5	1	2

(i)

0	5	10	15	20
10	15	20	0	5
20	0	5	10	15
5	10	15	20	0
15	20	0	5	10

(ii)

1	7	13	19	25
14	20	21	2	8
22	3	9	15	16
10	11	17	23	4
18	24	5	6	12

(iii)

Not only does this square have 9 in the middle, not 13, but it can easily be manipulated to produce a magic square with any one of the numbers 1 to 25 in the middle.

Any cyclic permutation of the rows or columns leaves the magic property invariant. So, for example, to get 8 in the middle it is necessary to cycle the columns 2 places to the left then rows one down.

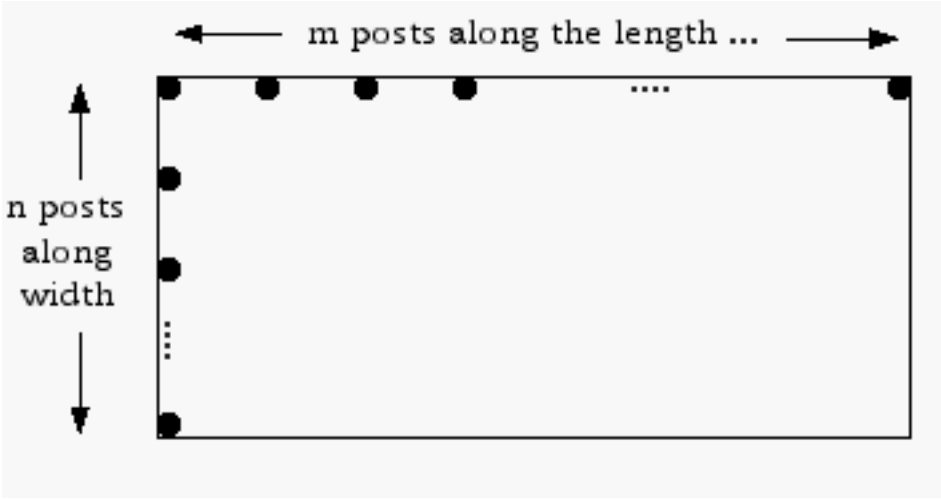
Another way to look at this property is to imagine a tessellation of squares where each square consists of the above magic square. Then any 5x5 square you pick out of this array will be magic and contain 1 to 25. Shown below is the square chosen with 14 in its centre.

1	7	13	19	25	1	7	13	19	25	1	7	13	19	25
14	20	21	2	8	14	20	21	2	8	14	20	21	2	8
22	3	9	15	16	22	3	9	15	16	22	3	9	15	16
10	11	17	23	4	10	11	17	23	4	10	11	17	23	4
18	24	5	6	12	18	24	5	6	12	18	24	5	6	12
1	7	13	19	25	1	7	13	19	25	1	7	13	19	25
14	20	21	2	8	14	20	21	2	8	14	20	21	2	8
22	3	9	15	16	22	3	9	15	16	22	3	9	15	16
10	11	17	23	4	10	11	17	23	4	10	11	17	23	4
18	24	5	6	12	18	24	5	6	12	18	24	5	6	12
1	7	13	19	25	1	7	13	19	25	1	7	13	19	25
14	20	21	2	8	14	20	21	2	8	14	20	21	2	8
22	3	9	15	16	22	3	9	15	16	22	3	9	15	16
10	11	17	23	4	10	11	17	23	4	10	11	17	23	4
18	24	5	6	12	18	24	5	6	12	18	24	5	6	12

Solution to September's problem

The problem can be solved either by trial and error or some fairly straightforward algebra and we got both of these types of solution from readers. Derek Smith sent in the algebraic method and Damian Redpath the trial and error one. We are going to divide the prize between the two of them.

Suppose Johnny used m posts along the length of his chicken run and n along the width as shown,



Then, since the posts are one metre apart, the length and width of the run are m -1 and n - 1 metres respectively, giving an area of (m - 1)(n - 1) square metres. The perimeter of the run is 2m + 2n - 4 metres and this, we are told, is numerically equal to the area.

So,	$2m + 2n - 4$	$= (m - 1)(n - 1)$
		$= mn - m - n + 1$
Rearranging gives,	$mn - 3m$	$= 3n - 5$
i.e.	$m(n - 3)$	$= 3n - 5.$

Making m the subject gives, $m = (3n - 5)/(n - 3)$ ……..**Formula A**

Since m and n represent a whole number of metres we can investigate **Formula A** by tabulating for a few values of n. The denominator tells us that n > 3. Trying n = 4 first gives m = 7, then n = 5 gives m = 5 and so on.

n	**m**
4	7
5	5
6	13/3
7	4

As we try larger values of n, m takes decreasing values approaching 3, so as far as the problem is concerned our table is complete. Fractional values of m are inappropriate as are the values m = n = 5 (the chicken run is a rectangle not a square). Since m and n are interchangeable, the top and bottom rows of the table represent the same solution.

Hence the number of posts along the length is 7 and along the width 4. Johnny used 18 posts to make his chicken run.

This Month's Problem

This problem can be solved by trial and error but try to shortcut using the mathematics you have learnt about whole numbers. The question is, how many numbers divide into 68 274 with remainder 9?

Each month we give a petrol voucher to one of the correct entries. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so we can send the voucher if you are the winner.