

NZ Maths Site Newsletter No. 20

# February 2003

Hi there again, welcome to 2003 which we hope will live up to all your best expectations. Nga mihi nui o te tau hou.

What's special about 2003, you may ask? Well, it has something in common with 503 and 4003 but not with 1003, nor 3003. Can you say what that is? I've included the answer with the answer to November's problem below.

## What's new on the nzmaths site this month?

We hope you have had a restful and relaxing Christmas break. You'll be pleased to know that while most of the country has been on their holidays we at nzmaths have been hard at work. If you are looking for new units of work, there are plenty. New units in Levels 1-4 include three around money. These are located in the Number section of the site, as money is more about operating with number than it is about measurement.

Strand	Level 1-4	Level 5-6	
Number	Money matters		
	Money marvels		
	Money for starters		
Geometry	Amazing mazes		
Statistics	Predict away	Probability trees	
	Planning a statistical investigation	Random processes	
		Probability distributions	
Measurement	Making benchmarks (length)	Practical measuring I	
	Making benchmarks (mass)	Practical measuring II	
		Babylonian fractions I	
		Babylonian fractions II	
Algebra		Holistic algebra	
		Why and how of general terms	
Problem	400 Problem	Consecutive numbers	
Solving			

There are now seven completed Bright Sparks investigations, which will all be on the site by the end of February – keep an eye out!

Frogs: Based on the problem where there are three brown frogs and three green frogs that want to get past each other on seven lily pads. There is also a staff seminar around this problem.

Round Table: Every second person around a table is removed until only one remains, and they are the winner of a grand prize.

Six Circles: Find all possible ways of arranging the numbers 1 - 6 in six circles arranged in a triangle such that the sum of the numbers on each side is equal

Sole Survivor: Based on Solitaire, where counters are jumped and removed from play. Jewels: A version of Noughts and Crosses, played in a 1xn grid and with only one type of counter.

Toni's Tiara: A geometry problem; what shapes of faces can be produced by one planar cut through a cube?

Double Trouble: A two-headed monster wants his toast cut to strict specifications.

#### Sums that aren't sums

Some children add some things up without thinking even when they can't be summed. Take square roots and fractions, for instance. Quick. True or false. Is  $\sqrt{3} + \sqrt{4} = \sqrt{(3+4)}$  or is  $1/3 + \frac{1}{4} = 1/(3+4) = 1/7$ ?

In both cases it's easy enough to make sure just by doing the calculation. Take the square root first. Now  $\sqrt{3}$  is about 1.732 and  $\sqrt{4}$  is exactly 2. So  $\sqrt{3} + \sqrt{4}$  is about 3.732. On the other hand  $\sqrt{7}$  is roughly 2.646. So the two square roots aren't equal.

This raises the question of whether two square roots like this are ever equal. Is  $\sqrt{a} + \sqrt{b}$  ever equal to  $\sqrt{(a + b)}$  for any values of a and b? Well let's put them equal and see if we can find some values for a and b.

Assume that  $\sqrt{a} + \sqrt{b} = \sqrt{(a + b)}$ . So we can square both sides. Squaring the right hand side gives a + b. Squaring the left hand side we get  $(\sqrt{a})^2 + 2\sqrt{a}\sqrt{b} + (\sqrt{b})^2 = a + 2\sqrt{a}\sqrt{b} + b$ . So we have

 $\mathbf{a} + 2\sqrt{\mathbf{a}}\sqrt{\mathbf{b}} + \mathbf{b} = \mathbf{a} + \mathbf{b}.$ 

This simplifies to  $2\sqrt{a}\sqrt{b} = 0$ . This can only happen if one or both of a and b is zero. So you can only take  $\sqrt{a} + \sqrt{b}$  and claim it's  $\sqrt{(a + b)}$  if there's a zero around. But then, of course, it's totally not worth doing.

The fraction one's also quickly sorted out by doing the calculation. What's 1/3? About 0.333. What's 1/4? It's 0.25. And so  $1/3 + \frac{1}{4}$  is about 0.333 + 0.25 = 0.583. On the other hand, 1/7 is about 0.143.

So you can't add this pair of fractions the 'easy' way. Can you add any pair of fractions this way? Is there an a and a b where 1/a + 1/b = 1/(a + b)? Try a few values and see what you get. Is  $\frac{1}{2} + \frac{1}{2} = \frac{1}{4}$  or  $\frac{1}{2} + \frac{1}{5} = \frac{1}{7}$ ?

But maybe there are some values of a and b for which 1/a + 1/b = 1/(a + b). Let's see. First let's play around with the left hand side. 1/a + 1/b = (b + a)/ab. So

(b + a)/ab = 1/(a + b) and that leads to  $(a + b)^2 = ab$  or  $a^2 + 2ab + b^2 = ab$ . This is the same as

 $a^2 + ab + b^2 = 0.$ 

If you remember about completing the square, this gives you  $(a + b/2)^2 + 3b^2/4 = 0$ . Now the two squared terms on the left have to add to zero. But squared terms are either zero or positive. The only way that they can sum to zero is if they are both zero. So a + b/2 = 0 and b = 0. This means that a and b are both zero. The only way that 1/a + 1/b can possible equal 1/(a + b) is if a and b are both zero! Again that's not very useful, especially as it's not clear what 1/0 actually is.

Somehow both of these additions are easy traps to fall into. But if one of a and b isn't zero, then  $\sqrt{a} + \sqrt{b}$  is never equal  $\sqrt{(a + b)}$ , and no matter what a or b are 1/a + 1/b is never equal to 1/(a + b).

### **Diary Dates**

The New Zealand Association of Mathematics Teachers is planning its 8th annual conference from the 8th to the 11th July 2003 in Hamilton. Plenary speakers so far announced are: Vaughan Jones, Kaye Stacey, Laurinda Brown, John Edwards, Jeff Witmer and Charles Lovitt.

For more information contact: Kathy Paterson Box 101, Cambridge New Zealand organiser@nzamt8.ac.nz

And just an early reminder that Maths week for 2003 is 10-16<sup>th</sup> August.

### Solution to November's problem

Here's a two-way addition problem using the nine different positive digits 1 to 9. The numbers form a correct sum as they stand or if they are looked at turning your head 90° anticlockwise.

	5	8	3
+	1	4	6
	7	2	9
		+	

The question is, are there any other such patterns using the digits 1 through 9?

With some trial and deduction you may have found that there is only one other such pattern, viz., 482 + 157 = 639.

Unfortunately there were no solutions sent to us so no one won the prize. Please don't let this happen again!

### 2003 revisited

What do 503, 2003 and 4003 have in common? Well, they are all prime numbers. The other two are not;  $1003 = 17 \times 59$  and  $3003 = 3 \times 7 \times 11 \times 13$ .

#### This month's problem

With my junior classes (Forms 3 and 4) I used to begin each year with the old problem based on the four 4s. You know, the one where you use four 4s and a prescribed set of mathematical symbols to express as many whole numbers as possible from one upwards. For example,

$$1 = 4 - 4 + 4 \div 4$$
  
2 = 4 × 4 ÷ (4 + 4)  
3 = (4 + 4 + 4) ÷ 4 and so on.

It is a popular problem with students and helpful in revising order of operations. As far as is known, it first appeared in the London weekly magazine *An Illustrated Magazine of Science* sometime in 1881 in the letter to the editor column. How successful you are at the puzzle, of course, depends on the operations allowed. These are usually taken as  $+, -, \times, \div, \sqrt{}$ , indices and brackets. For higher numbers to be obtained concatenation, i.e. putting fours side-by-side, 44 for example, and factorials are often allowed.

Factorials arise from the mathematics of arrangements. The factorial symbol '!' denotes the product of a number and all the whole numbers below down to one. Thus  $3! = 3 \times 2 \times 1 = 6$  and  $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ . Factorials arise when numbers of arrangements are needed. For example, in how many ways can the letters of the word CAT be arranged? Listing them gives six. How about the number of arrangements of the letters of the word MATHS? Listing and counting them is time-consuming but there are 120. A quicker method is to note that there are five letters for the first choice. When that has been taken there remain four choices for the second, giving  $5 \times 4$  choices for the two letters. There are three choices for the third letter, two for the fourth and one for the fifth giving  $5 \times 4 \times 3 \times 2 \times 1$  or 5! altogether. (NOTE: the number of ways of arranging no items is defined to be one, thus 0! = 1).

The version of the puzzle that a number of teachers use at the start of each year is the one where the digits of the year are used instead of four 4s. So this year you have to use the digits 2, 0, 0, 3 (in that order) and the symbols given above. I used to give a small prize for the student who could express the most whole numbers between 1 and 100 in this way.

We won't set you that monstrous task, just the numbers from 1 to 20 and we'll start you off. The solutions, of course, are not necessarily unique and we will give a prize for the one which uses the least factorials.

So, here are a couple of starters:  $1 = 2^0 + 0 \times 3$  $2 = 2 + 0 + 0 \times 3$ 

Each month we give a petrol voucher to one of the correct entries. Please send your solutions to <u>derek@nzmaths.co.nz</u> and remember to include a postal address so we can send the voucher if you are the winner.