A Parabolic Investigation

Purpose:

The purpose of this multi-level task is to engage students in an investigation of parabolas in a practical context.

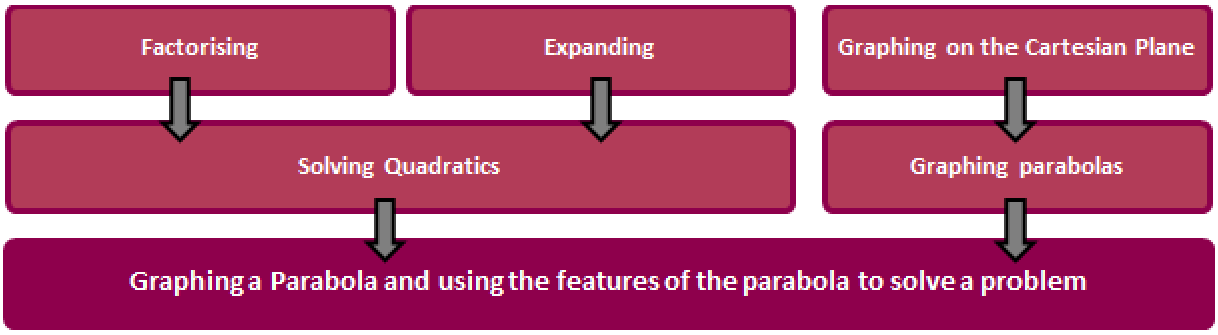
Achievement Objectives:

NA5-7: Form and solve linear and simple quadratic equations.

NA5-9: Relate tables, graphs, and equations to linear and simple quadratic relationships found in number and spatial patterns.

Description of mathematics:

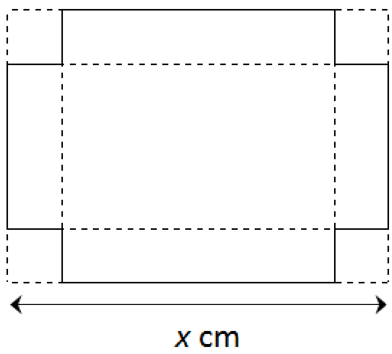
The background knowledge presumed for this task is outlined in the diagram below:



This investigation task can be presented with graded expectations to provide appropriate challenge for individual learning needs.

Activity:

Task: a rectangular sheet of card, with perimeter = 80 cm is made into an open-topped box, by folding in 2 cm x 2 cm squares from each corner. Investigate the relationship between x, the length of one side of the card and C, the capacity of the box.



The arithmetic approach

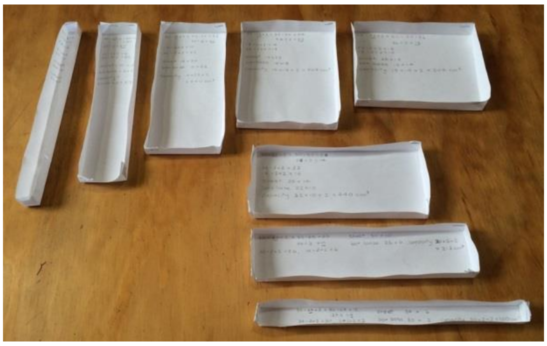
The student makes and measures concrete examples to lead them to solve the problem.

The student links the practical task to a generalised model

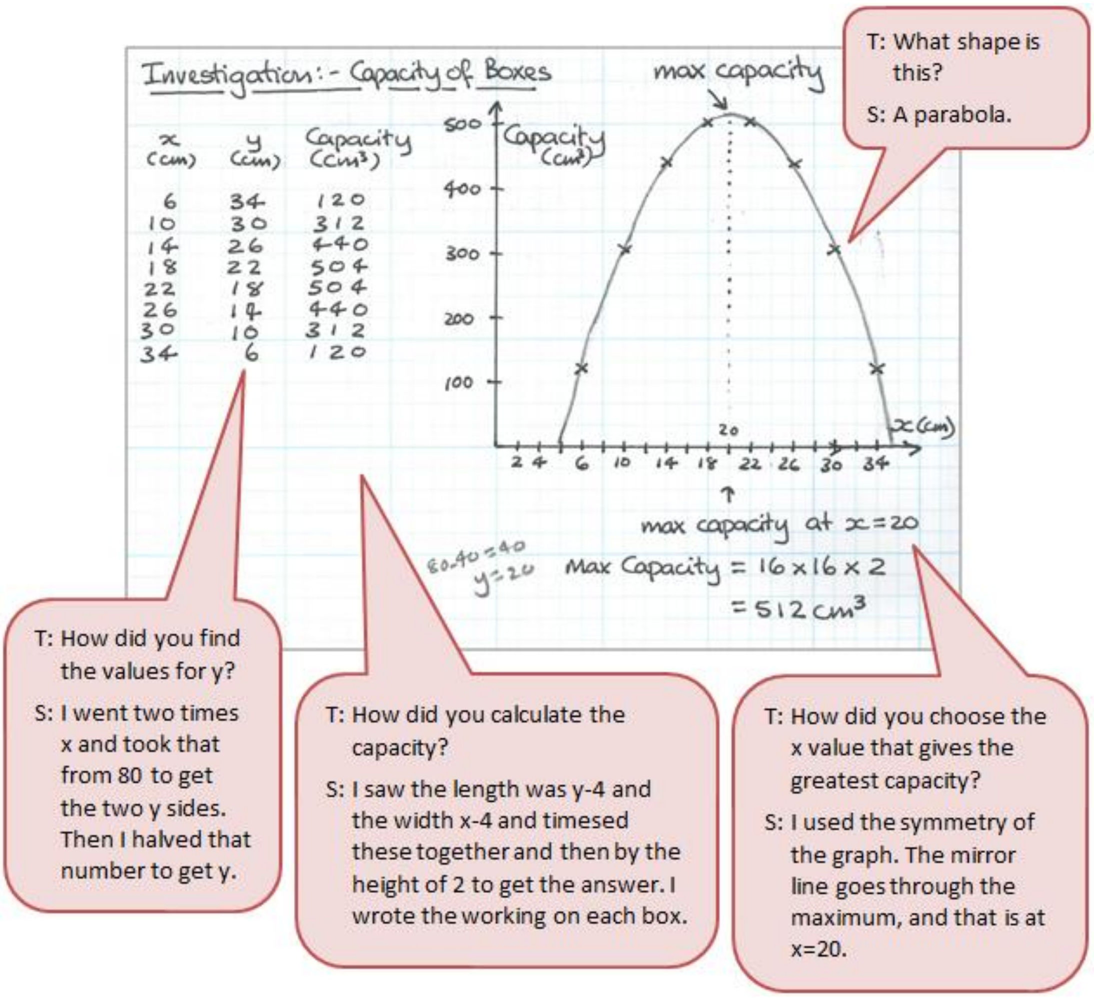
To focus on the linking the practical task to a generalised model, the students should be directed to make and measure.

Prompts from the teacher could be:

1. First assemble the following sheets of card, allowing you to make and measure a range of boxes for this investigation.
   x = 6 cm
   x = 10 cm
   x = 14 cm
   x = 18 cm
   x = 22 cm
   x = 26 cm
   x = 30 cm
   x = 34 cm
   What do you notice about these rectangles?



2. Now make the boxes and measure their capacity (either by weight of dry rice the box can hold, or by measuring and calculating).
3. Graph capacity (or weight of rice held) against x. Describe the shape of this graph.
4. What is the maximum capacity of all the possible boxes?



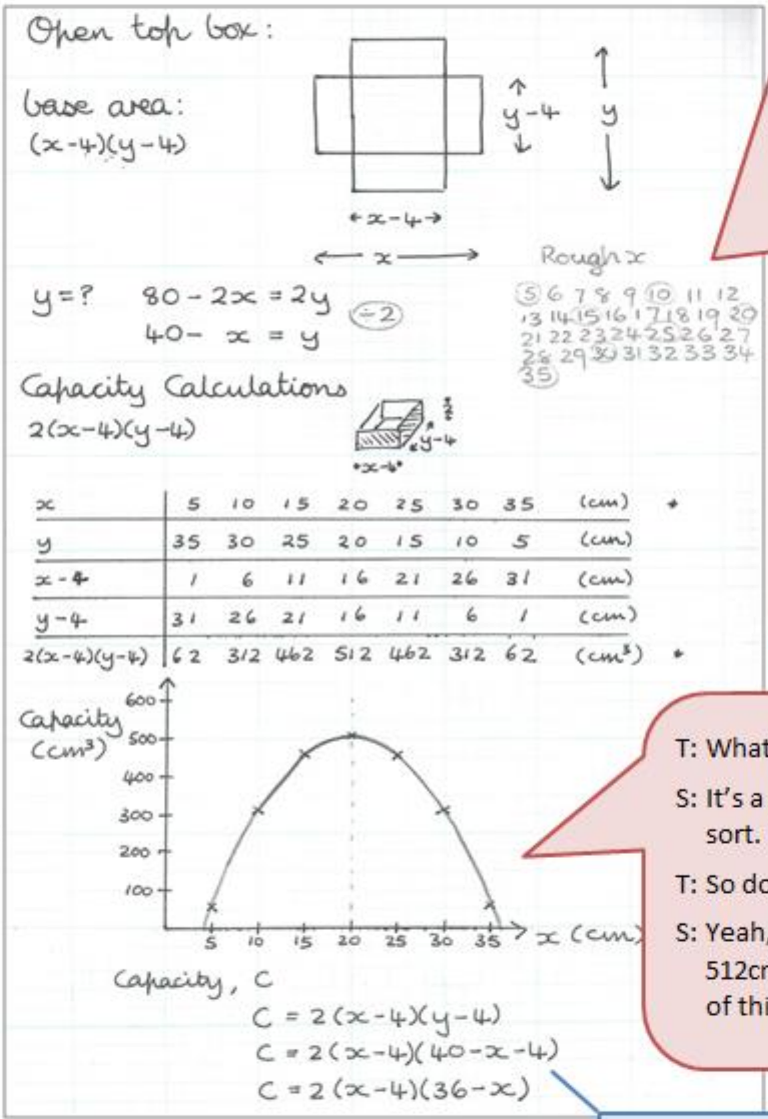
The procedural algebraic approach

The student carries out directed calculations that will lead them to find a quadratic relationship.

To engage the students in directed calculations that will lead them to find a quadratic relationship.

Prompts from the teacher could be:

1. Look at the net in the task and sketch the box it will make. Label the dimensions of that box.
2. Think of some of the values that x can be. Think also of what values x cannot be.
3. Now make up a table that will allow you to work out capacity, C for several different values of x. At least six different values of x should be used and these should be as wide a range as possible.
4. Calculate the capacity for each of three values.
5. Graph the capacity against x for these boxes. What shape is the graph?
6. Try to write out the steps of your calculation for capacity in terms of x.



T: Tell me about this rough working.

S: I need to make a range of boxes. x has to be bigger than 4 so I started listing x's that start at 5. Then I stopped at 35 because y has to be bigger than 4 and x+y=40.

T: And the circled numbers?

S: Well, once I had all my x's I could use I saw that going up in 5s would give me a good number of boxes to make up.

T: What shape is this graph?

S: It's a parabola, but an upside down sort.

T: So does it have a maximum value?

S: Yeah, the biggest box holds 512cm³. That's the point at the top of this graph.

Substitution for y

The conceptual algebraic approach

The student carries out an algebraic investigation that allows them to describe a quadratic relationship.

To engage the students in an algebraic investigation that would allow them to describe a quadratic relationship.

Prompts from the teacher could be:

1. Instead of making a model, or trying out a few values, use algebra to attempt this task. Translate the task into a series of equations that summarise the information given.
2. Give a general equation to describe the perimeter.
3. Give a general equation to describe the base area of the box.
4. Give a general equation to describe the capacity of the box.
5. Can the equations you've written be combined to give the capacity of the box in terms of only x?
6. What does your relationship between capacity and x tell you about the values x can be?
7. What does your relationship between capacity and x tell you about the maximum capacity of the box?

Further exploration of the quadratic relationship can be encouraged, with extended questioning:

1. Sketch and describe the features of the graph of capacity against x.
2. Discuss the limits of the values that x can take.
3. Use the symmetry of your graph to find the maximum capacity of the box.