Newsletter No. 24 **June 2003**

Listening to a news item on National Radio recently I heard the statement, “It cost less than average but it must be remembered that the average was higher than usual”. It almost passed me by. At first I was fairly confident I knew what it meant, now I’m not so sure. It reminded me of another comment that I almost took at face value. I was reading about Isaac’s Newton’s childhood. The author was describing how as a child Newton kept mostly to himself, occupying his time by reading or building miniature devices, working models and so on. He wrote, ‘It is said that he built a small working windmill driven by a mouse upon a treadmill.’ Well, that seemed straightforward enough, no ambiguity surely. Or was there? Shouldn’t a working windmill be wind driven not mouse driven? Maybe I’m being too pedantic.

In the March 2002 newsletter I wrote about misuse of the word average and gave some examples. Sometimes the misuse is due to a lack of understanding of the concept, at other times the word average can be a source for humour. For example, Oscar Wilde wrote in *Critic as Artist*, ‘If you confront a statistician with a man with one foot in a bucket of boiling water and the other foot in a bucket of ice-cold water, he will say that, on average, the subject is comfortable.’ In his classic book *Facts from Figures,* M. J. Moroney quotes from Punch magazine; ‘The figure of 2.2 children per adult family was felt to be in some respects absurd and a Royal Commission suggested that the middle classes be paid money to increase the average to a rounder and more convenient number.’ That, I think, shows a lack of understanding of the term but Mark Twain in *Life on the Mississippi* was certainly being humorous when he wrote, “In the space of one hundred and seventy years, the Lower Mississippi has shortened itself two hundred and forty two miles. That is an average of a trifle over one mile and a third per year. Therefore any calm person, who is not blind or idiotic, can see that in the old colitic silurian period, just a million years ago last November, the Lower Mississippi river was upwards of one million three hundred miles long, and stuck out over the gulf of Mexico like a fishing rod. And by the same token any person can see that seven hundred forty two years from now the Lower Mississippi will be only a mile and three-quarters long, and Cairo and New Orleans will have joined their streets together, and be plodding comfortably along under a single mayor and a mutual alderman. There is something fascinating about science. One gets such wholesome returns on conjecture out of such trifling investment of fact.’

You and your students will certainly get wholesome returns from the nzmaths website.

What’s new on the nzmaths site this month?

Nice Dice

This month we have completed another new Bright Sparks activity – Nice Dice.
In this activity you (or your students) are challenged to help Brad and Bess make pairs of 'Nice Dice', which are equally likely to sum to any number from 1 to 12. It's not too hard to find one pair, but can you find all the possible essentially different pairs?

Figure it Out Links

We are in the process of updating all the existing Level 1-4 units on the site to include links to appropriate activities from the Figure it Out series where there are any. If you are using the units, have a look at the end (near the Homelink) – you might find a handy follow-up or extension activity.

Diary Dates

Just another reminder that the New Zealand Association of Mathematics Teachers is planning its 8th annual conference in Hamilton from the 8th to the 11th of next month. Plenary speakers are: Vaughan Jones, Kaye Stacey, Laurinda Brown, John Edwards, Jeff Witmer, Harry Henderson and Anthony Huffadine.

For more information contact;
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And just an early reminder that Maths week for 2003 is 10-16th August.

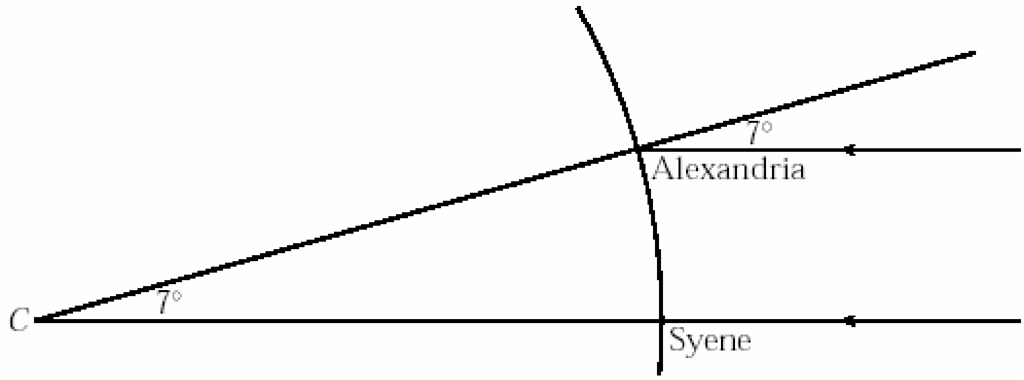
The Size Of The Earth by John Stillwell

How do we know that the earth is round? Contrary to popular belief, the roundness of the earth was not discovered by Christopher Columbus; it has been known since ancient times. One way to observe it is during eclipses of the moon, when the moon passes through the earth's shadow. No matter how the earth is oriented when the eclipse occurs, it always casts a round shadow on the moon. Since the only object that always casts a round shadow is a sphere, the only reasonable conclusion is that the earth is a sphere.

This argument is sometimes credited to Pythagoras (around 500 BCE) and it occurs in Aristotle's *On the Heavens*, Book II, Part 14, so the roundness of the earth was certainly known in ancient Greece. Indeed, the earth was known to be not terribly large. Aristotle pointed out that some stars seen in Cyprus could not be seen in Greece, so the earth curves appreciably in this short distance from south to north. Aristotle was writing around 350 BCE, and a century later his compatriot Eratosthenes refined this idea to actually measure the earth.

Eratosthenes (276–194 BCE) lived in Alexandria, then a Greek colony in Egypt. He knew that on midsummer's day the sun was directly overhead in the town of Syene, due south of Alexandria at a distance of 5000 stadia. The "stadium" was a unit of length then in use, representing the size of a standard athletic stadium. Eratosthenes was also able to measure the inclination of the sun at noon on the same day it was overhead in Syene. He found that in Alexandria it was about 7 degrees from the vertical.

Assuming that the sun is so far away that its rays to Syene and Alexandria are essentially parallel, this gives the following picture. *C* marks the centre of the earth and the arrows are the sun's rays.



We see that the sun's 7 degree deviation from the vertical at Alexandria is also *the angle between Syene and Alexandria, viewed from the centre of the earth*. Therefore, since there are 360 degrees in a full circle, *the distance between Syene and Alexandria is* 7/360 *of the circumference of the earth.* Using the measured distance of 5000 stadia between the two towns, this gives a distance of about 250000 stadia for the circumference of the earth.

We don't know the exact length of a stadium, but Eratosthenes' figure of 250000 stadia is, at worst, out by about 20%, and it could be within 1% of the correct value (around 40000 kilometers). Not bad for 250 BCE! Actually, I'm not sure why scholars are worried about the length of the stadium in this calculation. It seems more important to know the correct angle between Syene and Alexandria, which also depends on whether Syene is really due south of Alexandria.

Well, this can be checked today in any decent atlas. Syene is the town now known as Aswan, and it has latitude 24.1 degrees north and longitude 32.9 degrees east. Alexandria has latitude 31.2 degrees north, so 7 degrees is a good estimate for the difference in latitude between the two towns. Unfortunately, Alexandria has longitude 29.9 degrees east, so Eratosthenes estimated the north-south line rather poorly. Even so, the correct angle between Syene and Alexandria is about 7.6 degrees, so his error in the angle is less than 10%.

Footnote: In case you were wondering how to pronounce "Eratosthenes", then it's close to "error toss the knees", with the stress on the "toss".

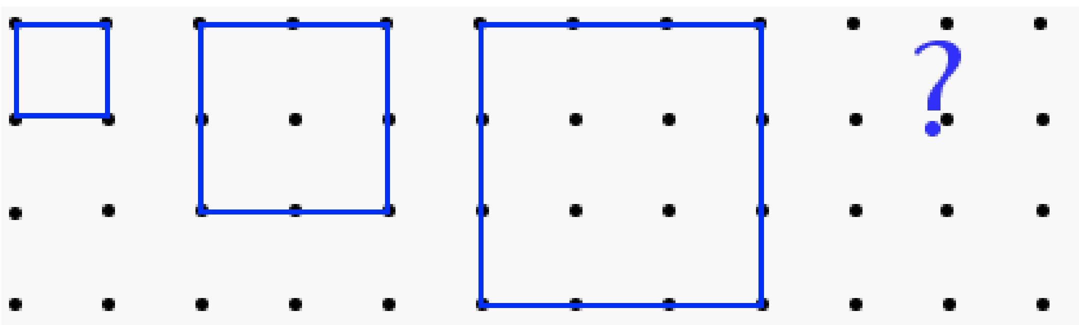
Two Announcements

We're glad to announce that we have received our first student's work on Bright Sparks. If you recall we are hoping that students who successfully get through any of the Bright Sparks puzzles, will send their reasoning to Derek at derek@nzmaths.co.nz. He then hopes to give them some feedback and possibly some extensions to think about. Not wishing to give the student's name, it's enough to say that he is a primary student and managed to make a great amount of progress on the Six Circles problem. Can Derek now expect a flood of solutions?

An interesting book has just been published simultaneously in the UK and the USA by Cambridge University Press, the British Mathematical Association and Mathematical Association of America. /The Changing Shape of Geometry/ is a book that aims to show a lively and modern side to geometry. There are about 50 chapters of varying sizes that have been written by a number of people, some of whom you may well have heard of. Some of the material could be used with primary and junior secondary classes and some is quite deep mathematics. At this point we are not sure of the local price but we'll find out for anyone who is interested.

Dotty Paper

Once upon a time it became fashionable to do hands-on mathematics in our classrooms (and not before time, I may add). Lots of geometrical facts were discovered using dotty paper, sheets of which were handed out to pupils by the dozen. One activity involved pupils exploring area and began with their being asked to draw, on the dotty paper, squares with areas one, four, nine units and so on.



It didn't seem to present much of a problem. Then the pupils were asked to draw a square with area two units. That seemed to stump them - and some of their teachers too!

Solution to May's problem

You were asked to place the whole numbers from 1 to 7 inclusive, 10, 12, 14, 15, 18, 20, 21, 24 and 28 in a four-by-four square such that the product of the numbers in each row, column and diagonal was the same. You were given four of the sites ensuring uniqueness of the solution and it was suggested you should look for a method other than trial and error.

Well, a good place to start is to express the sixteen numbers as the product of primes,

$4 = 2 \times 2$, $6 = 2 \times 3$, $10 = 2 \times 5$, $12 = 2 \times 2 \times 3$, $14 = 2 \times 7$, $15 = 3 \times 5$,
$18 = 2 \times 3 \times 3$, $20 = 2 \times 2 \times 5$, $21 = 3 \times 7$, $24 = 2 \times 2 \times 2 \times 3$, $28 = 2 \times 2 \times 7$
(the others are prime or the number 1).

Since the products of the numbers in each row, for example, are the same, x say, then the product of all sixteen numbers is x^4. Taking the fourth root of the product of the sixteen numbers gives the product of the four numbers in each row, column of diagonal. This product constant works out to be 5040.

Here's the square as it was originally given, with the numbers expressed in prime factors,

		3x5	
			5
		2x2x7	2x3x3

Now, $5040 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$ and this set of prime factors is the same as the set of prime factors of the numbers in each line. That is, each row, column and diagonal has the factors 2, 2, 2, 2, 3, 3, 5 and 7 in it somewhere.

For ease of explanation, label the columns from left to right as A, B, C and D the rows from bottom to top as a, b, c, d, each square can then be labelled by a pair of coordinates. Call the top left to bottom right diagonal X and the other diagonal Y.

Now we can start to fill in the rest of the square.

Looking at row a; Since three 2s, two 3s and a 7 have been allocated, we have 2 and 5 left to place. The 5 and 2 must be together since 5 on its own is already allocated to square Db. 5 cannot go in square Aa since there is already a 5 in diagonal X. Hence, $2 \times 5 = 10$ goes in square Ba next to the 28. That only leaves the number 1 to go into square Aa.

Now, looking at the whole square, each row and each column must have a factor 5 in it somewhere. 5, 10 and 15 are already allocated so 20, which has a factor of 5, must go into square Ad.

So, this is where we are so far,

2x2x5			
		3x5	
			5
1	2x5	2x2x7	2x3x3

Now looking at diagonal Y, there are only a 2 and a 7 to place so one of them must go into each square. Since column C already has a 7, the 7 goes into square Bc and then the 2 into Cb.

Continuing in a like manner completes the square (without any trial and error) and multiplying out the factors gives the solution as,

20	3	6	14
12	7	15	4
21	24	2	5
1	10	28	18

We're pleased to say that this month we had more solutions sent in than for any previous monthly problem. Can we do even better next month?

And this month’s winners are …. four students from Matawai School. Matawai is just off State Highway 2 on the way from Opotiki to Gisborne. As we haven’t got their permission to publish their names we can’t tell you who they are but we hope to be able to in next month’s newsletter.

This Month's Problem

Debbie and Wiremu were playing Patball, an outdoor game of their own invention. In skill they were evenly matched.

The winner of each game gained one point and they had decided to see who would be the first to achieve 21 points.

Debbie had the sun in her face. "Look here, Wiremu," she said, "with the sun where it is now you're scoring about three points to my one. I ought to have a handicap."

"Okay," said Wiremu. "Why not change over at half time? Say when I've scored 10 points."

"Right," said Debbie. She considered. "After all, Wiremu, you did win the toss. Let's change over when you've scored 12 points."

"A generous offer," said Wiremu. "I'll accept it."

What should be the outcome of the game?

Each month we give a petrol voucher to one of the correct entries. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so we can send the voucher if you are the winner.