

NZ Maths Site Newsletter

September 2003

Why is it, I wonder, that many of us don't use the history of mathematics in our teaching? After all, it is rich in accessible material and there is a lot that can be learnt by working through problems met and solved by our predecessors. Three reasons come to mind - lack of interest, lack of time and a perceived lack of expertise. Not much can be done about the first of those, while the second seems more like a problem of priorities than time management. For those of you who feel you don't know enough about the history of mathematics to confidently lead classroom discussion on the subject, I hope these newsletters will be of help. Previous issues have included a number of items of historical interest. In July there was Pythagoras' Theorem with a proof and John Stillwell's articles are always good reading. In this issue we've included an idea which might provide a start on how you could introduce the history of mathematics in your classroom. We will continue to include regular items to help you on your way.

It's use is not just that history may give everyone its due and that others may look forward to similar praises, but also that the art of discovery may be promoted and its method known through illustrious example.

Leibniz

What's new on the nzmaths site this month?

<u>New Units</u>

There are 7 new units available on the site this month, mostly in the Algebra strand. They are: Guzzinta – Number, Level 6 Tilted Squares and Right Triangles – Algebra, Levels 5&6 Arithmagons – Algebra, Level 5 Cups and Cubes – Algebra, Level 3 The Truth About Triangles and Squares - Algebra, Level 2 Supermarket Displays – Algebra, Level 2 Dino Containers - Number/Measurement, Level 1

An Historical Start

One thing you might like to try to introduce the history of mathematics in your classroom places the first step in your pupils' hands. It doesn't require any esoteric knowledge of the subject and creates a lot of interest. Why not give each class member a mathematician to research - his life, times and works? Look for a short biography (up to only a page in length) which covers context - when and where s/he lived - the area of work for which they are best remembered and any other interesting points about their lives. Emphasise that you don't want your students to write about mathematical material they don't understand. Mark it broadly; don't make a chore for yourself. Assume students have the facts correct and just enjoy what you read. Give a tick for each point covered from a list you've previously compiled - for example, when, where, the general mathematical area in which the mathematician worked, extra titbits, bonuses and so on - perhaps five items in all. Grade accordingly. That way, your pupils exercise their research skills and you get to learn a little about the history of maths. I ask my students to read some of the biographies out in class and a couple of the neater ones to put their efforts together for some sort of classroom wall presentation. The students love it!

How do you decide on the mathematicians for your students to research? Well, I write some names on pieces of paper and pop them in a hat for students to choose one each. I tend to include quite a lot of the early Greeks because the maths was simpler then. Here's some names for you that I've successfully tried: Thales, Pythagoras, Archytas, Zeno, Anaxagoras, Hippias, Hippocrates (of Chios - not to be confused with his contemporary Hippocrates of Cos who gave his name to medical ethics), Eudoxus, Menaechmus, Aristotle, Euclid, Aristarchus, Archimedes, Apollonius, Eratosthenes, Hipparchus, Hero, Ptolemy (of Alexandria), Diophantus, Hypatia, Alkarismi, Cardan, Vietta, Descartes, Pascal, Fermat, Newton, Leibnitz, Euler, Lagrange. A worthy New Zealander you might like to include is Alexander Aitken. He was a pupil of Otago Boys' High School and went on to gain the chair of Pure Mathematics at the University of Edinburgh among other glittering prizes of his profession.

It is true that for a number of reasons most of the important earlier mathematicians were men but there were a number of notable women including Emmy Noether, Sonya Kovalevski and Maria Agnesi as well as Hypatia mentioned above. If you'd like to know more about women mathematicians in particular, check the website <u>http://www.agnesscott.edu/lriddle/women/alpha.htm</u>

Some further items on the history of mathematics which we included in earlier newsletters are:

Issue 12: April '02	Infinitesimals
Issue 16: Aug. '02	Pi
Issue 18: Oct. '02	Editorial and Sundaram's Sieve
Issue 19: Nov. '02	Robert Recorde and multiplication
Issue 22: April '03	Pythagorean triples
Issue 23: May '03	Editorial

Issue 24: June '03	The size of the Earth
Issue 25: July '03	Pythagoras' Theorem
Issue 26: August '03	Fibonacci Numbers, more on Pythagoras' theorem.

There are a number of links between the history of mathematics and the history of astronomy. Many of the early mathematicians were also solving problems of astronomy. John Stillwell's article in Issue 24, for example, described the work of Eratosthenes in determining the size of the Earth. One of our team, who sings in cafes as a sideline, was recently giving a lecture on the subject for a U3A course on astronomy. U3A is the acronym for University of the Third Age, a learning organisation for retired people. At the end of his talk he sang a song he'd written to summarise some of the points he'd made. We include it for your entertainment.

Wrong Turnings

- The ancients thought the Earth was flat Like a disc, think of that, Around the land they thought water To go that far they didn't oughter. Wrong turnings, wrong turnings, the gods got in the way When priests rule minds then you'll find, progress fades away.
- Eclipses come, eclipses go
 When and where they didn't know,
 Then came Thales with his science
 Picked the time with some reliance.
 Wrong turnings, wrong turnings, gods had the power
 But when science is applied, reason will flower.
- Planets round the earth they spun Sun and moon as well had done But some of them backwards flew Eudoxus was sure he knew. Wrong turnings, wrong turnings, oh what a strain Spheres round spheres just weren't right, have to try again.
- 4. Hipparchus thought Eudoxus wrong Epicycles were the thing, Ptolemy said it best In his book the Almagest. Wrong turnings, wrong turnings, will we ever get it right Throw out prejudice and then maybe we might.
- 5. Nothing new for two thousand years Scientists repressed by fears, Copernicus was the one

	Who made the Earth go round the sun. Wrong turnings, wrong turnings, but there's hope in sight Think again, then again and we'll get it right.
6.	Tycho Brahe's observations And Johann Kepler's machinations, Gave us laws which hold today And helped Newton on his way. Wrong turnings, wrong turnings, prejudice gone for good Science rules the cosmos now - as of course it should.

More on Pythagoras' Theorem

Try the following game but first make up the some of the shapes below. You should make four of the right-angled triangles, one square of side length 3, one of side length 4 and one of side length 5. The dimensions are shown on the diagram. Perhaps the easiest way to do this is to use squared paper but any convenient unit of measurement would be fine - centimetres perhaps.



The puzzle is to make as many squares as you can from the pieces you have made. Not every piece has to be used in each case but you have to use more than one.

There are several ways to do this but in this newsletter I'll concentrate on two of them. (I'd be interested in knowing how many squares you were able to produce.) The two squares I want for now are shown in the diagram below.

If you look at the squares below, you will see that they are both 7 by 7. This means that the two areas have to be the same.

Now the one on the left is made up of four right-angled triangles and two squares, while the one on the right is made up of four right-angled triangles and one square. This means that the two small squares have the same area as the bigger square. In equation form we have

$$3^2 + 4^2 = 5^2$$
.

But that is what Pythagoras' Theorem would have told us.



But having discovered this we can repeat the exercise with *any* right-angled triangle and squares that correspond to its sides. So take a triangle with sides a, b and c. Using these in the appropriate combinations we can make two squares of side length a + b. The first of these has four right-angled triangles, an $a \times a$ square and $a b \times b$ square. The second has four right-angled triangles and one $c \times c$ square. Since both of these a + b squares clearly have the same area and four triangles in common, the $a \times a$ square together with the $b \times b$ square have the same area as the $c \times c$ square. So

$$a^2 + b^2 = c^2.$$

Since another way to read this is to say that, in a right-angled triangle, the squares on the two non-hypotenuse sides have the same area as the square on the hypotenuse, we have just proved Pythagoras' Theorem.

Magic Squares of Order Three by Brian Bolt

Most children now meet this 3x3 magic square early in their schooldays and are intrigued by its property that the numbers in each row, column and both diagonals all add up to 15.

8	1	6
3	5	7
4	9	2

But how many spot that the numbers along each line through the centre are in arithmetic progression, or that 5 is midway between the two outer numbers. Few know that the sum of the squares of the numbers in the first and third rows balance.

$$8^2 + 1^2 + 6^2 = 101 = 4^2 + 9^2 + 2^2$$

as do those in the first and third columns

$$8^2 + 3^2 + 4^2 = 89 = 6^2 + 7^2 + 2^2$$

If your pupils haven't met this magic square you might like to introduce it as a puzzle to solve. If they seem to be taking rather a long time on it then you might tell them that the number 5 should be placed in the centre box. This is the key to solving the puzzle and that the magic total is three times 5. This fact can be neatly established for <u>all</u> 3x3 magic squares whatever the centre number.

Let the nine numbers in the square be a, b, c,....i and the magic total T. Then by considering the numbers along each diagonal and the middle row we can write down three equations;

$$T = a + e + i$$
$$T = d + e + f$$
$$T = g + e + c$$

Adding these equations together gives

$$3T = (a + d + g) + 3e + (i + f + c)$$

but the letters bracketed correspond to two columns and so total T each. Taking 2T from both sides leaves

$$T = 3e$$

	а	b	c
	d	e	f
	g	h	i
/			

This establishes the fact that the magic total must be three times the number in the central square.

This one fact allows us to construct as many different magic squares as we please. Start with any number in the centre, say 7, and two numbers in the top corners, say 3 and 10. The magic total must be $3 \times 7 = 21$, so using this fact we can systematically deduce all the other numbers for the square.



Note again that

 $3^2 + 8^2 + 10^2 = 4^2 + 6^2 + 11^2$ and $3^2 + 14^2 + 4^2 = 10^2 + 0^2 + 11^2$

When constructing magic squares in this way you may find some numbers are repeated, some may be negative or even fractional, but it will still be a magic square.

Complete the following to see what can happen:

5		11
	7	





[If you'd like to have a go at other problems related to magic squares, see activity 128 in Brian's book *The Mathematical Funfair* and activity 72 in his book *Mathematical Cavalcade*.]

Solution to August's Problem

The problem poser for this month's problem gives this as the solution. There are two ways the numbers 1 to 9 can be collected in groups of three which have the same sum, namely (2, 4, 9), (1, 6, 8), (3, 5, 7) and (1, 5, 9), (2, 6, 7), (3, 4, 8). Only the first of these satisfies the other given information that no two of any triple should be consecutive. Hence the runner who came next-to-last, gaining two points, ran for the same club that had the first runner home (gaining nine points), Achilles' Arrows.

An alternative quicker method assumes, not unreasonably, that the question has an answer. Since the problem is symmetrical in terms of Hermes' Harriers and Zeno's Zephyrs (there is insufficient information in the problem for us to distinguish which of these two clubs a runner represents) any specific place-getter we are asked to identify must run for Achilles' Arrows.

It was actually difficult to find a winner this month as we had many answers sent in and all were correct. We saw a variety of methods but, after considerable deliberation, we gave the voucher to Leigh Walker, a 15 year old from Gisborne Girls' High School for this well argued solution.

I worked out the maths problem by working out that there was a total of 45 points and they were all tied so each had 15. Then I read that Achilles had a first place so that gave them 9 points, since they had 9 points they couldn't have either 8, 7 or 6 points, I knew that one of the other teams had to have 8 and 6 and the other team have 7. The team with 8 and 6 had to have the one point to make it up to 15. So that means that Achilles couldn't have 5 because then they wouldn't need the 1 point. So then I knew that the team with 7 had to have 5 then making them need 3 to make it add to 15. That meant that it left me with 4 points and 2 points. So that left 2 points being awarded to Achilles.

This Month's Problem

Johnny fenced off a rectangular area for his chickens. He placed the posts one metre apart all round the edge before tacking the wire netting in place. When he'd finished the job he noticed that the area of the chicken run was numerically the same as the perimeter. How many posts did Johnny use in making the run?

Each month we give a petrol voucher to one of the correct entries. Please send your solutions to <u>derek@nzmaths.co.nz</u> and remember to include a postal address so we can send the voucher if you are the winner.