Newsletter No. 15 **July 2002**

Well, things seem to be back to some semblance of order now with laptops running and communication channels open. That's good and as we mentioned last month, thanks for your letters – both brickbats and bouquets. We really want to know how you feel about the newsletter and other issues relating to mathematics.

The film 'A Beautiful Mind' has certainly raised peoples' awareness of mathematics although I suspect it reinforces the idea that mathematicians are all slightly mad and possess few social skills. Although I know a few who are like that, I assure you it's not generally the case! John Nash, the subject of the film, made important contributions to game theory, a branch of mathematics which uses ideas based on recreational games to analyse the way individuals or groups of individuals interact. If you're trying to track down the book on which the film was based it was written by Sylvie Naser who has recently edited, with Harold Kuhn, a more mathematically oriented tome on Nash's work entitled 'The Essential John Nash'.

Before you read on, remember the words of the mathematical historian E. T. Bell:

> *'If some earnest individual affecting spectacular clothes, long hair, a black sombrero, or any other mark of exhibitionism, assures you that he is a mathematician, you may safely wager that he is a psychologist turned numerologist.'*

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WHAT'S NEW ON THE NZMATHS SITE THIS MONTH?

Fourteen new units of work have been written for the site including thirteen at levels five and six. Eleven of them are on the site now (25/6/2002), and the rest will be shortly. The new units are outlined below.

Strand	Level	Title	Description
Number	5	Rounding numbers	Choosing and rounding to a sensible level of accuracy for different contexts.
Number	5	Square and Cube Roots	Geometrical measuring of square roots and cube roots and methods of calculating them when a scientific calculator is not used.
Number	6	Exponent Power	Solving multiplication and division problems using powers to meaningfully practice students' skills with the laws of exponents.
Number	6	Estimating for Accuracy	A wide range of estimating methods for checking calculator answers for errors.
Number	6	Egyptian Fractions	Adding and subtracting fractions in the context of Egyptian Fractions.
Geometry	4	Building with triangles	Drawing triangles and constructing three-dimensional objects with triangles.
Geometry	5	How High? and Other Problems	Using ruler and compass constructions to draw a variety of shapes and to construct angles such as 90°, 60°, 45° and 15°.
Geometry	5	Space Tiling with Captain Planet	Tessellations are used as an application of angle properties of polygons. Interior angle properties of polygons are used to justify the existence of the five platonic solids.
Statistics (Probability)	5	Fair Games	Investigation of fairness of several games of chance.
Geometry	5	Introducing Trig	Explores the importance of triangles, particularly right-angled triangles, in the real world.
Measurement	5	Pythagoras' Theorem	An introduction to Pythagoras' Theorem, including history, proofs, and practise in application of the Theorem.
Problem Solving	5	400 Problem	Looks at conditions under which two subtraction sums are equal.
Problem Solving	5	V-Numbers	Looks at numbers that fit into a V-arrangement of circles so that the sums of each arm are equal.
Problem Solving	5	Six circles	Looks at numbers that fit into a triangle arrangement of 6 circles so that the sums of each side are equal.

Note: Highlighted units have been completed but are not yet on the site as of 25/6/2002

There are now six units of work available in the Rauemi Reo Mäori component of the site, they are:

- Hautau
- He Hanga i te Tekau
- He Këmu Häkinakina
- He Tapatoru
- Te Ahunga me te Taunga
- He Këmu Tüpono

DIARY DATES

MERGA 25
The annual conference of the Mathematics Education Research Group of Australasia (is
scheduled for July 7-10, 2002 at Auckland University. The 2002 MERGA conference will provide opportunities for mathematics teachers, educators and curriculum developers to contribute and listen to research presentations and be actively involved in workshops,
symposia and special interest groups developed around the conference theme of Mathematics Education in the South Pacific. For further details look on the conference website: www.math.auckland.ac.nz/MERGA25

Applications for **NZ Science Mathematics and Technology Teacher Fellowships**
close 16 July 2002. For more details on these Fellowships see our newsletter No.9 of last November and/or contact Peter Spratt at spratt.p@rsnz.org.nz or look at the website www.rsnz.govt.nz/awards/teacher_fellowships/index.php

Maths Week runs from the 12th to the 16th of August. For more information contact www.nzamt.org.nz

UNEXPECTED PROPORTIONS

Benford's law of numbers was accidently rediscovered in the 1960s by a student checking sales figures from his brother's hardware store. He had needed some real data for a commercial statistics project. What he found was anything but real - over ninety percent of the figures began with the digit one. His brother had cooked the books to defraud the tax office.

You might expect such data to follow a random pattern in which the first digits occur in equal proportion but this is not the case. Data from a wide range of sources like this, including stock market prices, some census data, the heat capacities of chemicals and river drainage areas follow a pattern in which first digits do not occur in equal proportions. In fact the digit one occurs over 30% of the time and the digit two just under 18%. The proportions of the remaining first digits steadily decrease to only 4.6% for the digit 9.

Benford's law was originally discovered over 100 years ago when the American astronomer Simon Newcomb noticed that in a book of logarithms the first pages were grubbier through use than later ones (logarithms were used for tedious arithmetic before the development of electronic calculators). For some reason people did more calculations with numbers beginning with the digit one than any other digit. Newcomb even devised a formula governing such phenomena.

In 1938 physicist Frank Benford rediscovered the phenomena and supplied a great deal of supporting evidence. It has been called Benford's Law ever since. In 1961 mathematician Roger Pinkham supplied a justification for Benford's Law and proved that it was scale-invariant. A law is scale-invariant if it holds true no matter what system of units are used for the measurements involved. It wasn't until 1996 that Theodore Hill from the Georgia Institute of Technology found its true origin.

The ubiquity of Benford's Law means it has many applications. It can be used to detect suspicious data, for example, in clinical trials or provide 'reality checks' for mathematical models used by demographers and economists.

For those of a mathematical bent, the law states that numbers beginning with digit D do so in the proportion $\log_{10}(1 + {}^{1}/_{D})$. So, numbers beginning with the digit one occur in proportion $\log_{10}2$ and numbers beginning with the digit two have proportion $\log_{10}1.5$. For numbers to follow Benford's Law they must confirm to a number of conditions. They must be free from artificial limits and come from a large sample. Don't expect the law to show itself when you look back over the tracks you've played from your last ten CDs.

THE (UK) MATHEMATICAL ASSOCIATION'S WEBSITE

This month we look at another mathematical association's web site. This time it's one from the UK, www.m-a.org.uk, the site of The Mathematical Association.

When you reach their home page you are faced with several options. Probably the most interesting for readers of this newsletter are Primary Education, 11-16 Education, Special Education, Periodicals, and Publications. But, if you are likely to be in Britain for any reason, then you should also look at Conferences and Local Activities. You would be more than welcome at any of these.

The MA is well known for its publications and periodicals and Primary Education lists two periodicals and several publications that you might find interesting. Each one is listed with an outline and cost. If there is any spare money in the kitty, it might be worth getting hold of some of these. (The Periodical and Publication buttons extend these lists to other levels of the school.) You might like to chase up the Primary Mathematics Challenge. While you may not want to formally participate, there are some good problems there for your students to attempt.

The Special Education section is concerned with children with difficulties in mathematics. The MA publishes a journal on this area called Equals. There are some articles from past issues on the web so that you can get some flavour of what this is about.

One warning, the site does contain a few acronyms that may not make any sense to you but you can still get a great deal from the site by pretending that they do.

SOLUTION TO MAY'S PROBLEM

Last month we repeated the problem given in the May issue of the newsletter which was to find the minimum perimeter when four unit squares are placed, non-overlapping, in a plane. The problem continued asking you to find the minimum perimeter for five squares, six squares and 200 squares.

We also wondered if you could generalise for shapes formed from n non-overlapping unit squares in a plane.

Cathy Walker sent in an elegant solution which covered all the cases asked for in the problem, although needed a little 'tweeking' for the general case. Congratulations Cathy, We're certainly going to send you the prize. Here's our attempt at a full solution…..

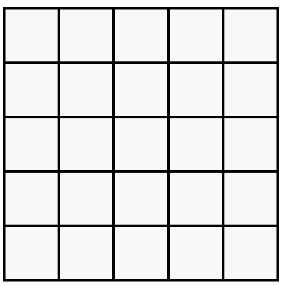
Well, a little bit of drawing will have given the first three answers as 8, 10 and 10 and perhaps the inkling of an emerging pattern.

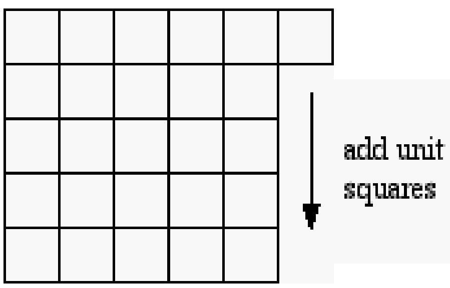
Investigating and tabulating for small values of the number of sides n, gives:

n:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
p:	4	6	8	8	10	10	12	12	12	14	14	14	16	16	16	16

(where p = minimum perimeter).

A formula for p in terms of n is not obvious but further exploration may suggest a way of obtaining p. The minimum perimeter certainly occurs when the shape is as close as possible to a square. When n is a square number, $n = d^2$, the minimum perimeter is obtained when the shape formed is a square. The perimeter is then 4d, as fig(i) shows for d = 5.



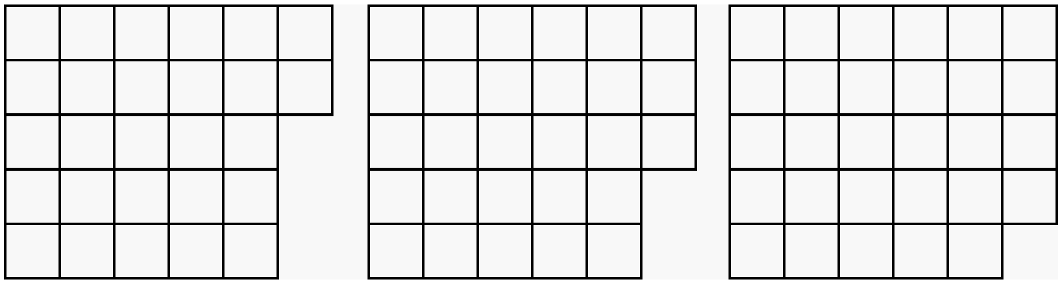


$p = 4d$ (in this case $4 \times 5 = 20$)
fig (i)

$p = 4d + 2$
fig(ii)

Adding another unit square to the outside edge, as shown in fig(ii) adds another two to the perimeter. Thus when $n = d^2 + 1$ the minimum perimeter is $4d + 2$.

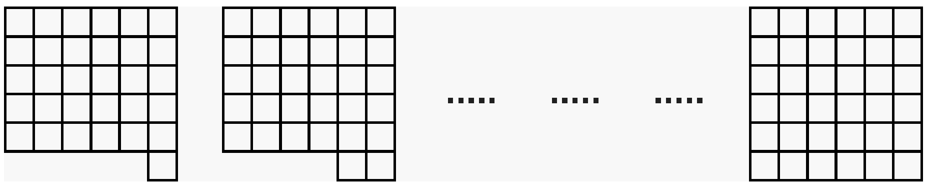
Adding further unit squares along the same edge adds nothing to the perimeter, see fig(iii). Hence for $n = d^2 + 1$ to $d^2 + d$ the minimum perimeter is $4d + 2$.



$p = 4d + 2$ $p = 4d + 2$ $p = 4d + 2$ and so on

fig(iii)

Now add unit squares along the longer edge, see fig(iv).



$p = 4d + 4$ $p = 4d + 4$ $p = 4d + 4$

fig(iv)

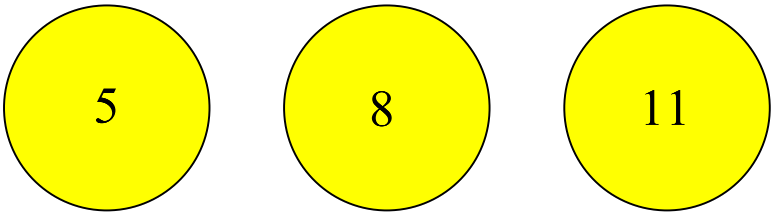
Hence for $n = d^2 + d + 1$ to $(d + 1)^2$ the minimum perimeter is $4d + 4$.

Now to solve the remaining part of the problem, the minimum perimeter for 200 squares,.
The largest square number less than 200 is 196 giving $d = 14$. In that case, $d^2 + 4 = 200$ and hence $n = 200$ lies between, $n = d^2$ and $d^2 + d$. So using $d = 14$ and $p = 4d + 2$ we see that the minimum perimeter for 200 squares is 58.

SOLUTION TO JUNE'S PROBLEM

We don't really have an answer to last month's problem because we've had two. So far two different answers have been sent in that are both correct! It would be nice to have a complete set of answers so we've left June's problem open. So the question now becomes the following.

I have three discs. They have a number on each side. Three of these numbers are shown in the diagram.



I toss the numbers up into the air and add the three numbers that I see. After a while I notice that all the totals I get are consecutive numbers. What numbers are on the other sides of these discs? FIND ALL POSSIBLE WAYS OF NUMBERING THE DISCS.

THIS MONTH'S PROBLEM

Three farmers Alan, Bob and Craig and their three sons Dan, Ernie and Fred (not necessarily in the same order) each buy calves. Each person buys as many calves as he gives dollars for one. Each farmer pays altogether $63 more than his son. Alan buys 23 calves more than Ernie and Craig buys 11 more than Dan. What is the name of each man's son?

Each month we give a petrol voucher to one of the correct entries. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so we can send the voucher if you are the winner.

Finally, a quote:

> *'If you're really concerned about your child's education, don't teach it to subtract, teach it to deduct.'*
> Fran Lebowitz

Enjoy your teaching - Gill, Derek, Russ and Joe.