

NZ Maths Site Newsletter

August 2003

Board games can provide a valuable educational tool in the mathematics classroom at all ages. Such games can be grouped loosely under various headings. War games like Chess and Draughts, race games like Backgammon, Ludo and Fox and Geese, games of position like Nine Men's Morris and I-go and mancala games are just four. All these games are useful for the insights they give in defining and testing strategies and perhaps, in some cases, the formation of proofs.

Many of the games, some played over centuries, have been forgotten and superseded by electronic alternatives more appealing to young people. It's interesting that when electronic games first came on the market, either on early home computers or hand-held ones, they were mostly electronic versions of traditional games. Some favourites are still popular, played either electronically, with boards and pieces, or with pencil and paper. The popularity of games like noughts and crosses, chess, draughts, backgammon, I-go and Chinese checkers is assured. Some teachers, I know, keep packs of games they've made up themselves available in the classroom for use by pupils who have completed pieces of set work. One you may not have met before and which is easy to make and play is described below.

At evening, when with pencil, and smooth slate In square divisions parcelled out and all With crosses and with ciphers scribbled o'er, We schemed and puzzled, head opposed to head In strife too humble to be named in verse.

William Wordsworth

What's new on the nzmaths site this month?

<u>New Units</u>

There are 5 new units available on the site this month. They are:

- What Happens on Average Algebra, Level 5
- Taller, Wider, Longer Measurement, Level 1
- Perplexing Perimeters Measurement, Level 3
- Triangles Measurement, Level 4
- Fences and Posts Measurement/Algebra, Level 5

Number Framework Links

We have introduced a new category of units this month; units from strands other than Number with a link to stages of the Number Framework.

There are 8 units available so far at

http://www.nzmaths.co.nz/Number/crossstrand.htm :

Number Framework Stage	Strand/Level	Unit Name
Count all	Statistical Investigations, Level 1	Greedy Cat
		I like trucks
Advanced Counting	Measurement, Level 1	Taller, Wider, Longer
	Algebra, Level 1	Counting on Counting
Early Additive	Statistical Investigations, Level 2	Party, Party, Party
Advanced Additive	Measurement, Level 3	Perplexing Perimeters
Advanced Multiplicative (Early Proportional)	Probability, Level 3	Predict Away
	Measurement, Level 4	Triangles

Numeracy Project Activities and games

In the Numeracy Project materials section of the site you can now find a collection of additional lessons and games to support the Numeracy Project material. http://www.nzmaths.co.nz/Numeracy/Other%20material/Lessons.htm

<u>New Games</u>

There are two new games around patterns within 5 located in the Number Knowledge section of the site. They are called Blast Off, and Party Time.

Diary Dates

A reminder that Maths week is 10-16th August. There's some really great material on the NZAMT web site. Make sure that your students get involved. Keep an eye on <u>www.mathsweek.org.nz</u>.

We have managed to get three articles from a range of people and places this month so the further proofs of Pythagoras promised in July will be held over till next month. (Did I hear a big sigh?)

The Board Game Pong hau k'i

A number of the so-called games of position are still played today. Versions of Nine Men's Morris have been played for more than three thousand years. The game is called Mill in Scandinavia and America and is a relative of the well-known Noughts and Crosses. Boards have been found in ancient Egyptian tombs and in more recent times scratched onto choir stalls in English cathedrals, presumably to help pass the time during those long, medieval sermons. The complicated board game Go or I-go originated in China but soon became the national game of Japan. It was also popular here. In the early 1900s some New Zealand newspapers, including the Otago Daily Times, even featured a Go column much like the Listener's chess column today. Backgammon is occasionally played in this country and is very popular in Turkey and parts of Europe.

Of all the many games of position Pong hau k'i which originated in Canton, China, is probably the simplest with just four pieces and five board positions. Each player has two stones of different colour, placed as shown below (on four of the possible board positions), and one is moved in alternate turns of play along any line to the next empty position. At the start, of course, only the position marked A is empty and so whether the first player to move is black or white that's where the first move is made. There doesn't seem to be any rule about who goes first so that can be decided on the toss of coin. If a player blocks his opponent and prevents him/her moving, the latter has lost the game. With only thirty different positions of the pieces possible and only four end games, two for each colour, the game is easy to analyse yet interesting to play. Why not draw up a board and have a go?



Fig: Pong Hau k'i board with the pieces placed at the beginning of the game.

The game is won, as explained, by blocking your opponent. In what is essentially only one position (one mirror solution and two equal options for your opponent make the four mentioned) you find that one player is unable to move a piece onto the adjacent empty position because there isn't one! The adjacent positions are taken by your opponent, the empty position is beyond your reach, so to speak. When you play the game it becomes clear.

Pages of Postage Stamps by Brian Bolt

Many years ago I came across the following puzzle in a magazine:

a	b	с
d	e	f

It is required to design a page of six postage stamps so that by allocating appropriate values a, b, c, d, e, f to the stamps it will be possible to tear off a single stamp or a connected set of stamps (connected by a common edge) whose total value is 1, 2, 3, 4,..., N. The idea is to make N as large as possible without leaving any gaps.

You might like to try this problem right away, but my experience with pupils/students is to start with the equivalent problem for a page of four stamps.

a	b	1	1
с	d	2	5

For example, by allocating a=1, b=1, c=2 and d=5 it is possible to tear off a stamp or set of connected stamps to the values 1, 2, 3, 4, 5, 6, 7, 8, 9. You don't need to look for long to see how to improve on this. But what is the optimum solution? "Please sir have I got the best solution?" is invariably the question I get faced with when using this investigation. With four stamps the answer lies in considering all the different ways in which a stamp or set of connected stamps can be removed from the page. This gives an upper bound to the value for N and is achievable in two ways which is very satisfying. The same spatial approach to the page with six stamps gives a target of N to aim for but in this case it can't be reached owing to the constraints on the way the stamps are connected. But a surprisingly high value for N is achievable, quite close to this spatial upper bound. You can find the optimum solution, again two possible, in my book Even More Mathematical Activities.

Suppose that instead of square stamps, a page consists of four triangular stamps. What would the optimum solution be in this case?



The original stamp puzzle suggested a whole range of similar problems to me. One which still intrigues me is to design a *strip* of n stamps so that it is possible to tear off a stamp or a connected set of stamps whose total values are 1 to N where N is to be as large as possible.

1 1	1	1		1
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Suppose, for example, the strip of stamps all had value 1, then it is easy to see how with n stamps it is possible to tear off strips with values 1, 2, 3,, n. But this is very inefficient. Suppose I wanted a strip with two stamps, then allocating 1 and 2 as their values it is possible to remove a stamp of stamps with totals 1, 2 or 3.

With a strip of three stamps totals of 1, 2, 3, 4, 5 and 6 are possible, and with four stamps consecutive totals from 1 to 9 are possible. Here are two of the four solutions I have found so far.

1 3 3 2 2 5 1 3

For quite sometime I had thought that the optimum solution would always be found by starting with 1 followed by a string of 3s and ending in 2. It certainly gives the optimum with a string of one, two, three, or four stamps but not thereafter. I made the mistake of generalising too quickly!

The upper bound can again be found from spatial considerations by seeing how many ways one stamp, two stamps, three stamps, etc., can be removed from the string. For example, with four stamps:

one can be removed in	4 ways
two can be removed in	3 ways
three can be removed in	2 ways
four can be removed in	1 way

This gives the fourth triangular number 10.

In the same way it can be seen that with a string of n stamps the upper bound is the nth triangle number, that is $\frac{1}{2}n(n + 1)$.

Here are two solutions for a string of seven stamps which allow consecutive totals from 1 to 23 to be removed.

1 3 6 6 2 3 2 1 1 9 4 3 3 2

This is 5 short of the seventh triangle number 28 so I may not yet have found the optimum. There doesn't seem any obvious pattern in the sequence either.

Investigate the best arrangements for five stamps and six stamps and see if you can better my solution for seven stamps.

(We should point out that this article is by THE Brian Bolt. If you have heard of the books Mathematical Activities, More Mathematical Activities, The Mathematical Funfair, A Mathematical Pandora's Box, Mathematical Cavalcade, and so on, these were all written by Brian and all published by Cambridge University Press. If you are short of several hundred maths problems for students of all ages, then you should make sure that these are in your school library. Brian is now retired and lives in Exeter where it is a short hop to his favourite walking spot – Dartmoor.)

The Fibonacci Numbers by John Stillwell

The numbers 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ... are called the *Fibonacci numbers* after the Italian mathematician Fibonacci (1170 - 1250). The name Fibonacci is actually a nickname, derived from "filius Bonaccii or "son of Bonacci", and he is more officially known as Leonardo Pisano or Leonardo of Pisa.

His numbers, as you have probably noticed, follow the rule that each (after the first two) is the sum of the previous two. Thus

$$2 = 1 + 1
3 = 1 + 2
5 = 2 + 3
8 = 3 + 5
...$$

and so on. This process quickly produces large numbers, and hence gives good practice in addition, which is what Fibonacci intended. The 20th Fibonacci number is 6765 and the 30th Fibonacci number is 832040.

Fibonacci was in favour of adding large numbers because he wanted to show people the advantages of something we now take for granted: Arabic numerals. In Fibonacci's time, Europeans used Roman numerals as names for numbers but they used the abacus for actual calculation, since adding and multiplying with Roman numerals was, and still is, diabolical.

Fibonacci learned Arabic numerals as a child in North Africa, where his father assisted Pisan merchants in their commerce with the Arab world. He introduced Arabic numerals to Europe in 1202 in a book called the *Liber abaci*, which means "book of calculation". It sounds like "book of the abacus", but only because the abacus was then synonymous with calculation. In fact, the *Liber abaci* is about calculating *without* the abacus, and the Fibonacci numbers turn up in one of its worked examples. It consists in adding numbers in the Fibonacci sequence for 12 steps past the initial 1, 1, but it is dressed up as a problem about the breeding of rabbits:

A certain man had one pair of rabbits together in a certain enclosed space, and one wishes to know how many are created from the pair in one year when it is the nature of them in a single month to bear another pair, and in the second month those born to bear also.

This just goes to show that bogus "applications" are as old as mathematics! In fact, Fibonacci numbers *do* occur in nature, though this was probably not known in Fibonacci's time. Some plants, such as the "sneezewort" actually grow new shoots very

like Fibonacci's rabbits breed new rabbits, so the number of shoots at appropriate intervals is 1, 2, 3, 5, 8, . . . Also, flowers such as the sunflower and coneflower have seeds in the centre arranged in spirals, and the number of spirals tends to be a Fibonacci number. A spectacular example is shown at the web site

http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html#seeds

in which there are 55 spirals going one way and 34 going the other.

Fibonacci numbers also turn up frequently in mathematics and computer science. One reason for this is that the sequence of Fibonacci numbers is quite complex, despite the simple rule for producing it, and it raises interesting mathematical questions. How do you compute the 100th Fibonacci number, say, without computing the 99 Fibonacci numbers before it? The answer was not found for more than 500 years after the *Liber abaci* appeared. It lies in the formula discovered by the English mathematician Abraham de Moivre in 1730:

*n*th Fibonacci number =
$$\frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2} \right)^n + \left(\frac{1-\sqrt{5}}{2} \right)^n \right].$$

Who would have thought that the Fibonacci numbers have anything to do with the square root of 5? The reason for this is another story, which I don't have time to tell right now, but you can read about it in books on the *golden ratio*.

(This is another article from John Stillwell who was a full-time member of the Maths Department at Monash University in Melbourne for many years. He has now decided that it's more fun to spend six months there and six months at the University of San Francisco. He has written a number of books but his book Mathematics and Its History, published by Springer-Verlag in 1987, is something you might like to get hold of.)

http://www.censusatschool.org.nz by Megan Jowsey

A statistical experience......

About the Project

CensusAtSchool NZ joins an international **educational** project designed to enhance statistical enquiry. It began in the UK, but was based on a trial project by Dr Sharlene Forbes of Statistics New Zealand, which took place in 1990. **The Royal Statistical Society (RSS) Centre for Statistical Education** (Nottingham Trent University), started the **CensusAtSchool** project in 2000 and it has since been joined by South Africa, Queensland, South Australia, New Zealand and most recently Canada.

CensusAtSchool NZ is an online children's survey for **school Years 5** through to **10**. Schools take part voluntarily, with students completing the short survey during lesson

time, then submitting their data to contribute to an international database. We have included some of the internationally common questions, to provide comparisons between countries, while tailoring the remainder of the questionnaire to reflect the interests of New Zealand children. Results and sample data will be made available to teachers once the 'census' is complete and classroom resources will be developed over time.

CensusAtSchool NZ is hosted by the Department of Statistics at the University of Auckland and is coordinated by a secondary Mathematics teacher as part of a NZ Science, Mathematics and Technology teacher fellowship, awarded by the Royal Society of New Zealand. During 2003 the team will plan, launch and complete the first phase of CensusAtSchool NZ. We are a non-profit, educationally motivated project.

When is it happening? 11 August – 12 September 2003

An international on-line children's census for Year 5 through to Year 10

What do schools gain? A ready made data collection experience for students, motivation and ideas for statistical learning, real and relevant multi-variate data sets.

What is involved? Register and find out more on-line, students do a 10 minute survey, results and data available soon after end of census. It will be a rich resource free to schools.

In association with 'Maths Week', hosted by Dept of Statistics, University of Auckland

Schools can register now for CensusAtSchool NZ 2003 on our website:

http://www.censusatschool.org.nz

(Megan Jowsey has a NZ Science, Mathematics and Technology Teacher Fellowship this year. So she is on leave from her job as HOD Maths at Birkenhead College. She would be interested to get in touch with other teachers who would like to work on the census project in the future.

We also would encourage teachers to apply for these fellowships. As anyone who has had one will tell you, they are an extremely stimulating and satisfying experience.)

Feedback

Last month's item on Murphy's Law and others in the same style encouraged a letter from Reg Alteo. He wrote that apparently Murphy's Law has undergone empirical testing on at least two different occasions in the classroom:

One day a teacher named Murphy wanted to demonstrate the laws of probability to his maths class. He had thirty of his students spread peanut butter on slices of bread, then toss them into the air to see if half would fall on the dry side and half on the buttered side. As it turned out, 29 of the slices landed peanut-butter side on the floor, while the thirtieth stuck to the ceiling.

In the second test, another teacher conducted the Murphy experiment and all of the slices landed on the floor with their buttered sides down. He ran straight away to his Head of Department to report this deviance from one of the basic rules of nature. At first the HOD would not believe him but finally became convinced that it had happened. However, he didn't feel qualified to deal with the question and passed it along to the School Principal. After weeks of waiting, the principal finally came up with an answer: "The bread must have been buttered on the wrong side."

We also received a few more examples of Murphy's Law, many thanks for those

Motorists Law: You never really learn to swear until you learn to drive.
Law of Manana: Hard work pays off in the future. Laziness pays off now.
New Newton's Law: For every action, there is an equal and opposite criticism.
Lost's Law: He who hesitates is probably right.
Green's Law: No one is listening until you make a mistake.
Cooper's Law: Success always occurs in private, and failure in full view.

And we had this response from Cathy Walker.

Once again I enjoyed reading the newsletter and found your info about the large numbers fascinating. Until you think about the maths involved its hard to believe that the desks in a class could be arranged in that many ways. Also your blurb on rounding. I used to use a number line with a hill between the numbers then talked about the ball rolling back down or if it had got to half way rolled over!!!!!

We encourage you to send in your comments or articles.

Solution to July's problem

Many people are surprised when they find the method is not fair, that although there seem to be two positions in which the roses may be arranged they are not equi-likely. If we ignore the yellow roses and assume Dido places a red rose in the vase at the back of the table and to the left, then the other red rose can be placed in one of three vases. However, in two of these positions the red roses are side-by-side (Fig 1). Only in one position are the red roses diagonally opposite each other (Fig.2). Since Tessa decided that she would agree to Kilo's proposal if the roses were side-by-side, the odds are two to one that she and Kilo will get together. We'll leave you to decide whether the method was fair!



Fig 1: Two side-by-side positions for the red roses.



Fig 2: One position for the red roses to be opposite.

Cathy Walker took off the prize again this month. Come on you readers out there. Let's have some competition next month. This year this voucher is becoming a benefit for Cathy and the students from Matawai.

This Month's Problem

Three athletics clubs; the Achilles' Arrows, Hermes' Harriers and Zeno's Zephyrs, each had three runners entered in the annual cross-country run. No other clubs took part. Nine points were awarded to the first runner home, eight to the second, seven to the third and so on down to one for the last runner home. All runners completed the race and when the results were worked out it was found that each of the three clubs gained the same total number of points. Achilles' Arrows were presented with the trophy since they had the first runner home.

If there were no dead heats and no teams had consecutive runners crossing the finishing line, for which club did the athlete gaining two points run?

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Each month we give a petrol voucher to one of the correct entries. Please send your solutions to <u>derek@nzmaths.co.nz</u> and remember to include a postal address so we can send the voucher if you are the winner.