

Accelerating Learning in Mathematics

DRAFT



Learning to: use decimals to solve problems

Target group: students in years 6–8

Focusing on:

- representing decimals on a number line
- understanding the effect that operations have on decimals
- using decimal numbers to solve problems.

Beliefs underpinning effective teaching of mathematics:

- Every student's identity, language, and culture is respected and valued.
- Every student has the right to access effective mathematics education.
- Every student can become a successful learner of mathematics.

Ten principles of effective teaching of mathematics:

1. An ethic of care
2. Arranging for learning
3. Building on students' thinking
4. Worthwhile mathematical tasks
5. Making connections
6. Assessment for learning
7. Mathematical communication
8. Mathematical language
9. Tools and representations
10. Teacher knowledge

See *Effective Pedagogy in Mathematics* by G. Anthony and M. Walshaw, Educational Practices Series 19, International Bureau of Education, available at: www.ibe.unesco.org

TEACHER OBSERVATION OVER A RANGE OF ACTIVITIES

The student has strategies for working with whole numbers and a sound knowledge of place value for whole numbers, with a fast and fluent recall of basic facts. They may struggle when working with decimal amounts, either ignoring them or treating them as whole numbers. The student may have difficulty converting simple decimals into fractions and vice versa.

POSSIBLE BARRIERS TO THE STUDENT'S PROGRESS

1	Difficulty placing decimals on a number line
2	Weak understanding of decimal place value
3	Difficulty multiplying decimals

EXPECTATIONS FOR NUMBER

AFTER 1 YEAR AT SCHOOL		AFTER 2 YEARS AT SCHOOL		AFTER 3 YEARS AT SCHOOL		BY THE END OF YEAR 4		BY THE END OF YEAR 5		BY THE END OF YEAR 6		BY THE END OF YEAR 7		BY THE END OF YEAR 8	
COUNTING FROM ONE		ADVANCED COUNTING		EARLY PART-WHOLE THINKING		EARLY ADDITIVE		EARLY ADVANCED ADDITIVE		ADVANCED ADDITIVE – EARLY MULTIPLICATIVE		EARLY ADVANCED MULTIPLICATIVE		ADVANCED MULTIPLICATIVE – EARLY PROPORTIONAL	
NZC EARLY LEVEL 1	NUMERACY STAGE 2 OR 3	NZC LEVEL 1	NUMERACY STAGE 4	NZC EARLY LEVEL 2	NUMERACY EARLY STAGE 5	NZC LEVEL 2	NUMERACY STAGE 5	NZC EARLY LEVEL 3	NUMERACY EARLY STAGE 6	NZC LEVEL 3	NUMERACY STAGE 6	NZC EARLY LEVEL 4	NUMERACY EARLY STAGE 7	NZC LEVEL 4	NUMERACY STAGE 7

BARRIER BEING ADDRESSED

1 DIFFICULTY PLACING DECIMALS ON A NUMBER LINE

DIAGNOSTIC QUESTIONS

1. Give the student four pieces of paper with the numbers 0.5, 1.99, 2.5, and 1.0 written on them. Ask the student to place the numbers in order of size.
2. Ask the student to position the numbers on a number line.

WHAT TO NOTICE IN THE STUDENT'S RESPONSE

- Can the student place the numbers in order?
- Do they attend to the whole numbers first, or do they order the numbers based on how many digits they have?
- Do they know that 0.5 is located halfway between 0 and 1?

DELIBERATE ACTS OF TEACHING

The most effective way to learn new information is to make connections with existing knowledge. In these exercises, a 0–10 number line is compared to a 0–100 number line and a 0–1 number line.

Place It! Label It!

Construct a large number line (at least 1 metre long) representing values from 0 to 10. Mark the halfway point and ask the student to label it. Mark the halfway points between 0 and 5 and between 5 and 10, asking the student which number goes at each point. If necessary, use decimats to model half of 5 wholes. Write in the whole numbers between 0 and 10.

Give the student sticky notes with various decimal numbers written on them and ask them to place them on the number line. Encourage the use of benchmarks to make decisions, for example, “3.2 would be a bit less than halfway between 3 and 3.5.” Give the student practice with this number line for a few days, asking them to place decimal numbers on it and to label different points.

Create a number line the same length as the first one, representing values from 0 to 100. With the student, mark in the halfway points (50, then 25 and 75).

Place one number line below the other and compare the values of the halfway points. Discuss the relationship between 7.5 and 75.

Ask the student to find halfway points between several pairs of consecutive whole numbers. Give the student values to place on the number line and places to label with values.

Make a third number line the same length as the other number lines, representing values from 0 to 1. Discuss with the student where the halfway points will be and use a decimat to check their values.

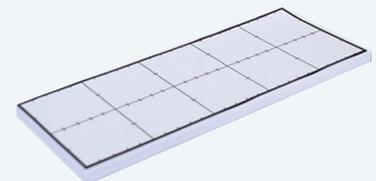
Place the 0–1 number line above the 0–10 number line and compare 0.25 to 2.5. Discuss how the change in scale has impacted on the place value of the 2 and the 5.

Compare the shift in scale to looking at a single section of a number line through a magnifying glass.

Use Digital Learning Object: Scale Matters for additional practice.

MATERIALS/LINKS

- Decimats (Material master 7-3)
- Long strips of paper
- Sticky notes
- Digital Learning Object: Scale Matters – Tenths



WHAT TO DO NEXT IF THE STUDENT IS STUCK

Continue working with the 0–10 number line. Model each number. Give the student repeated practice placing values on the number line and labelling points.

INITIATING HOME-BASED ACTIVITIES

Provide a tape measure and ask the student to make linear measurements in their home environment, for example, heights of family members or the lengths and widths of windows.

NEXT TEACHING STEPS BACK IN THE CLASSROOM

Measurement work, particularly length, is useful in strengthening the concept of number lines.



BARRIER BEING
ADDRESSED

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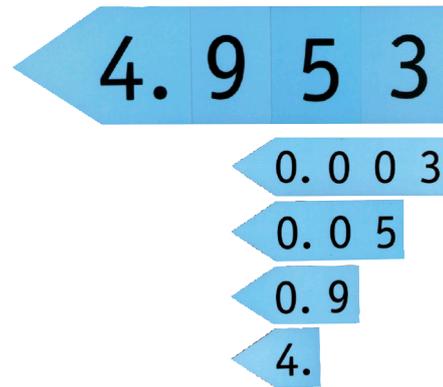
WEAK UNDERSTANDING OF DECIMAL PLACE VALUE

DIAGNOSTIC QUESTIONS

1. a. Draw a square on a piece of grid paper and write 0.5 inside it. Tell the student that the square is 0.5 of a larger rectangle. Ask the student to draw the larger rectangle. If the student is struggling, tell them that the square is half the size of the larger rectangle and see whether this helps.
2. a. Show the student a second square with 0.1 written on it. Tell the student that the square is 0.1 of a larger rectangle. Ask the student to draw the larger rectangle. If the student is struggling, tell them that the square is one-tenth of the size of the larger rectangle and see whether this helps.
- b. Draw four identical squares with 0.1 written on them and ask the student what decimal they represent.
- c. Ask the student what fraction of a larger rectangle the four squares represent.
3. a. If the student demonstrates an understanding of 0.5 and 0.1, draw a square with 0.25 written on it and ask them to show you the size of the larger rectangle. If necessary, tell the student that the paper is one-quarter of the original rectangle.

WHAT TO NOTICE IN THE STUDENT'S RESPONSE

- Does the student know that 0.5 is equivalent to $\frac{1}{2}$?
- Do they know that two halves make a whole?
- Does the student know that 0.1 is equivalent to $\frac{1}{10}$?
- Can the student identify that 4×0.1 is 0.4?
- Does the student know that 0.25 is equivalent to $\frac{1}{4}$?



DELIBERATE ACTS OF TEACHING

Use whole numbers to review the rules for place value. For example, 10 of one unit creates a new grouping or room in a place value house; a place has 10 times the value of the place that comes after it and $\frac{1}{10}$ of the value of the place that precedes it.

Use these rules to identify the value of the first decimal place.

Decimats

Use one colour for the mat on which the decimals will be modelled and different colours to represent tenths, hundredths, and thousandths.

Cut out tenths and use them to model different decimals (1 d.p.).

Model a decimal such as 0.3 and ask the student how many tenths are needed to make a whole. Make connections to making-tens skills.

Use the decimat to make visual links between $\frac{1}{2}$ and 0.5, $\frac{1}{4}$ and 0.25, $\frac{1}{5}$ and 0.2, and $\frac{1}{10}$ and 0.1.

Explore place value to 3 decimal places.

Introduce problems involving whole numbers, for example, $2.3 + \square = 4$. Relate the decimat model to the way decimal values are written or to arrow cards.

Use Digital Learning Object: Decimaster to model numbers and to locate them on number lines.

MATERIALS/LINKS

Decimats (Material master 7-3)

Place value house or place value flip chart for decimals

Decimal Arrow Cards (Material master 7-2)

Digital Learning Object: Decimaster (Number, level 3)

**WHAT TO DO NEXT IF THE STUDENT IS STUCK**

Check whether the student understands whole number place value and unit fractions. Do not move on to the second and third decimal places until the concept of tenths is secure. Use other objects that can be divided into ten pieces.

INITIATING HOME-BASED ACTIVITIES

The student can make a copy of decimats or use Digital Learning Object: Decimaster to practise at home.

Encourage the student to record decimal numbers that they find in their home environment.

Send home a copy of Decimal Place Loopy.

NEXT TEACHING STEPS BACK IN THE CLASSROOM

Give the student problem-solving activities involving decimals, with equipment available if required.

Measurement and statistics work are useful contexts within which to explore decimals.

BARRIER BEING
ADDRESSED

3

DIFFICULTY MULTIPLYING DECIMALS

DIAGNOSTIC QUESTIONS

- Three friends each have 1.5 litres of juice. How many litres do they have altogether?
Answer: 4.5 litres
- If an eel weighs 0.6 kg, how much would 3 eels weigh?
Answer: 1.8 kg

WHAT TO NOTICE IN THE STUDENT'S RESPONSE

Can the student combine halves to create new wholes?
Do they see 3×1.5 as $(3 \times 1) + (3 \times 0.5)$ or know that $1.5 + 1.5 = 3$ and another 1.5 makes 4.5?
Does the student use a known fact such as $3 \times 6 = 18$ but get an incorrect answer such as 0.18?

DELIBERATE ACTS OF TEACHING

Learning how to halve or double a decimal number will help students to avoid overgeneralising whole number strategies to decimals.

These activities will challenge the common misconception that "multiplication makes things bigger".

Double or Half the Loot

The context for this activity is a group of pirates dividing up gold bars.

Make a set of cards with decimal amounts on them, using only even numbers initially. Create yellow decimatats to represent gold bars. Label a dice so that three sides have "x 2" (double) and three sides have "x 0.5" (half). Take turns drawing a card, modelling the amount on the decimat, and then rolling the dice. Discuss how to double or halve the amount. Record the result as a running total.

Read aloud the total as whole bars and fractional amounts, for example, "3 whole bars and 2 tenths of a bar and 5 hundredths of a bar".

Increase the difficulty by using odd numbers and by changing the operations on the dice, for example, $\times 4$, $\times 0.1$, $+ 0.5$...

MATERIALS/LINKS

Decimatats (Material master 7-3)

Dice labelled with $\times 2$ and $\times 0.5$

WHAT TO DO NEXT IF THE STUDENT IS STUCK

Spend several lessons exploring decimat representations. Simplify the operations dice by using $+ 0.1$ or $- 0.1$.

INITIATING HOME-BASED ACTIVITIES

Encourage parents to use games to help the student practise working with decimals. One example is "How much more to make 1?" The student takes a playing card and uses the number to represent a decimal, for example, 8 becomes 0.8. The student states which decimal is required to make 1 (0.2).

NEXT TEACHING STEPS BACK IN THE CLASSROOM

Decimal Cards (www.nzmaths.co.nz)

Most level 3–4 maths activities involve working with decimal amounts. Measurement and statistics are common applications.

Encourage the student to use their understanding of decimals to make estimates when solving problems in order to check whether their final answers are reasonable.

