Newsletter No. 21 **March 2003**

Last year was the 150th anniversary of the four-colour map problem. In 1852 mathematician Francis Guthrie noticed that on a map only four colours are necessary to ensure that no two adjacent regions are the same colour. Adjacent here means having a common border, what topologists call an edge. Common corners (vertices) are O.K.

For over 100 years no-one found an exception to the conjecture or, to put it another way, no-one could construct a map that required five colours. The search for a proof captured the imagination of lay people and professional mathematicians alike, almost as much as that of Fermat's Last Theorem. Every week, somewhere round the world, a university maths department would be sent a 'proof', which of course was in error. In 1976 three mathematicians and a computer came up with a solution that has divided mathematicians ever since. The proof rested on a computer programme of well over 100 pages that ran for 1,200 hours. It couldn't be checked 'by hand' either because the calculations were too complex and the time needed to painstakingly check all possibilities was too long.

Many mathematicians are unhappy with computer-assisted proofs which they feel lack elegance. There is always the thought that there is a more concise, elegant solution just around the corner. A number of long-term problems have recently succumbed to algorithmic proofs - see Newsletter 18, for example. (You will note that I wrote algorithmic proofs there and not algorithmic 'proofs'. I am not one of those who has trouble with the concept.) Some mathematicians feel that if an algorithm can be written to solve a problem, we should accept it and move on. Maybe acceptance of the proof will bring to light further concepts that need exploring.

By the way, if you are interested in the Four-colour Map problem and its solution Allen Lane/Penguin published a book about it last year by Robin Wilson called *Four Colours Suffice*: *How the map problem was solved*.

> *' Teachers should present the modern precise idea of an algorithm as among the great ideas in human intellectual history.'*
>
> ***S.B. Maurer***

P.S. You won't need an algorithmic proof to solve this month's problem!

What's new on the nzmaths site this month?

There are several new additions to the site this month:

Numeracy PA
In the Numeracy section of the site you will find a link to the Numeracy Planning Assistant, which can be used to help plan for teaching Numeracy to groups of students. The Numeracy PA not only helps by organizing activities into stages of the Number Framework, types of activity and Learning Outcomes (there are currently 60+ activities, aimed mainly at the first 4 stages of the Framework), it also provides you with a printable planning sheet for your records. A help document is available to guide you through the functions of the Planning Assistant.
Our server will save your planning sheet, and you can return to it (or let other teachers access it) by entering a code, which is given on the printable sheet.

Books/Material Masters
The updated versions of all the books and Material Masters for the Numeracy Development Projects are now online under the Project materials link from the Numeracy section of the site.

Diary Dates

Don't forget that the New Zealand Association of Mathematics Teachers is planning its 8th annual conference from the 8th to the 11th July 2003 in Hamilton. Plenary speakers so far announced are: Vaughan Jones, Kaye Stacey, Laurinda Brown, John Edwards,
Jeff Witmer and Charles Lovitt.

For more information contact;
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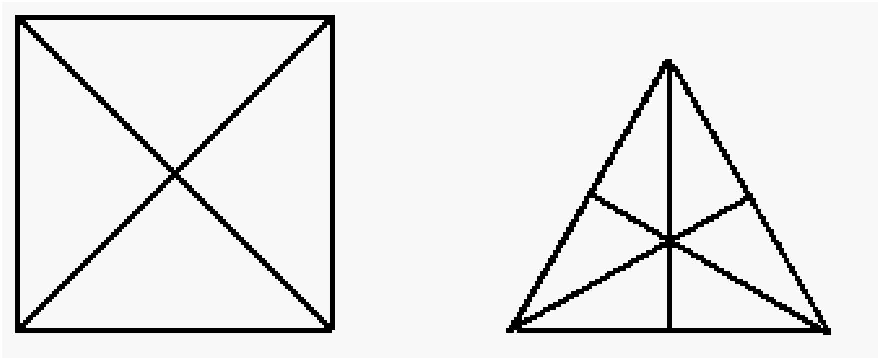
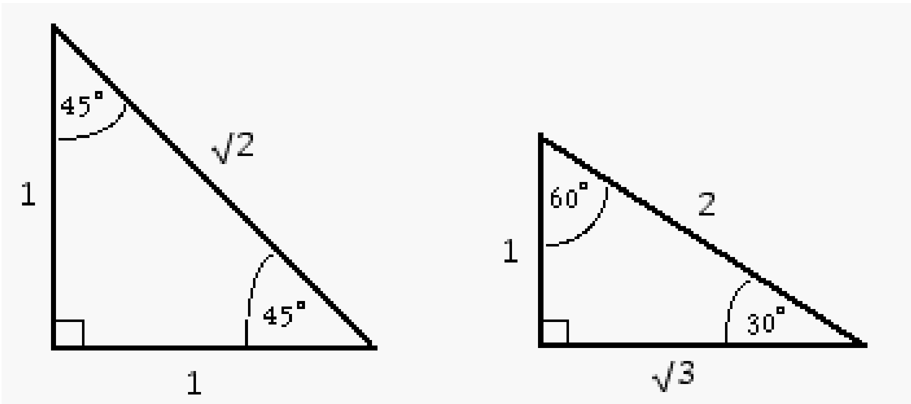
And just an early reminder that Maths week for 2003 is 10-16th August.

Beautiful Triangles.

Plato knew a thing or two. Amongst other things, he was fascinated by what he called his
'most beautiful triangles'. These are obtained from the symmetrical divisions of the square and equilateral triangle.



In each case congruent right-angled triangles are formed, namely,



Bearing in mind that in mathematics the word 'beautiful' has connotations of simplicity,
elegance and wider applications, it might be noted that these two triangles were once much beloved by writers of mechanics texts for the simplicity of their trig ratios.

$\sin 30° = \cos 60° = {}^{1}/_{2}$

$\cos 30° = \sin 60° = {}^{\sqrt{3}}/_{2}$

$\tan 60° = \sqrt{3}$

$\tan 30° = {}^{1}/_{\sqrt{3}}$

$\sin 45° = \cos 45° = {}^{1}/_{\sqrt{2}}$ and so on.

Plato would have been enthralled. They don't come much simpler!

With the extensive use of calculators in the classroom, the knowledge of these simple ratios and the relationship to Plato's most beautiful triangles is likely to be lost. That's a shame.

360

The other day, we were talking about why there were 360º in a complete turn. Someone suggested that the reason that there are 360 in a complete turn is that some ancient culture had 60 as the base for their number system. It's clear that whoever invented the idea liked 60 because there are 60 minutes in a degree and 60 seconds in a minute. What's more, the whole thing has got intimately entangled with time. That may not be surprising because time is often recorded by things rotating – the sun around a sundial and the hands around a clock face. But if we took to 360º in a complete turn, wouldn't it have been more natural to have had 360 minutes in an hour?

If you are interested in this sort of thing, then follow the discussions on http://mathforum.org/epigone/historia_matematica

The 360 may have come about because there are almost 360 days in a year. If the ancients thought about the year as going around a circle they may well have connected the two. Now I know that there are about 365 days in a year but 360 is nicer – it has more divisors – than 365. When you think about it 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 180 and 360 are all divisors of 360.

But how can you decide if a number has a particular factor? Now it's easy enough to tell when a number is divisible by 2. You just look at the last digit. If it's even the number is even. If it's odd the number is odd.

The test for divisibility by 3 is different but nice. A number is divisible by 3 if, when you add its digits, the sum is divisible by 3. You can see this by looking at 360. 3 + 6 + 0 = 9 and that's divisible by 3. This works because of the properties of using base 10. Follow this.

$360 = 3 \times 100 + 6 \times 10 + 0 = 3 \times (99 + 1) + 6 \times (9 + 1) + 0 = 3 \times 99 + 6 \times 9 + 3 \times 1 + 6 \times 1 + 0 = 3 \times (3 \times 33 + 6 \times 3) + (3 + 6 + 0)$.

The first term on the left clearly has a factor of 3. So 360 is divisible by 3 if and only if (3 + 6 + 0) is.

It's actually a pity that I chose 360 here. Take another number 417 and repeat what we just did.

$417 = 4 \times 100 + 1 \times 10 + 7 = 4 \times (99 + 1) + 1 \times (9 + 1) + 7 = 4 \times 99 + 1 \times 9 + 4 \times 1 + 1 \times 1 + 7 = 3 \times (4 \times 33 + 1 \times 3) + (4 + 1 + 7)$.

Then we can repeat the argument. 417 is divisible by 3 if 4 + 1 + 7 is. Since 4 + 1 + 7 = 12, 417 is divisible by 3.

Solution to February's problem

You were asked to express the numbers from 1 to 20 using the digits of 2003, in that order, and a number of mathematical operations and procedures. Remembering that the solutions are not necessarily unique, here is one set of solutions:

1 = $-2 + 0 + 0 + 3$
2 = $2 + 0 + 0 \times 3$
3 = $2 \times 0 + 0 + 3$
4 = $2^0 + 0 + 3$
5 = $2 + 0 + 0 + 3$
6 = $(2 + 0) \times (0 + 3)$
7 = $2^0 + 0 + 3!$
8 = $2 + 0 + 0 + 3!$
9 = $((2 + 0!) + 0) \times 3$
10 = $((2 + 0!)! + 0! + 3$
11 = $((2 + 0!)! - 0! + 3!$
12 = $(2 + 0! + 0!) \times 3$
13 = $20 - 0! - 3!$
14 = $20 + 0 - 3!$
15 = $20 + 0! - 3!$
16 = $20 - 0! - 3$
17 = $20 + 0 - 3$
18 = $20 + 0! - 3$
19 = $20 - (0 \times 3)!$
20 = $20 + 0 \times 3$

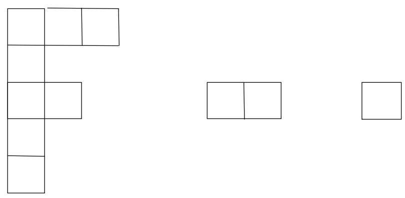
I'm sorry to say that there were no solutions sent in last month.

If you thought that was difficult think back to the year 2000. An extra operation (the greatest integer function) was allowed in that case and some persistent students managed to solve up to 100.

As an aside, I once set the problem of approximating π using the digits of the then current year and the mathematical operations used above. It was amazing how close solvers got to the true value - well within 0.1%. There is a straightforward way of approximating π using the digits of the year 2003 in order and the operations given. It is within 0.2 % of the true value. Oddly enough, it has been known for over 1000 years. You might like to see if you can find it (solution next newsletter).

This month's problem

There are a number of ways the following three shapes can be placed so they all touch (no overlapping) and the resulting shape has line symmetry, i.e. a mirror line. Most people can find five but can you find all seven?



Each month we give a petrol voucher to one of the correct entries. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so we can send the voucher if you are the winner.