Newsletter No. 25 July 2003

Large numbers have always intrigued people. Questions like how many millimetres there are to the moon, how many grains of sand on the beach, or how many possible chess moves there are, create immense interest in the classroom. Answering them gives an excellent opportunity for using approximations and standard form.

Here's one that I've tried successfully. How fast are you moving when you're sitting still? This, not surprisingly, elicits a lot of preliminary discussion until someone realises that our planet is rotating and we're rotating with it. The question requires a look at latitudes, circle circumference and the speed = distance/time equation. Then someone remembers that the Earth travels round the Sun and off we go again!

Another I've tried is; which is greater, the number of arrangements of the desks in the room or the number of drops of water that make up the oceans? Assuming the American definition of a drop as 0.1ml, the answer involves handling large numbers in standard form and using the formula for the volume of a sphere. It turns out that if the whole Earth were water it's number of drops would be a lot less than the number of arrangements of 30 desks which we can take as 30 factorial, or 30! Students find this awesome.

Incidentally, the number of millimetres from here to the Moon is very approximately 4×10^{11} and the number of different moves in a typical game of chess about 10^{118}.

You'll find out all about large numbers and standard form on the nzmaths website.

To teach I would build a trap such that to escape my students must learn.
Robert M. Chute

What's new on the nzmaths site this month?

Units

Three new senior units have been completed and are available on the site. They are:
Discovery, Number, Level 6
Proof, Algebra, Level 6
The Power of Algebra, Algebra, Level 6

There is also a new Level 1 Algebra unit – Counting on Counting.

Numeracy Material

Due to the increasing volume of material associated with the Numeracy Development Projects, we have streamlined the menu for the Numeracy Project Material. This area of the site now includes:

- The nine Numeracy Project books (Book 8 'Teaching Number Sense and Understanding Number Properties' has been newly added in the last month).
- The Material Masters associated with Numeracy activities (Including 32 new Material Masters associated with Book 8).
- Four equipment animations, available in English and Māori versions.
- Global Strategy Assessment forms.
- A guide to BSM.

Diary Dates

A reminder that Maths week for 2003 is 10-16th August.

It's in the Language! The Rounding Problem Type by Doug McFarland

Setting up the problem

When teaching rounding it is crucial to the trajectory of a child's conceptual understanding that the teacher not only understand this concept herself, but that she uses the precise *language* that will affect a true understanding of rounding. In this article selected vocabulary terms will be emboldened to emphasize their importance in this lesson and *when* they are used in The Lesson.

Rounding is an estimation skill based on one's knowledge of place value. So, this skill, as you can see, requires the learner to bring a mixture of background knowledge with them. In the following figure:

1 4, 6 7 5

it is first important to be certain that the student recognizes the value present in each column. Begin by asking, "What is the total value in the ones column? [5] The tens column? [70] The hundreds column? [600] The one thousands column? [4000] The ten thousands column? [10,000]

It's good to write out these values as you ask the questions as a way of diagramming the problem:

$$10{,}000 + 4{,}000 + 600 + 70 + 5$$

Write the figures from RIGHT to LEFT. Math is different from Reading in this respect—math works from right to left, and reading and language, of course, work from left to right. This confuses some kids. It should!

The Lesson

The first item to discuss is *what place does the problem ask you to round to?* This idea is not self-evident to many children. I tell students that you can't answer a rounding problem if all that the problem says is "Round 14,675." A rounding problem *must* have two parts to function properly, like a bicycle has to have two wheels to function properly. The problem must call for what value they want you to round.

Now, here's the process at this point. Let's say the problem says *Round 14,675 to the nearest thousand.* Ask, "What is the **total value** in the **one thousand's column**?" If the student says "4" say, "No, not in that **column**. There's a lot more **value** in that **column** than only 4." Usually they self correct and say, "Oh, 4,000." It's hard to get students to think in terms of *total* value. Point out the 4,000 in your "diagram" from earlier in the lesson. You might review the **total value** in the other columns as well and then come back to the one thousands column.

Put your pencil or chalk under the 4 in the one thousands column. Ask, "Is this **value** closer to 4,000 or 5,000?" If they are not sure, guide them to study the column to the right (hundreds). This next question is very important to the vocabulary of the lesson. Ask, "Is 600 half way or more to 1,000?" Do not use the old, "Is the number to the right 5 or more? Or 4 or less?" You can review that idea directly at a later point. That is *not* the math concept. That is a trick or a convenience that lubricates the process but does not add any math value to the lesson concept. Avoid that mistake!

The question rephrased another way might be, "Well, there's 600 in this column. Is there enough value there to say that this figure is closer to 5,000?" Hopefully you see the importance of both the vocabulary and the questioning format.

To extend this lesson you could examine each of the other total values and go over the same question format. I have found that this simple application of vocabulary and questioning greatly enhances students' true conceptualization of rounding and promotes long-term retention of the rounding problem type.

Note: You meet some interesting people on the internet. Doug MacFarland is someone who emailed me out of the blue to talk about the nzmaths web site. He is a doctoral student at the University of Kansas who also teaches children who have trouble learning in the general education classroom.

Alternative Laws

Mathematics and humour only occasionally go together but when they do it can give rise to even more fun. There are a lot of rules and ideas in mathematics and science which have been plagiarised, perhaps based on anecdotal evidence, to give amusing rules of thumb like the well-known Murphy's Law that 'if anything can go wrong, it will.' Here's a few more we've collected for you:

Marshall's Generalised Iceberg Theorem: Seven-eighths of everything can't be seen.

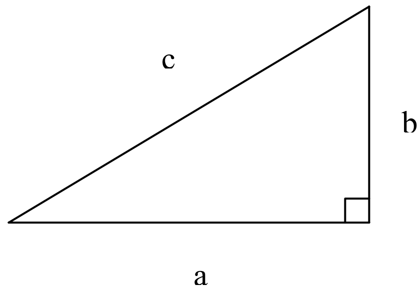
Finagle's Law: The likelihood of something happening is inversely proportional to its desirability.
Friedman's Law: Never walk across a river just because it has an average depth of one metre.
Parkinson's First Law: Work expands to fill the time available.
Parkinson's Second Law: Officials promote subordinates, not rivals.
Ryan's Law: Anyone making three correct guesses consecutively will be established as an expert.
Roddenberry's Law: Ninety percent of everything is junk.
Block's Law of the Letter: The best way to inspire fresh thoughts is to seal the letter.
Fiedler's Rule: When presenting a forecast, give a number or a date but never both.
Runvon's Law: In all human affairs, the odds are always six to five against.
Cornford's Law: Every public action which is not customary is either wrong, or, if it is right, is a dangerous precedent. It follows that nothing should ever be done for the first time.
Terman's Law: If you want your track team to win the high jump find a person who can jump two metres not two people who can jump one metre.
The Peter Principle: In every hierarchy - government, business or whatever - each employee rises to a level of incompetence (as long as people are doing well they will be promoted).
Knebel's Law: Smoking is one of the leading causes of statistics.
Ginsberg's Conjecture: Confusion begins with the introduction of pencil and paper.
Brown's Law: 99 percent of lawyers give the rest a bad name.
Karran's Conjecture: The view from the mountain top can be different.
Student's Conjecture: No mathematics taught in school will be of any use in everyday life.
Harvey's Law: If there is a 50-50 chance something will go wrong, then nine times out of ten it will.
Postie's Law: Bills travel through the mail at twice the speed of cheques.
Ford's Law: When everything's coming your way, you're in the wrong lane.
Felix's Law: The early bird gets the worm but the second mouse gets the cheese.
New Darwin's Law: Change is inevitable, except from vending machines.
Likeit's Law: The sooner you fall behind, the more time you'll have to catch up.

If you've come across any others we'd love to hear about them.

Pythagoras' Theorem

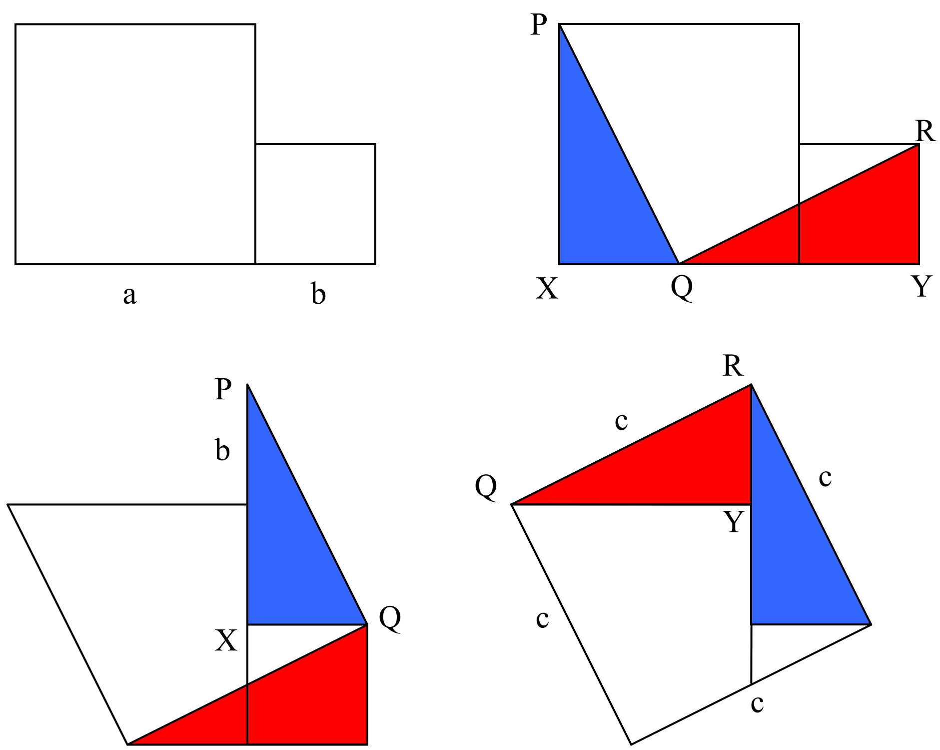
If you asked anyone on the street to name a mathematical theorem, the one that they would come up with would almost certainly be Pythagoras'. They might even be able to make a fist of saying what it was about. "It has to do with right angled triangles; and squares on sides; …"

Pythagoras is famous for this result more than 2500 years after his death but the Babylonians and the Chinese knew that about right angled triangles with sides of length 3, 4, 5 and 5, 12 , 13, existed (see the April newsletter for an article on Pythagorean triples). Pythagoras' contribution was that he (or one of his group), proved that in any right angled triangle, the area of the square on the hypotenuse is the same as the4 sum of the areas of the squares on the other two sides. From the diagram $a^2 + b^2 = c^2$.



Proof is important in maths. It makes sure that we are not just guessing. This is one part of maths that sets it apart from other subjects.

It turns out that there are over 100 proofs of Pythagoras' Theorem. We'll give a few over the next few months. The one we start with below requires only a knowledge of area.



In the first picture we have two squares, one with side length a and the other with side length b. The area of these two squares is the $a^2 + b^2$ of the Theorem.

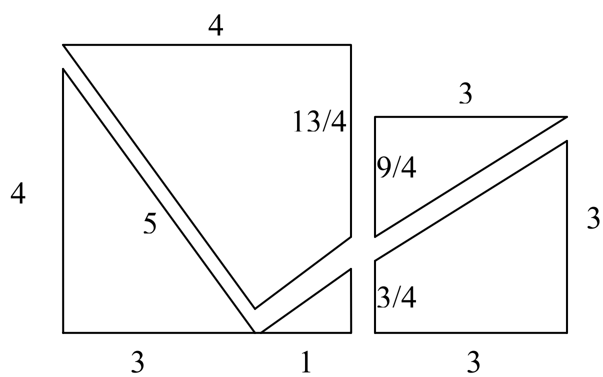
In the second picture we have drawn in PQ and QR, where Q is b from X. The effect of that is to make one right angled triangle PQX (the blue one) with sides PX = a and XQ = b. So, from the right angled triangle earlier, PQ must be of length c. But QRY is also a right angled triangle (the red one) with sides a, b and c.

Now move the blue triangle so that the point Q coincides with R (see the third picture). The point P protrudes by an amount b above the large square.

We can then move the red triangle so that Q coincides with P. The shape that we have formed has all sides of length c. Its corner angles are all right angles so it is a square. The area of this square is c^2.

Now in the process of moving the triangles we haven't created or destroyed area. So we must have $a^2 + b^2 = c^2$.

But there is a bonus here: you can make quite a nice puzzle with the pieces from this proof. We set the pieces up as they are in the second picture so that you can see how they fit together. The sides have been given specific lengths to make them easier to make.



So now you have 5 pieces that will make two small squares (one of side 4 and one of side 3) and one big square (of side 5). This might be a useful problem for your own students to try. You could also get your class to make these and then try the puzzle out on another class (or parents).

Feedback

Following on from the item on the Four 4s and 2003 problem in issue 20 of the newsletter (February 2003) it appears that two Japanese mathematicians have solved the generalised problem, that of using four x's and a selection of mathematical operations to write any number. They also managed it with fewer than four x's. The solution involves using logarithms to different bases and would be accessible to students preparing for university. If you'd like to know more about it you can email Derek at the email address below.

Correspondent Linda Bonne reminded us of the TV advertisement that enjoins us to think of New Zealand's population as 100 and the consequences of that; for example, 35 people will get married and five divorced. She thinks there must be some very odd couples in New Zealand! It certainly seems like that until you realise that the ad is using percentages to describe the population. Why they don't mention the word percentages is a bit of a worry. Is it perhaps because they have no faith that the New Zealand population understands the word? That in itself, of course, is a reflection on our education system.

Solution to June's problem

Debbie's 'generosity' is overrated. When Wiremu has scored 12 points, Debbie should, at worst, have scored 3. Then they change over. While Debbie scores the 18 points necessary for her to win the game, Wiremu scores, at most only 7. So Debbie should win by at least 2 points.

This week we have two students who cracked it. They are both in the extension class at Matawai School (between Opotiki and Gisborne). Congratulations!

This Month's Problem

Madame Serena, or Tessa as she was usually called, sat on her couch looking out at the flower table beside her neighbour's caravan. It was early and Dido Fleur hadn't yet started loading the table. As she sat, Tessa pondered Kilo's question. Should she accept his proposal of marriage, or not?

Deep in thought, she idly watched as Dido appeared at her caravan door and sniffed the air. In her hands were bunches of yellow and red single-stemmed roses which she would eventually place individually in narrow glass vases for sale to the townspeople who came to the fair.

If only she could see into her own future, Tessa thought. Every day she gazed into her crystal ball to capture fleeting images for customers who had crossed her palm with silver, well, gold coins these days. Never had she espied scenes of Kilo and herself, no future for them outlined in the glinting glass.

Dido skipped down the steps of her caravan and reached under the table for her vases. It was her custom to set up her display by first placing a vase on each corner of the table and then placing a single red or yellow rose in each one. It was always two of each colour, Tessa had noted. As Dido began to select the roses Tessa quickly squeezed her eyes shut and covered them with her hands.

Decision time, she thought. There are really only two different positions for the roses to be placed in the vases - with the red ones side-by-side or diagonally opposite each other. Tessa quickly made up her mind. If it was the former she would say yes to Kilo, if the latter, she would say no. That seemed a fair way to settle the issue.

Assuming Dido placed the four roses, two red and two yellow, in the vases at random, was Tessa correct? Is it a fair way to decide whether she should marry Kilo? Explain your answer.

Each month we give a petrol voucher to one of the correct entries. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so we can send the voucher if you are the winner.