**Newsletter No. 22** **April 2003**

Last month we left you with the problem of finding an approximation of π using the digits of the year 2003, in order, and common mathematical operations. We added, that the one we were looking for was straightforward and within 0.2 % of the true value. The approximation we had in mind was $\sqrt{2} + 0 + 0 + \sqrt{3}$ which is accurate to about 0.15 %.

Last year, in the August issue of the newsletter we listed a few approximations of π that have been used throughout the ages. The one that has always impressed me the most is that determined by the Chinese astronomer Tsu Ch'ung Chih in about 480 CE. Making successive use of Pythagoras' theorem to polygons with up to 24 576 sides he came up with $^{355}/_{113}$ - that's accurate to about 0.000008 %, truly amazing.

Even that accuracy pales beside the current most accurately known value of π. In September 2002, a team from the Information Technology Centre of Tokyo University, led by Prof. Yasumasa Kanada broke the world record (their own) for the number of calculated digits of π - a staggering 1.2411 trillion! It represents six times the number of places of the record currently recognised by Guinness World Records. Dr. Kanada's team of ten spent five years designing the programme to make the calculation. It took the Hitachi supercomputer, which is capable of handling two trillion calculations per second, over 400 hours to complete the operation. Wow!

> *The data with which mathematics starts out is concrete, whereas*
> *The objectives it strives for are abstract*
>
> ***Saadia Gaon (c. 910 CE)***

This month we have an article by John Stillwell who used to be full time in the Maths Department at Monash but now spends half of his year working in San Francisco. He has written many books for Springer-Verlag. One of these is one of the best books on the history of mathematics in print. We hope to have a series of contributions from him over the course of the year.

## What's new on the nzmaths site this month?

### Numeracy PA

More activities have been linked to the Numeracy Planning Assistant, especially at the higher levels of the framework. There are now a total of over 130 activities linked to this resource.

### Staff Seminar

There is a new staff seminar in the Info Centre. It is around the idea of Angles, and addresses many areas of this topic that may be misunderstood or not well taught.

### Bright Sparks

We did mention Bright Sparks in a previous newsletter but there are now three of these puzzles up and running. These are Frogs, Round Table, and Six Circles. Frogs is an animated version of the old problem of getting 3 frogs on one set of three lily pads to change places with three frogs on another set of three lily pads. There is an empty lily pad between the two lots of frogs to make movement possible.

When I first heard about the Round Table it was couched in a King Arthur and His Knights of the Round Table scenario. King Arthur had a dragon to be killed (or some such) and he was going to pick the knight to do the job by counting knights out alternately round the table. Our Round Table has a less dramatic backdrop but the idea is the same.

Six Circles is a combination lock. Solve the problem and advance through the doors. How many times can you do this before there are no more combinations that will work?

These problems can be used with any students in your class. However, there is one drawback with the computer and that is that it can't check students' work. If you would like to get some of your students to send us the reasons that they think they can solve the problems, then please send their work to Derek at derek@nzmaths.co.nz. He'll check them out and follow up with appropriate comments.

**Diary Dates**
Another reminder that the New Zealand Association of Mathematics Teachers is planning its 8th annual conference from the 8th to the 11th July 2003 in Hamilton. Plenary speakers are: Vaughan Jones, Kaye Stacey, Laurinda Brown, John Edwards,
Jeff Witmer and Charles Lovitt.

We understand that they are looking for people to give workshops or presentations. If you have some ideas please get in touch with them.

**For more information contact;**
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**N.B.** Maths week this year is 10-16th August.

**More about Primes**

There are all sorts of unexpected relationships relating to prime numbers, some of which such as Sundaram's sieve (Newsletter No. 18, October 2002), we've looked at before. The first few prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, …. and all of them, apart from 2, give remainder 1 or 3 when divided by 4.

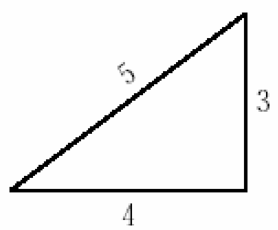
First set (remainder 1) = {5, 13, 17, 29, 37, 41, … }
Second set (remainder 3) = {3, 7, 11, 19, 23, 31, … }

The interesting thing here is that all the first set and none of the second can be expressed as the sum of two integral squares, for example,

$5 = 1^2 + 2^2$, $13 = 2^2 + 3^2$, $17 = 1^2 + 4^2$, etc

## Pythagorean triples by John Stillwell

What is special about the triple of numbers 3, 4, 5? If you make a triangle with sides of these three lengths, it has a *right angle* between sides 3 and 4.

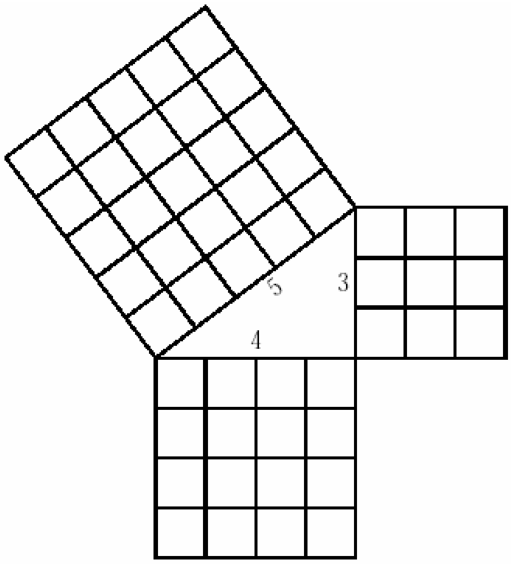



So, 3, 4, 5 are not just three successive numbers, *they make a right angle*. If we take a loop of string 12 centimeters in length, with knots in it 3, 4 and 5 centimeters apart, and stretch it into a triangle with knots at the corners, then we necessarily get the triangle shown above, with a right angle between the two shorter sides.

This fact has been known to builders since ancient times (it is mentioned by the ancient Roman writer Vitruvius in his book on architecture), but it was first discovered by mathematicians. Nearly 4000 years ago, in Mesopotamia, this property of the 3, 4, 5 triple was known, and it was rediscovered in India, China, and Greece.

But that is not all. These ancient civilizations discovered how to recognize precisely which triples of numbers “make right-angles”. They are the ones for which *the squares of the two smaller numbers sum to the square of the largest number*.

This property can be seen in the following picture of the 3, 4, 5 triangle, with squares on the three sides.




The “square of” 3 is 3 _ 3 = 9, represented by the 9 little squares making up the square on the side of length 3. Similarly, the square of 4 is 4 _ 4 = 16, and the sum of these two squares is 25, which equals 5 _ 5, the square of the longest side.

The beauty of the test, however, is that it is *not* necessary to actually draw pictures of triangles with squares on their sides. The squares can be found, and added, by arithmetic.

For example, the sides 5, 12, 13 also make a right-angled triangle because:

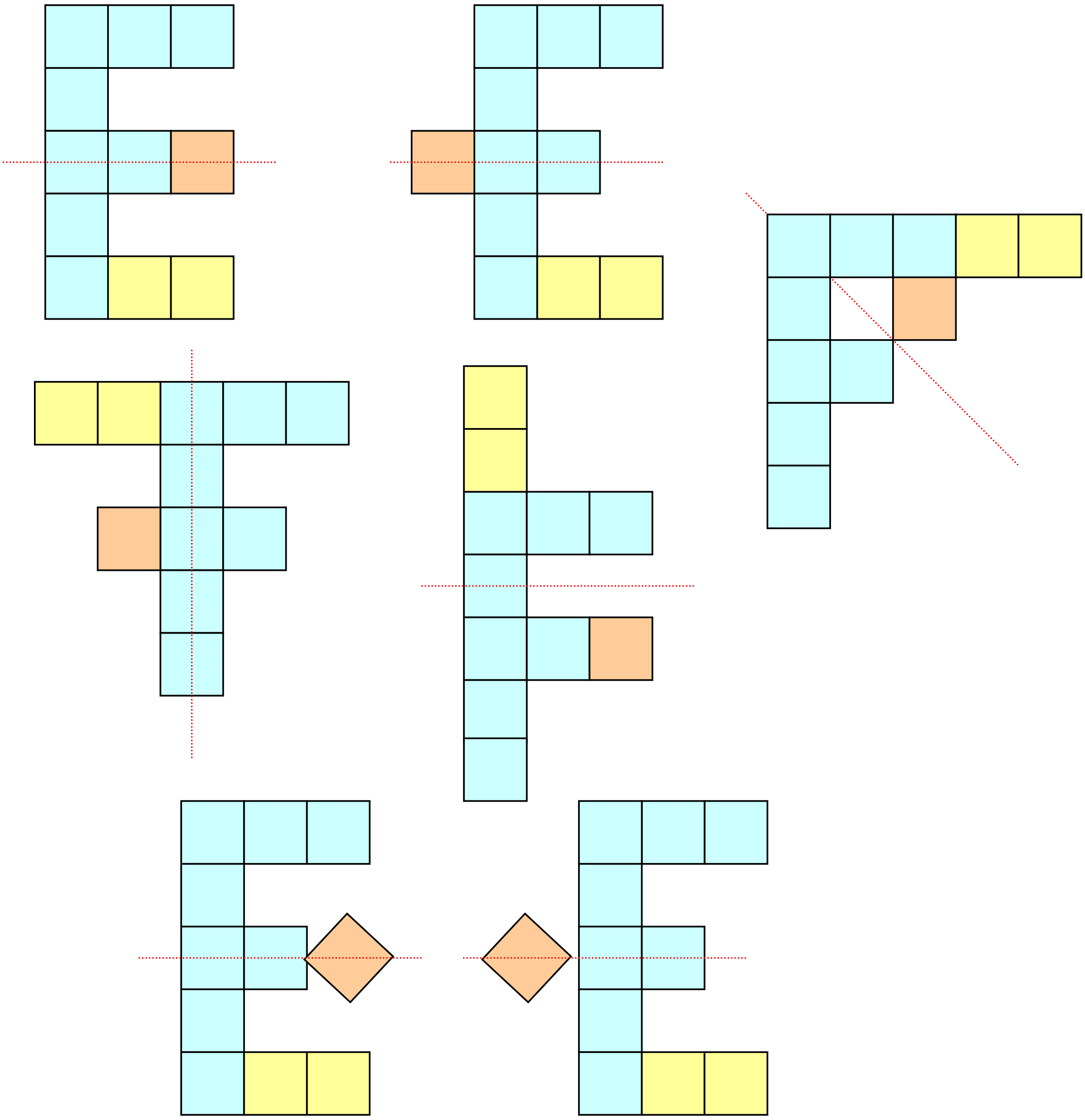
square of 5 + square of 12 = 5 x 5 + 12 x 12
= 25 + 144
= 169
= 13 x 13
= square of 13

Triples of whole numbers that make right angles are called *Pythagorean triples*, and they are quite rare. The next simplest ones after 3, 4, 5 and 5, 12, 13 are 8, 15, 17 and 7, 24, 25. Thus it is hard work to find Pythagorean triples by guessing the numbers and then working out their squares. Yet 4000 years ago, the Mesopotamians were able to find huge Pythagorean triples, such as 12709, 13500, 18541! We don’t know how they did it. This is one of the great mysteries in the history of mathematics.

Pythagoras lived about 2500 years ago, so in fact he was not the first to discover Pythagorean triples. However, he (or his followers) was the first to *prove* that the sum of squares on the shorter sides of a right-angled triangle equals the square on the largest side. This is why we remember Pythagoras, because he showed that Pythagorean triples belong to the world of *ideas*, not just to construction sites.

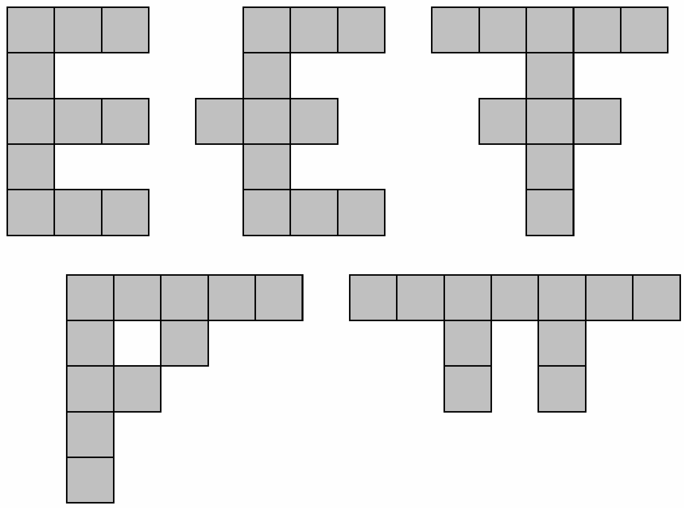
**Solution to March's problem**

This month's winner is Alison Talmage from Milford, Auckland 1330.
Congratulations Alison.

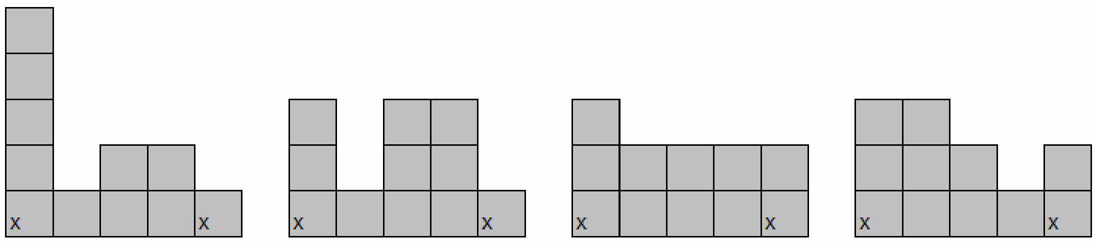


Most people find the first five solutions but the last two are tricky and elude most people.

Perhaps the most original answer that we received was from Cathy Walker. She moved into 3-dimensions to get some extra possibilities. Have a look at her solutions.



These were the 2 dimensional shapes I could find with the 3 shapes




These were the 3 dimensional shapes I could find with the 3 shapes. Imagine the shapes wrapped around and joined at the crosses. They all have line symmetry!!!

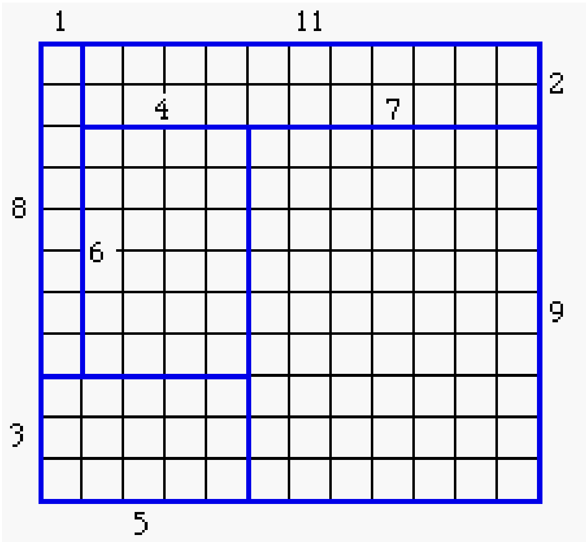
As we were only thinking in the plane we didn't think that we could give her the prize.

**This month's problem**

In Newsletters 17 and 18 (September and October 2002) we looked at problems dissecting squares into smaller squares. The classic dissection problem on squares is called 'Squaring the square' in which a square is cut into a finite number of smaller squares, no two of which are the same size. Although it had been known for sometime that the best solution, i.e. the one needing the least number of squares, required 21 of them, it wasn't until 1978 that this solution was found, by the Dutch mathematician A.J.W. Duijvestijn.

An analogous problem we might call 'Rectangling the Rectangle': That of dividing a non-square rectangle into the minimum number of smaller rectangles in such a way that no two sides of two different rectangles have the same length.  It is understood that all lengths are whole numbers.

Here is such a dissection.  Well, at least the constituent rectangles have sides all of different lengths but could the problem be solved using fewer than five rectangles?  I don't think so.




Here's your problem for this month.  It is to solve the 'Rectangling the Rectangle' problem so that the five constituent rectangles have sides 1 through 10.  That is, dissect a rectangle into five rectangles such that no two of their sides have the same length and only the lengths 1 to 10 are used.  I'm not sure at this stage whether there is a unique solution but we'll help you on your way to one solution by telling you that one of the rectangles has sides of length three by five units and another is six by eight.

Each month we give a petrol voucher to one of the correct entries.  Please send your solutions to [derek@nzmaths.co.nz](mailto:derek@nzmaths.co.nz) and remember to include a postal address so we can send the voucher if you are the winner.