Exploring Issues in Mathematics Education

An Evaluation of the Advanced Numeracy Project 2002

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Executive Summary

The Advanced Numeracy Project (Years 4 to 6) is part of a key government initiative aimed at raising student achievement by building teacher capability in numeracy teaching. It is one of four projects implemented in 2002. The other projects were the Early Numeracy Project (Years 1 to 3), the Numeracy Project (Years 7 to 10), and the Māori-medium Pilot Project (Te Poutama Tau).

The previous evaluations of the Advanced Numeracy Project (ANP) specifically investigated effective teaching, effective facilitation, and knowledge of how students acquire number concepts. This previous evaluation informed the 2002 delivery of the project. This third evaluation of the ANP reflects the continued development and the success of the project.

The Number Framework groups students' mathematical thinking into broad stages and forms an important part of the project. The framework is also the basis for assessing students' thinking and informing the next stage of learning. Using the framework also helps teachers to deepen their understandings of mathematics content and how students learn mathematics.

This evaluation report evaluates the impact of the ANP on approximately 54,000 students and 2,000 teachers from eight regions (Northland, Auckland, Waikato, Massey, Wellington, Canterbury, Otago, and Southland). The report identifies changes in student achievement and in teacher knowledge and practice, that can be attributed to the professional development provided by the ANP. A particular focus of the report is the case study in Chapters Six and Seven, which look at issues and challenges that have been found in two low-decile schools in implementing the project.

The research on which this report is based addressed eight main questions:

- In what ways does the Advanced Numeracy Project impact on facilitators' subject and pedagogical content knowledge?
- What characterises "effective" classroom practice in decile 1 schools?
- In what ways does the Advanced Numeracy Project impact on teachers' subject and pedagogical content knowledge?
- How do teachers see changes in their subject and pedagogical content knowledge impacting on their classroom practice?
- What was it that the facilitators did that had most impact on improving teachers' classroom practice
- What progress do students make on the Number Framework?
- How is progress linked to age, ethnicity, gender, geographical region, or school decile level?
- To what extent and in what way has student performance in number improved over the duration of the Advanced Numeracy Project in 2002?

Key Findings

- The ANP created an increase in student achievement in number. As in 2001, this growth was irrespective of students' age, gender, ethnicity, and decile. Students participating in the project developed sophisticated strategies for solving number problems. A shift to part-whole thinking characterises this advancement as this enables students to solve more complex number problems.

- The changes to the diagnostic interview to provide a choice of forms (based on students' strategy knowledge) may have unintentionally created a ceiling effect. This is shown in the comparison of the 2001 and 2002 data. Teachers may have inadvertently used the same form for the initial and final diagnostic interviews, when students probably should have been given a more advanced form for the final interview. Changes to the diagnostic interview in 2003 may address this issue.
- While all students advanced in their thinking of number, the gains of Asian and New Zealand European students were greater than for Māori and Pasifika students.
- The support that facilitators gave to individual teachers in their classroom was most important because it enabled teachers to refine their existing practices and develop new ones in their everyday teaching context. Working together in the classroom gave the facilitator and teacher a common basis for discussion of the issues that arose in the classroom implementation of the project.
- Teachers appeared to extend their views of active learning to include discussion as well as manipulation of equipment. More teachers commented on their adoption of group work as their key teaching approach.
- Teacher attitudes towards mathematics and the teaching of mathematics improved during the course of the project, with teachers becoming more enthusiastic and confident because of their participation.
- Student attitudes towards learning mathematics (as judged by teachers, principals, and facilitators) improved, with students more often expressing enthusiasm and confidence.
- Factors identified as inhibiting change in low-decile schools include teacher expectations of and beliefs about students in low-decile schools, the high proportion of students with English as a second language, the capacity of parents to provide additional assistance to students who are struggling, and student transience.
- Successful strategies for enhancing achievement in low-decile schools include a focus on literacy and numeracy and the provision of additional staff (both trained and untrained) in school.

Recommendations

As a result of this study, it is recommended that:

- further opportunities should be given to schools that have already participated in the ANP to continue with the project in order to consolidate the substantial changes to teachers' practice at the year 4 to 6 levels;
- further investigation should be made into the components of effective facilitation;
- the use of the strategy window that determines which form of the diagnostic interview to use in assessing student achievement should be further investigated (facilitators need to ensure that teachers understand the importance of using the correct form of the diagnostic interview in the final assessment);
- the adoption of an instructional model that complements the teaching model based on the work of Pirie and Kieren (1992) should be investigated;
- ways of raising student achievement should be investigated, with a particular emphasis on raising the achievement of Māori and Pasifika students;

- strategies for bringing about change in low-decile schools should be investigated;
- the project should be continued across all sites of teacher education from pre-service and in-service training to postgraduate courses.

Chapter One: Introduction

In 2001 the ANP was effective in raising student achievement across six aspects of number monitored at years 4 to 6 irrespective of students' age, gender, ethnicity, school region, or decile ranking (Higgins, 2002). An important goal for year 4 to 6 students is to shift from counting-based problem-solving strategies to more sophisticated and efficient strategies that are based on part-whole thinking or the partitioning of numbers. The shift to part-whole thinking is critical to success in later mathematics (Thomas and Ward, 2002; Young-Loveridge, 2001). By the final assessment in 2001, most students had achieved this.

This report evaluates the impact of the ANP in 2002 on participating students, teachers, principals, and facilitators. In particular, it examines the impact of the project on lower decile schools, which appeared (from the 2001 data) to face greater challenges in shifting their students to using part-whole thinking. In policy terms, the project fits within the government's Literacy and Numeracy Strategy, which aims to raise student achievement, develop teacher capability, and increase community involvement in the areas of literacy and numeracy.

Raising Student Achievement and Developing Teacher Capability

The Numeracy Project has been informed by current research and the development of theoretical models to explain students' acquisition of number concepts (see Jones et al., 1996; Fuson et al., 1997; Wright, 1998; Young-Loveridge, 1999). These models aim to capture the complexities of place-value and multi-digit understanding for instructional purposes by identifying the different conceptual structures or key constructs in number development. There is general agreement that the emphasis on counting to solve problems needs to be replaced with an emphasis on the part-whole relationships among numbers. It is also agreed that this shift is very difficult for some students to make (Young-Loveridge, 2001). This emphasis encapsulates the aim of the ANP that students should develop increasingly sophisticated and efficient ways of partitioning numbers.

A key part of the literacy and numeracy strategy is the improvement of teacher learning and facilitation of teacher change. Facilitators have worked alongside teachers to support and guide them in establishing more effective teaching and learning models informed by sound mathematics content knowledge. The Number Framework (Ministry of Education, 2002a) provides a structure for teachers to use when identifying the extent to which a student is employing part-whole strategies in solving number problems.

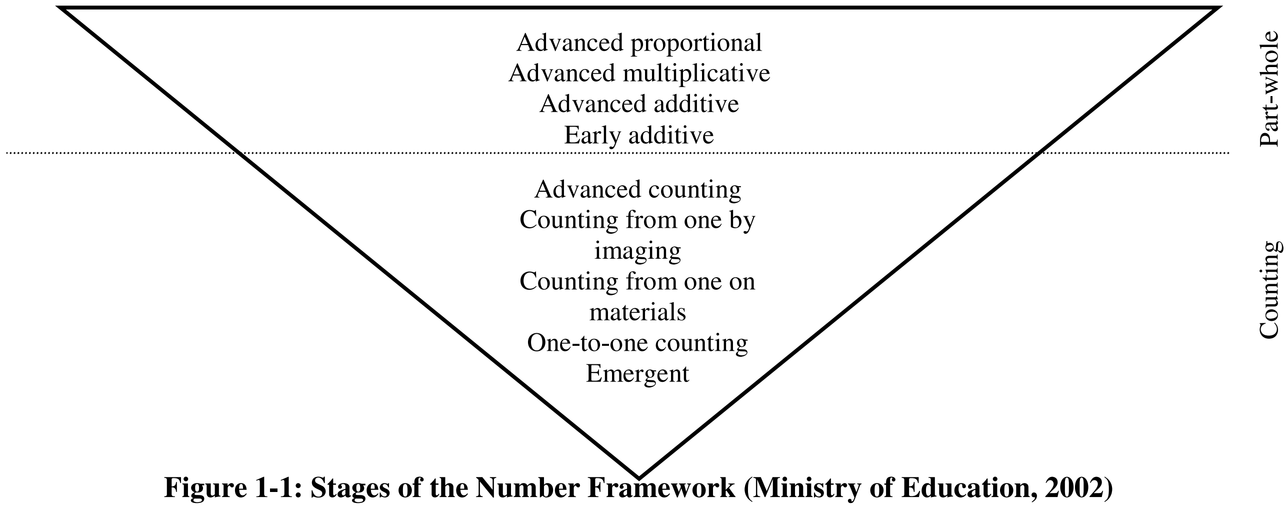


Figure 1-1: Stages of the Number Framework (Ministry of Education, 2002)

The Teaching Model (Ministry of Education, 2002b), which draws on the work of Pirie and Kieren (1989), highlights potential difficulties in abstracting mathematical ideas from equipment. The emphasis in the Numeracy Project has been on using equipment to help students think about mathematical ideas rather than following the more traditional approach (which uses procedural models to solve a problem).

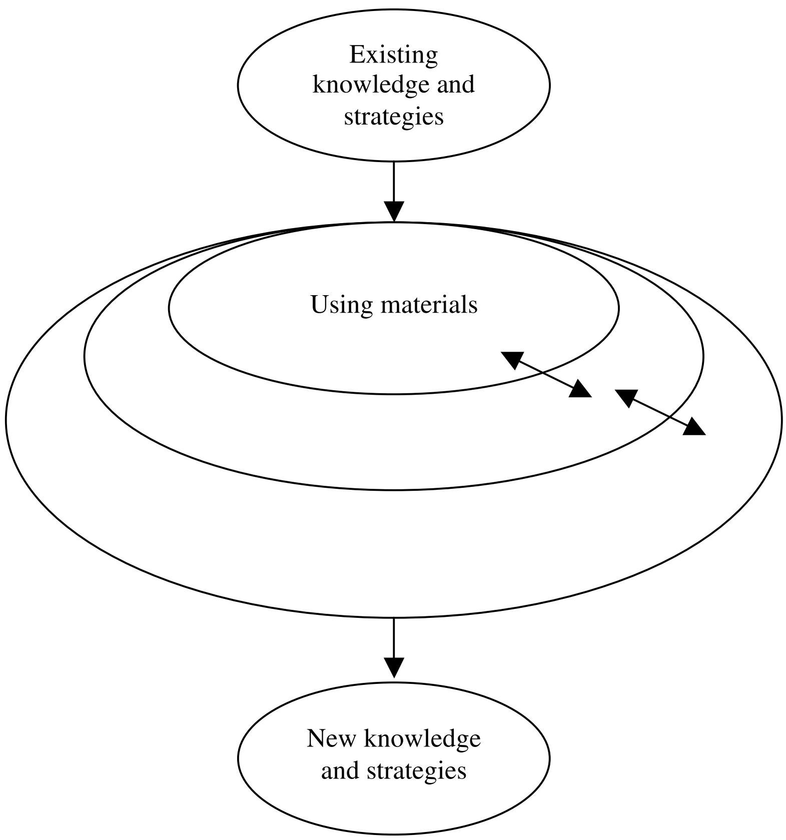


Figure 1-2: The Teaching Model (Ministry of Education, 2002)

The learning environment within which the teaching model is implemented is an important factor when considering students' opportunities to learn. Alton-Lee's (2002) best-evidence synthesis of recent research on quality teaching for diverse students suggests that variance in student achievement is attributable more to teaching than to schools per se. This suggests that we need to pay more attention to the instructional practices of teachers. Comments from teachers, facilitators, and principals reported in 2001 (Higgins, 2002) suggested an increase in discussion and group work.

Work by Cobb and colleagues has been influential in providing guidelines for instructional practices that foster and support interaction, such as small-group instruction. They argued that the negotiation of social norms is key to the setting-up of effective small-group interactions (Yackel and Cobb, 1996). Two intertwined levels of discourse, one for "talking about talking about mathematics" and one for "doing and talking about mathematics" appear to foster interaction (Cobb, Wood, and Yackel, 1993). "Talking about talking about mathematics" is the process of negotiating social norms that regulate the activity of "doing and talking about mathematics". The teacher's role is to establish the expectation that the students co-operate by listening to each other's ideas and expressing their own ideas fully. By contrast "doing and talking about mathematics" is about negotiating mathematical meaning.

More recent work by Fraivillig, Murphy, and Fuson (1999) described a pedagogical framework that supports the development of students' understanding. It identifies three key components: eliciting students' solution methods, supporting students' conceptual understanding, and extending students' mathematical thinking as means of Advancing Children's Thinking (ACT). This framework builds on the work of Cobb and colleagues by incorporating the social norms into a model that teachers can use to advance students' mathematical thinking (see Figure 1-3). The aim of the model is to identify and articulate "instructional strategies used by skilful teachers to support flexibility in students' reasoning about solution methods and to encourage participation in intellectually sophisticated mathematical discussions" (p. 3).

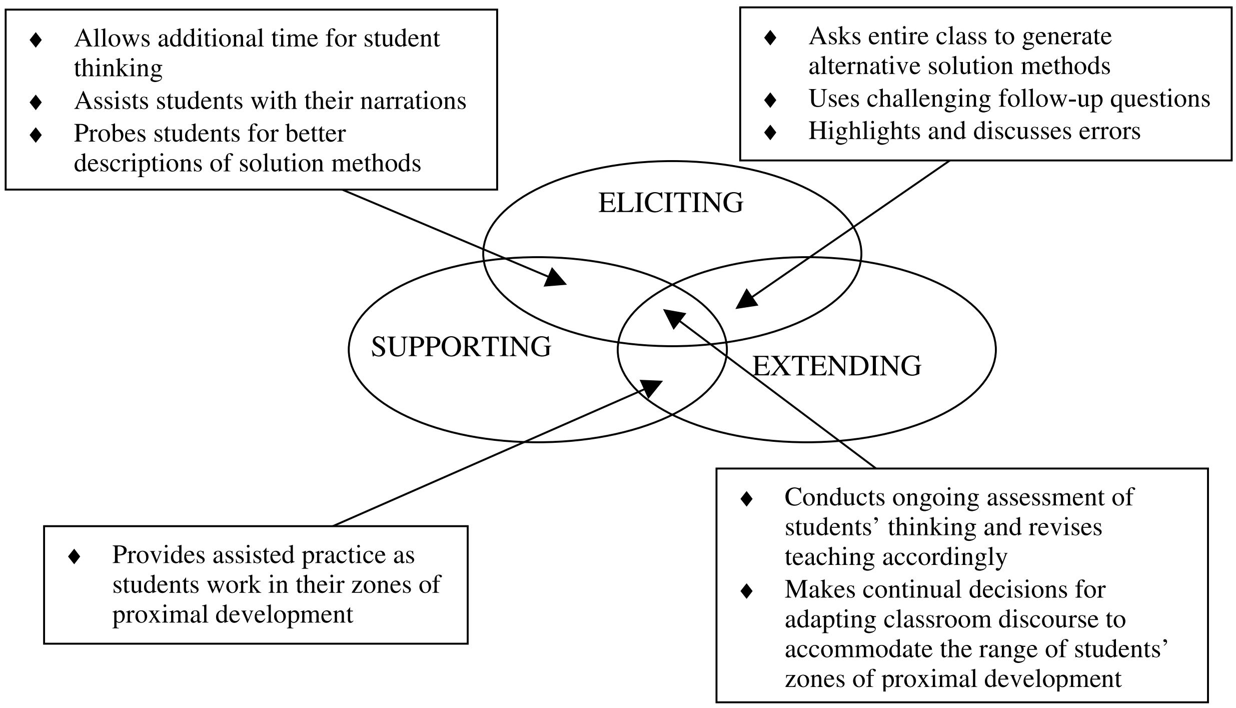


Figure 1-3: Diagram of the Advancing Children's Thinking (ACT) framework and the teaching strategies that reside in the intersections between and among the instructional components of the ACT framework.

Issues and Challenges for Low-decile Classrooms

Results to date (Higgins, 2002, Thomas and Ward, 2001, 2002) suggest that there are special issues and challenges for student achievement in low-decile schools. In a broad sense, a similar programme of work appeared to be implemented in all schools, with teachers reporting that their teaching had become more focused through a greater emphasis on mental strategies to solve number problems. They also reported that changes to questioning and instructional grouping provided greater opportunities for students to explain their problem-solving strategies. However, in 2001, the results for the ANP showed that 62% of students in decile 1 schools did not make the shift to part-whole thinking. The next highest proportions of student not making that shift were those students in decile 7 (42%) and those in decile 3 (41%) schools.

Further investigation into the pedagogical considerations around teaching students in lower decile schools needs to take account of a number of additional factors. These include the transient nature of high proportions of students and the high proportions of students from non-English-speaking backgrounds.

Report Structure

Chapter Two discusses the methodology used to investigate the ANP in 2002.

Chapter Three presents the analysis of student results from the initial and final diagnostic interviews.

Chapter Four discusses the shift from counting-based to part-whole strategies.

Chapter Five provides evidence from the questionnaires of the perspectives of adult participants on the impact of the ANP on all participants. It discusses the extent to which teachers' professional knowledge and attitudes have been enhanced by participation in the project.

Chapter Six uses interview data to present views of active learning in low-decile schools. The case study suggests that the teachers need to make special pedagogical efforts to ensure that students benefit from discussion of mathematical ideas.

Chapter Seven focuses on low-decile classrooms and schools, and, in particular, examines teacher-led group discussion.

Chapter Eight summarises the results of the research.

Chapter Two: Methodology

Aim of the Investigation

The investigation examined the impact of the ANP in 2002 on participating teachers and students. An increase in the professional knowledge of teachers and increased levels of numeracy for students were intended outcomes of the project. The investigation focused especially on the impact of the ANP on decile 1 schools.

The first part of the evaluation examines the impact of the ANP on students' understanding of number concepts as detailed on the Number Framework. The following research questions were addressed:

- What progress do students make on the Number Framework?
- How is progress linked to age, ethnicity, gender, geographical region, or school decile?
- To what extent and in what way has student performance in number improved over the duration of the ANP in 2002? In particular, what are the characteristics of the groups of students who are successfully making the shift to part-whole thinking?

The second broad area examined is the impact of the ANP on teachers' subject and pedagogical content knowledge and classroom practice. Specific questions that will be addressed are:

- In what ways does the ANP impact on teachers' subject and pedagogical content knowledge?
- How do teachers see changes in their subject and pedagogical content knowledge impacting on their classroom practice?
- What was it that the facilitators did that had the most impact on improving teachers' classroom practice?
- What characterises "effective" classroom practice in decile 1 schools?

The third broad area investigated is the characteristics of effective facilitation. These were studied by looking at the ways in which the ANP impacts on facilitators' knowledge and the effectiveness of the facilitator training. The project is designed to answer the following research questions:

- In what ways does the ANP impact on facilitators' subject and pedagogical content knowledge?

Design and Methodology

The approach comprised three components:

- the quantification of student progress,
- an evaluation of the impact of the professional development programme through questionnaires to facilitators, principals, and teachers,
- case studies of classroom practice in low-decile classrooms.

Before data was gathered, ethical approval was sought from the Wellington College of Education Ethics Committee, which operates under the NZARE Code of Ethics. The data gathering was guided by the principles of this code.

Summary of the data-gathering process

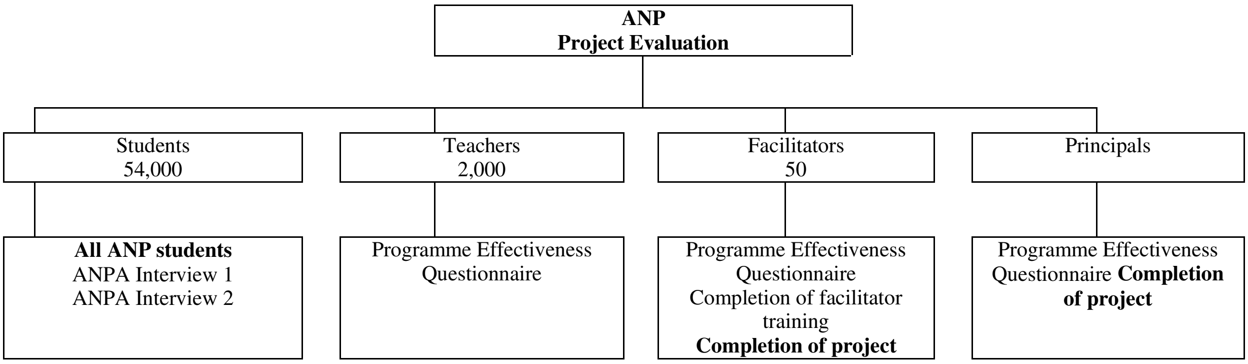


Figure 2-1: Evaluation of the ANP – impact on participants

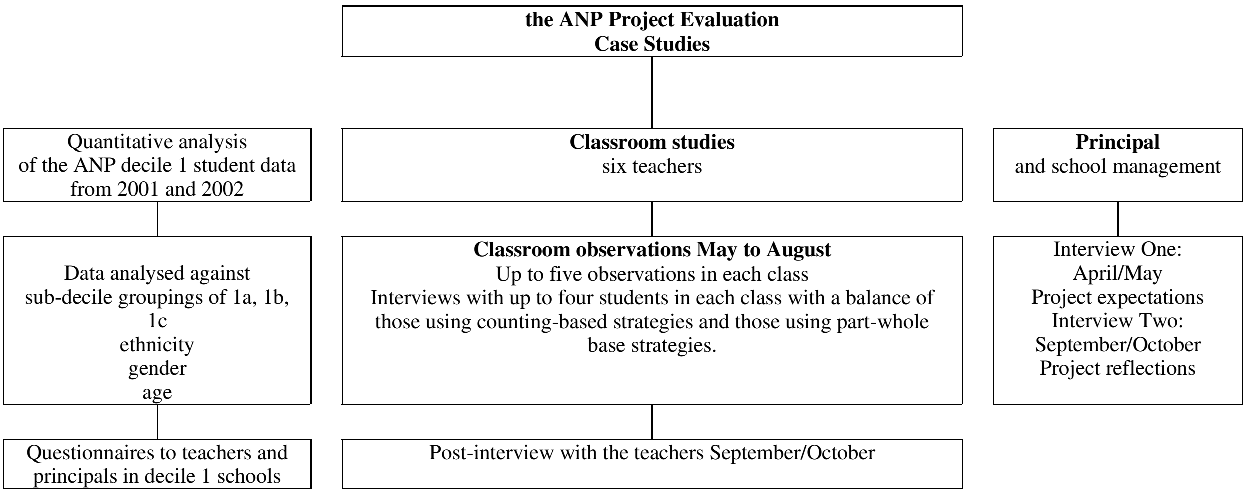


Figure 2-2: Implementation of the ANP in urban decile 1 schools – case studies

Although there were 54,000 students in the project, data for the purposes of this evaluation was only available on approximately 33,000 students.

The Participants

In 2002, the Ministry of Education offered the ANP to schools throughout New Zealand, with a particular emphasis on those classified as being at lower decile levels. The ANP involved approximately 54 facilitators, 2,000 teachers, and approximately 54,000 students in 2002.

All the facilitators were offered three days' training in December 2001 and another three days' training in February 2002. There were also regional seminar days throughout the year and visits and communication from the national co-ordinator of the project. At the end of the year, the facilitators in each region were invited to complete a questionnaire of open-ended questions on aspects of the ANP (see Appendix C). The questionnaires were completed anonymously and a stamped addressed envelope was included for the return of the questionnaire to ensure confidentiality for those participating in the study. The findings are presented in Chapter Five, which discusses the impact of the project on adult participants.

The approximately 2,000 teachers who participated in the ANP worked with 54 facilitators in eight regions throughout New Zealand. A similar approach was used to that of previous years, with participating teachers being organised into clusters of a few schools for after-school or whole-day workshops. Continuing the approach adopted in 2001, the teacher development had a prime focus on the ANP Assessment (ANPA) diagnostic tool. At the conclusion of the project, the 2,000 teachers were invited to complete a questionnaire on the project's effectiveness. The analysis of these questionnaires focused particularly on how the programme might be improved in subsequent years. School principals were also invited to complete questionnaires on the effectiveness of the project from their perspective. Additional questions were added for those teachers and principals in low-decile schools.

A similar process was followed to that used in 2001, with teachers assessing students twice during the project using the Numeracy Project Assessment (NumPA) tool: first at the start of the project, just after the teachers had received their initial training, and then again, after 15 weeks teaching, at the completion of the project. On the basis of the initial results, facilitators worked with teachers to plan their instructional programmes. These results were also used to track the achievement of students using a web-site set up especially for the project. Teachers completed a profile for each student, including their date of birth, their year level, and their ethnicity (defined as the ethnic group with which the student mostly identifies).

Classroom Interactions: Case Studies

The case studies examined the teacher-led group discussion in six classrooms in two lower decile South Auckland schools. The teacher/student interactions were recorded and the participating teachers, students, and senior management interviewed. Full details of the method and analysis are included in the introduction of Chapter Six.

Overview of the Student Participants

Ethnicity			Decile										Total
			1	2	3	4	5	6	7	8	9	10	
	New Zealand European	N	529	947	1,879	2,899	2,602	1,186	1,764	2,356	2,217	2,004	18,383
		%	2.8%	4.9%	9.8%	15.1%	13.6%	6.2%	9.2%	12.3%	11.6%	10.5%	100%
	Māori	N	2,165	1,326	1,109	1,065	826	229	296	300	155	98	7,843
		%	27.6%	16.9%	14.1%	13.6%	10.5%	2.9%	3.8%	3.8%	2.0%	1.2%	100%
	Pasifika	N	1,376	340	490	226	167	87	51	52	40	25	3,078
		%	44.7%	11.0%	15.9%	7.3%	5.4%	2.8%	1.7%	1.7%	1.3%	.8%	100%
	Asian	N	113	51	237	109	99	60	83	170	101	451	1,662
		%	7.8%	3.1%	14.3%	6.6%	6.0%	3.6%	5.0%	10.2%	6.1%	27.1%	100%
	Other	N	71	138	176	143	158	103	77	88	114	330	1,459
		%	4.9%	9.5%	12.1%	9.8%	10.8%	7.1%	5.3%	6.0%	7.8%	22.6%	100%
Total		N	4,254	2,802	3,891	4,442	3,852	1,665	2,271	2,966	2,627	2,908	33,209
		%	12.8%	8.4%	11.7%	13.4%	11.6%	5.0%	6.8%	8.9%	7.9%	8.8%	100%

Table 2-1: Numbers of students by ethnicity and decile

NB: Data is missing, so the totals are incomplete.

Gender			Region								Total
			Auckland	Christchurch	Massey	Northland	Otago	Southland	Waikato	Wellington	
			1	2	3	4	5	6	7	8	
	F	N	5,269	1,542	2,114	1,252	807	789	2,767	1,332	16,045
		%	32.8%	9.6%	13.2%	7.8%	5%	4.9%	17.1%	8.3%	100%
	M	N	5,666	1,656	2,163	1,360	864	835	3,024	1,397	17,164
		%	33%	9.6%	12.6%	7.9%	5%	4.9%	17.4%	8.1%	100%
Total		N	10,935	3,198	4,277	2,612	1,671	1,624	5,791	2,729	33,209
		%	32.9%	9.6%	12.9%	7.9%	5%	4.9%	17.5%	8.2%	100%

Table 2-2: Numbers of students by gender and region

NB: Data is missing, so the totals are incomplete.

Chapter Three: Patterns of Achievement within Year Groups

Overview of the Findings

Most students improved during their participation in the ANP. This accords with the 2001 results of the ANP (Higgins, 2002) and the 2000 and 2001 results of the ENP (Thomas and Ward, 2001, 2002).

An important goal of the Numeracy Project is to help students to develop strategic thinking in mathematics. The shift from counting-based thinking to part-whole thinking enables students to use more sophisticated strategies when problem-solving. Comparing the percentages of students at particular stages on the Number Framework before and after the instructional period is a way of tracking progress. This chapter focuses on pre- and post-instruction comparisons within the three year levels that participated in the ANP. The results have been grouped by year level for teachers' convenience (given teachers' usual focus on a particular year level). These comparisons are presented in three operating domains:

- addition and subtraction,
- multiplication and division,
- proportions and ratios.

The stages in the operating domains map increasing sophistication in the problem-solving strategies used.

Patterns of development in knowledge are also presented for the six aspects of knowledge:

- forward number word sequence (FNWS),
- backward number word sequence (BNWS),
- number identification,
- grouping,
- fractions,
- decimals and percentages.

These knowledge aspects underpin the improvement of strategic thinking, and, in turn, more sophisticated thinking prompts further knowledge development. This relationship between knowledge and strategy requires students to make progress in both domains.

Achievement of year 4 students

The achievement of year 4 students over the instructional period of the project is evident. The results for addition and subtraction were slightly better than for the other operational domains. Tables 3-4, 3-5, and 3-6 show that the majority of students in year 4 were at the advanced counting stage in addition and subtraction, multiplication and division, and proportions and ratios at the start of the project. At this stage, students' strategies problem-solving are based on counting, albeit in different forms.

For addition and subtraction (see Table 3-1), the percentage of students using any part-whole strategies before instruction was 37%. By the end of the project, the percentage had increased to 63%. In 2001, the percentages of students at the more sophisticated part-whole stage, advanced additive, before and after instruction were roughly double those in 2002, being 8% (before) and 25% (after). Those not assessed have been considered to have no more than counting-based strategies for solving multiplication and division problems and probably had insufficient knowledge to develop any strategy at all.

For multiplication and division (see Table 3-2), most students were using counting-based strategies before instruction, with only 22% using part-whole strategies. The percentage using part-whole strategies after instruction had increased to 50%.

Proportions and ratios (see Table 3-3) is a more difficult domain for students, and this is reflected in the stages at which students started. The majority of students were either using counting-based strategies (65%) or were not assessed (21%). Only a small proportion (14%) had part-whole strategies. This percentage was more modest than for the other operational domains.

Stage	Initial	Final
Emergent	1% (129)	1% (71)
One to one counting	2% (160)	1% (57)
Counting from one on materials	8% (811)	3% (309)
Counting from one by imaging	4% (465)	4% (383)
Advanced counting	49% (5,195)	30% (3,162)
Early additive part-whole	32% (3,392)	50% (5,301)
Advanced additive part-whole	5% (506)	13% (1,376)
Total	100%[1] (10,659)[2]	100% (10,659)

Table 3-1: Year 4 addition and subtraction – students' initial and final assessment

Stage	Initial	Final
Not assessed	20% (2,165)	10% (1,035)
Counting from one by imaging	16% (1,711)	7% (689)
Advanced counting	41% (4,390)	35% (3,718)
Early additive part-whole	16% (1,693)	29% (3,108)
Advanced additive part-whole	6% (612)	16% (1,741)
Advanced multiplicative part-whole	1% (87)	4% (368)
Total	100% (10,659)	100% (10,659)

Table 3-2: Year 4 multiplication and division – students' initial and final assessment

[1] Due to rounding, not all percentages add to 100.
[2] Any variations in total are the result of minor errors in data entry.

Stage	Initial	Final
Not assessed	21% (2,208)	10% (1,070)
Counting from one by imaging	25% (2,706)	8% (849)
Advanced counting	40% (4,232)	42% (4,488)
Early additive part-whole	11% (1,156)	26% (2,799)
Advanced additive part-whole	3% (308)	10% (1,093)
Advanced multiplicative part-whole	0% (44)	3% (340)
Advanced proportional part-whole	0% (4)	0% (20)
Total	100% (10,659)	100% (10,659)

Table 3-3: Year 4 proportions and ratios – students' initial and final assessment

Tables 3-4, 3-5, and 3-6 and Figures 3-1, 3-2, and 3-3 illustrate patterns of improvement within the year groups track achievement against the students' starting point and provide another way of examining the shifts in thinking made by the students while participating in the project. This is in contrast to the previous set of tables, which looked at the percentages of students before and after instruction irrespective of their starting point.

In addition and subtraction (see Table 3-4), 19% of the year 4 students who were at the emergent stage at the start of the project were still emergent at the end; 23% had moved to advanced counting and 17% moved to early additive (a move of at least four stages); and a smaller number (3%) moved to advanced additive (five stages). As students move up the stages, the number of students remaining at the same stage at the end of the project increased. Forty-one percent shifted from early additive part-whole to advanced additive part-whole. While each of these shifts is only one stage, they represent increasing levels of sophistication in student thinking.

In multiplication and division (see Table 3-5), 22% of year 4 students who were counting from one by imaging at the start remained at this stage after instruction. The larger shifts were made by the 58% who moved from counting from one by imaging to advanced counting by the end of the project (a move of one stage) and the 40% who moved from advanced counting to advanced additive part-whole.

The percentages of students shifting were similar for proportions and ratios (see Table 3-6). However, the percentages of students making the shifts each time were slightly lower than for the other operational domains. In proportions and ratios, 18% of students remained at the counting from one by imaging stage at the end of the project. The larger shifts were from counting from one to advanced counting (58%), from advanced counting to early additive part-whole (35%), and from early additive part-whole to advanced additive part-whole (30%). This pattern of improvement is consistent with that found for the addition and subtraction and the multiplication and division operational domains.

Final	Initial						
	Emergent	One to one counting	Counting from one on materials	Counting from one by imaging	Advanced counting	Early additive part-whole	Advanced additive part-whole
Emergent	*19% (24)*	1% (1)	0% (3)	0% (1)	1% (27)	0% (12)	1% (3)
One to one counting	9% (12)	*23% (36)*	0% (3)	1% (4)	0% (1)	0% (0)	0% (1)
Counting from one on materials	23% (30)	29% (47)	*27% (217)*	2% (10)	0% (5)	0% (0)	0% (0)
Counting from one by imaging	6% (8)	16% (25)	25% (204)	*22% (100)*	1% (45)	0% (1)	0% (0)
Advanced counting	23% (29)	29% (46)	40% (324)	54% (253)	*47% (2,434)*	2% (73)	0% (2)
Early additive part-whole	17% (22)	3% (5)	7% (58)	21% (96)	50% (2,571)	*75% (2,526)*	5% (23)
Advanced additive part-whole	3% (4)	0% (0)	0% (2)	0% (1)	2% (112)	23% (780)	*94% (477)*
Total	100% (129)	100% (160)	100% (811)	100% (465)	100% (5,195)	100% (3,392)	100% (506)

Table 3-4: Year 4 patterns of improvement through the stages in addition and subtraction

Final	Initial					
	Not assessed	Counting from one by imaging	Advanced counting	Early additive part-whole	Advanced additive part-whole	Advanced multiplicative part-whole
Not assessed	45% (973)	1% (22)	1% (26)	1% (10)	1% (3)	1% (1)
Counting from one by imaging	12% (256)	22% (368)	1% (63)	0% (2)	0% (0)	0% (0)
Advanced counting	31% (680)	58% (997)	45% (1,986)	3% (51)	1% (3)	0% (0)
Early additive part-whole	9% (190)	16% (268)	40% (1,764)	50% (851)	5% (33)	2% (2)
Advanced additive part-whole	3% (59)	3% (55)	12% (512)	41% (687)	69% (422)	7% (6)
Advanced multiplicative part-whole	0% (7)	0% (1)	1% (39)	5% (92)	25% (151)	90% (78)
Total	100% (2,165)	100% (1,711)	100% (4,390)	100% (1,693)	100% (612)	100% (87)

Table 3-5: Year 4 patterns of improvement through the stages in multiplication and division

Final	Initial						
	Not assessed	Counting from one by imaging	Advanced counting	Early additive part-whole	Advanced additive part-whole	Advanced multiplicative part-whole	Advanced proportional part-whole
Not assessed	46% (1,010)	1% (37)	0% (14)	1% (6)	1% (2)	2% (1)	0% (0)
Counting from one by imaging	13% (278)	18% (494)	2% (75)	0% (2)	0% (0)	0% (0)	0% (0)
Advanced counting	32% (705)	58% (1,558)	52% (2,184)	3% (37)	1% (3)	0% (0)	0% (0)
Early additive part-whole	7% (164)	18% (478)	35% (1,494)	56% (646)	6% (17)	0% (0)	0% (0)
Advanced additive part-whole	2% (39)	5% (126)	9% (394)	30% (343)	61% (189)	5% (2)	0% (0)
Advanced multiplicative part-whole	1% (12)	1% (13)	2% (71)	10% (116)	31% (94)	77% (34)	0% (0)
Advanced proportional part-whole	0% (0)	0% (0)	0% (0)	1% (6)	1% (3)	16% (7)	100% (4)
Total	100% (2,208)	100% (2,706)	100% (4,232)	100% (1,156)	100% (308)	100% (44)	100% (4)

Table 3-6: Year 4 patterns of improvement through the stages in proportions and ratios

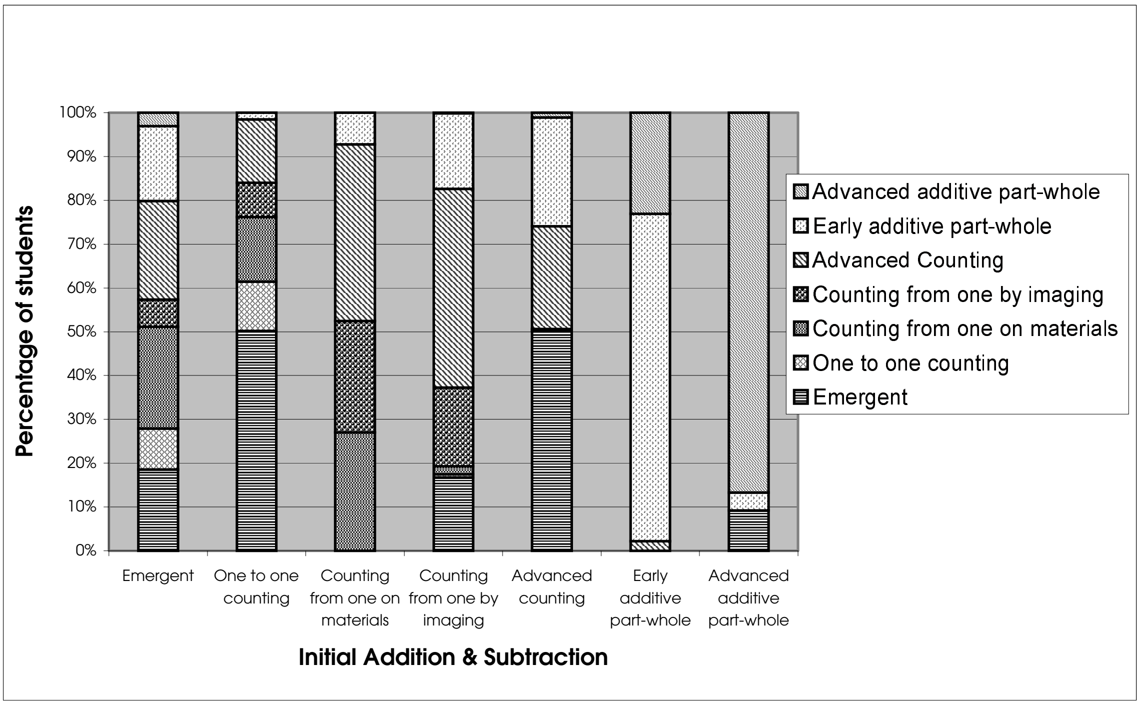


Figure 3-1: Year 4 patterns of improvement through the stages in addition and subtraction

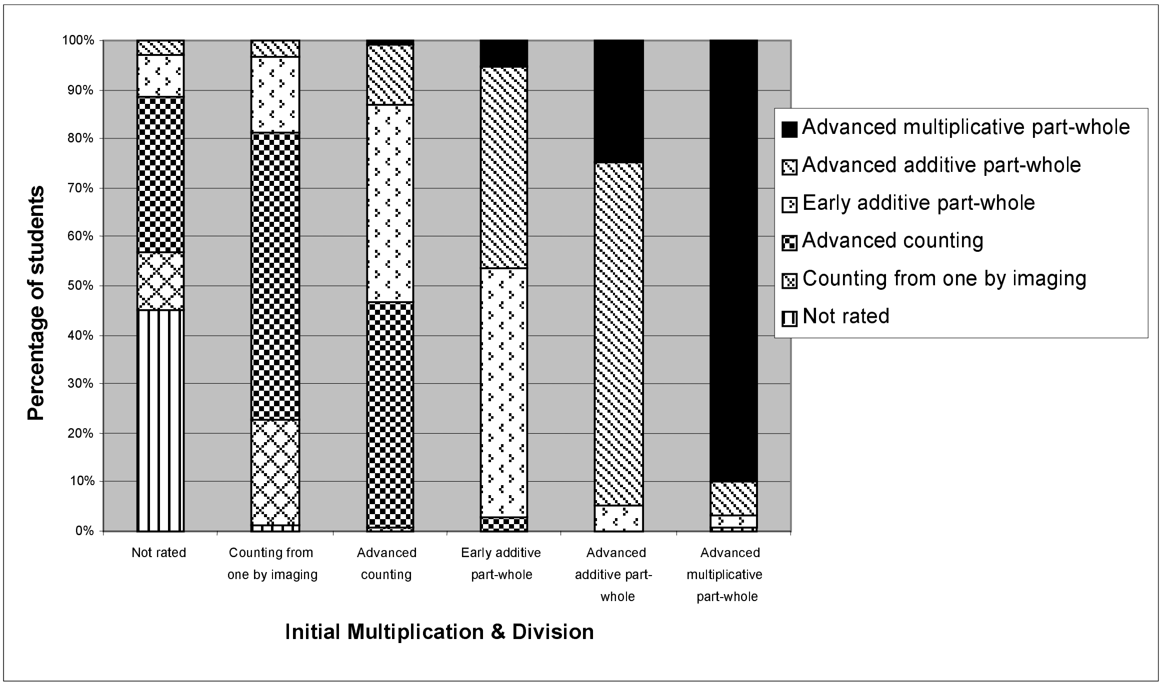


Figure 3-2: Year 4 patterns of improvement through the stages in multiplication and division

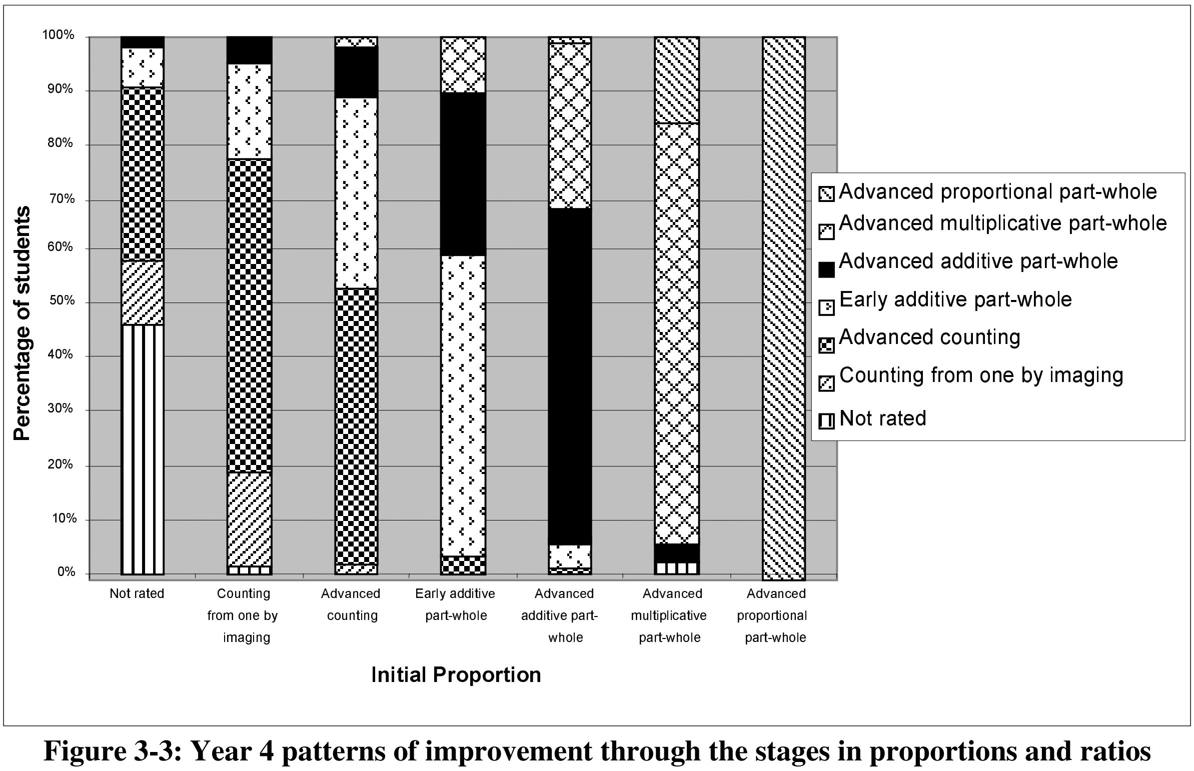


Figure 3-3: Year 4 patterns of improvement through the stages in proportions and ratios

Tables 3-7, 3-8, and 3-9 illustrate the pattern of achievement for the knowledge aspects was similar in terms of stages gained over the period of instruction – with the exception of number identification and decimals, where the majority of students were not assessed.

In FNWS (see Table 3-7) the largest shift related to FNWS up to 1,000 and above (initially 53% were at this stage, changing to 77% at the final assessment). For BNWS (see Table 3-8) the pattern was similar with the largest shift being from 50% at BNWS up to 1,000 and above prior to instruction to 73% after instruction.

In grouping, the main movement of students was at the stages of grouping with tens and tens in 100. Prior to instruction, 59% of students were at these stages. By the end of instruction, this had increased to 80%.

For fractions, the movement of students (77%) was greatest for ordered unit fractions, with 7% at this stage prior to instruction and 33% after instruction.

Stage	Initial	Final
Emergent FNWS	2% (180)	2% (160)
Initial FNWS up to 10	1% (57)	0% (18)
FNWS up to 10	2% (157)	0% (37)
FNWS up to 20	5% (531)	2% (179)
FNWS up to 100	38% (4,069)	20% (2,096)
FNWS up to 1,000	47% (4,970)	55% (5,840)
FNWS up to 1,000,000	7% (694)	22% (2,329)
Total	100% (10,659)	100% (10,659)

Table 3-7: Year 4 FNWS – initial and final assessment

Stage	Initial	Final
Emergent BNWS	2% (229)	2% (191)
Initial BNWS up to 10	1% (115)	0% (29)
BNWS up to 10	4% (428)	1% (101)
BNWS up to 20	6% (673)	3% (275)
BNWS up to 100	36% (3,874)	22% (2,288)
BNWS up to 1,000	44% (4,725)	53% (5,659)
BNWS up to 1,000,000	6% (614)	20% (2,116)
Total	100% (10,659)	100% (10,659)

Table 3-8: Year 4 BNWS – initial and final assessment

Stage	Initial	Final
Not assessed	80% (8,482)	87% (9,278)
Emergent number identification	1% (91)	0% (40)
Numerals to 10	1% (80)	0% (19)
Numerals to 20	1% (123)	1% (49)
Numerals to 100	6% (668)	3% (265)
Numerals to 1,000	11% (1,214)	9% (1,007)
Total	100% (10,659)	100% (10,659)

Table 3-9: Year 4 number identification – initial and final assessment

Stage	Initial		Final	
Not assessed	2%	(209)	2%	(182)
Non-grouping with 5s and within 10	15%	(1,571)	4%	(390)
With 5s and within 10	27%	(2,893)	12%	(1,315)
With 10s	43%	(4,529)	42%	(4,502)
10s in 100	12%	(1,266)	32%	(3,399)
10s and 100s in whole numbers	1%	(147)	6%	(644)
10s, 100s, and 1,000s in whole numbers	0%	(37)	2%	(191)
10ths, 100ths, and 1,000ths in decimals	0%	(6)	0%	(35)
Total	100%	(10,659)	100%	(10,659)

Table 3-10: Year 4 grouping – initial and final assessment

Stage	Initial		Final	
Not assessed	21%	(2,214)	11%	(1,168)
Non-fractions of regions	50%	(5,301)	15%	(1,644)
Assigned unit fractions	21%	(2,211)	29%	(3,095)
Ordered unit fractions	7%	(710)	33%	(3,513)
Co-ordinated numerators and denominators	2%	(198)	10%	(1,093)
Equivalent fractions	0%	(13)	1%	(93)
Ordered fractions	0%	(11)	1%	(53)
Total	100%	(10,659)	100%	(10,659)

Table 3-11: Year 4 fractions – initial and final assessment

Stage	Initial		Final	
Not assessed	95%	(10,146)	90%	(9,596)
	0%	(32)	1%	(62)
Emergent decimal identification	4%	(368)	5%	(481)
Decimal identification	1%	(82)	3%	(351)
Ordered decimals	0%	(21)	1%	(117)
Rounded decimals	0%	(9)	0%	(44)
Decimal conversions	0%	(0)	0%	(6)
Total	100%	(10,659)	100%	(10,659)

Table 3-12: Year 4 decimals – initial and final assessment

Achievement of year 5 students

The achievement of year 5 students can also be tracked over the course of the project. As with year 4 students' achievements, the results for addition and subtraction were slightly better than for the other operational domains, with about the same number of students using counting-based strategies (51%) as used part-whole strategies (49%). By the end of the project the percentage of those using part-whole strategies had increased to 74% in addition and subtraction.

Tables 3-14 and 3-15 show that in multiplication and division and in proportions and ratios the majority of students in year 5, as in year 4, were at the advanced counting stage at the start of the project. Those not assessed were about half as numerous as in year 4. For multiplication and division, most students were using counting-based strategies prior to instruction, with only 40% using part-whole strategies. The percentage using part-whole strategies increased to 66% after instruction.

In proportions and ratios, the majority of students were either using counting-based strategies (60%) or were not assessed (11%). This was slightly better than for the year 4 students. A smaller proportion (29%) used part-whole strategies. The number using part-whole strategies by the end of the project had increased to 57%, slightly lower than for the other operational domains.

Stage	Initial	Final
Emergent	1% (139)	1% (69)
One to one counting	1% (79)	0% (35)
Counting from one on materials	4% (391)	1% (152)
Counting from one by imaging	3% (290)	2% (194)
Advanced counting	44% (4,858)	22% (2,460)
Early additive part-whole	38% (4,198)	50% (5,572)
Advanced additive part-whole	10% (1,140)	24% (2,613)
Total	100% (11,095)	100% (11,095)

Table 3-13: Year 5 addition and subtraction – students' initial and final assessment

Stage	Initial	Final
Not assessed	11% (1,166)	5% (522)
Counting from one by imaging	11% (1,233)	4% (443)
Advanced counting	39% (4,336)	26% (2,825)
Early additive part-whole	25% (2,732)	31% (3,380)
Advanced additive part-whole	12% (1,326)	26% (2,905)
Advanced multiplicative part-whole	3% (302)	9% (1,020)
Total	100% (11,095)	100% (11,095)

Table 3-14: Year 5 multiplication and division – students' initial and final assessment

Stage	Initial	Final
Not assessed	11% (1,245)	5% (560)
Counting from one by imaging	20% (2,234)	6% (610)
Advanced counting	40% (4,403)	33% (3,617)
Early additive part-whole	19% (2,097)	29% (3,169)
Advanced additive part-whole	7% (800)	19% (2,092)
Advanced multiplicative part-whole	3% (297)	9% (941)
Advanced proportional part-whole	0% (19)	1% (106)
Total	100% (11,095)	100% (11,095)

Table 3-15: Year 5 proportions and ratios – students' initial and final assessment

Patterns of improvement in the three operational domains at year 5 follow a similar pattern to that for year 4 students. In addition and subtraction (see Table 3-16), 12% of the year 5 students who were emergent at the start of the project were still emergent at the end; 22% had moved to advanced counting and 26% to early additive (a move of at least four stages); and a smaller number (19%) had moved to advanced additive (five stages). As with year 4 students, the number of students remaining at the same stage at the end of the project increased with each advancing stage.

An important shift is from advanced counting to early additive part-whole strategy use. Fifty-three percent of year 5 students made this shift. Fewer students (31%) moved between early and advanced additive part-whole.

In multiplication and division (see Table 3-17), 11% of year 5 students remained at the stage of counting from one by imaging after instruction; about half as many as in year 4. The larger shifts were made by the 52% who moved from counting from one by imaging to advanced counting by the end of the project (a move of one stage) and the 43% who moved from advanced counting to early additive part-whole strategy use. These percentages are similar to those for the year 4 students. However, the percentages of year 5 students shifting from early additive part-whole (47%) to advanced additive part-whole and from advanced additive part-whole to advanced multiplicative part-whole were higher (33%) than for year 4.

The percentages of year 5 students shifting to more advanced stages of thinking about proportions and ratios (see Table 3-18) are similar to the percentages of year 4 students – in each case, the percentage is slightly lower than for the other operational domains. In proportions and ratios, 17% of students remained at the counting from one by imaging stage at the end of the project. Again the larger shifts were from counting from one to advanced counting (53%), from advanced counting to early additive part-whole strategy use (37%), and from early additive part-whole to advanced additive part-whole strategy use (37%).

The difference between the year 5 and the year 4 students was shown in the percentages who moved from advanced additive part-whole to advanced multiplicative part-whole strategy use (32%) and from advanced multiplicative part-whole to advanced proportional part-whole strategy use (18%). A small proportion of students (16%) also moved back to the previous level at this most advanced stage. Again, the pattern of improvement is consistent with that in the addition and subtraction and the multiplication and division operational domains, but with fewer students shifting stages.

Final	Initial													
	Emergent		One to one counting		Counting from one on materials		Counting from one by imaging		Advanced counting		Early additive part-whole		Advanced additive part-whole	
Emergent	12%	(16)	0%	(0)	1%	(5)	1%	(3)	1%	(27)	0%	(14)	0%	(4)
One to one counting	9%	(12)	22%	(17)	1%	(3)	0%	(1)	0%	(2)	0%	(0)	0%	(0)
Counting from one on materials	7%	(10)	30%	(24)	28%	(109)	2%	(5)	0%	(3)	0%	(1)	0%	(0)
Counting from one by imaging	4%	(6)	18%	(14)	20%	(77)	22%	(64)	1%	(27)	0%	(5)	0%	(1)
Advanced counting	22%	(31)	29%	(23)	39%	(153)	48%	(138)	41%	(2,013)	2%	(98)	0%	(4)
Early additive part-whole	27%	(37)	1%	(1)	11%	(41)	25%	(71)	53%	(2,588)	67%	(2,799)	3%	(35)
Advanced additive part-whole	19%	(27)	0%	(0)	1%	(3)	3%	(8)	4%	(198)	31%	(1,281)	96%	(1,096)
Total	100%	(139)	100%	(79)	100%	(391)	100%	(290)	100%	(4,858)	100%	(4,198)	100%	(1,140)

Table 3-16: Year 5 patterns of improvement through the stages in addition and subtraction

Final	Initial											
	Not assessed		Counting from one by imaging		Advanced counting		Early additive part-whole		Advanced additive part-whole		Advanced multiplicative part-whole	
Not assessed	40%	(460)	2%	(18)	1%	(29)	0%	(7)	1%	(6)	1%	(2)
Counting from one by imaging	12%	(134)	23%	(279)	1%	(29)	0%	(1)	0%	(0)	0%	(0)
Advanced counting	32%	(376)	52%	(646)	40%	(1,732)	3%	(67)	0%	(4)	0%	(0)
Early additive part-whole	11%	(128)	17%	(208)	43%	(1,848)	42%	(1,144)	4%	(51)	0%	(1)
Advanced additive part-whole	5%	(52)	7%	(80)	15%	(652)	47%	(1,273)	63%	(833)	5%	(15)
Advanced multiplicative part-whole	1%	(16)	0%	(2)	1%	(46)	9%	(240)	33%	(432)	94%	(284)
Total	100%	(1,166)	100%	(1,233)	100%	(4,336)	100%	(2,732)	100%	(1,326)	100%	(302)

Table 3-17: Year 5 patterns of improvement through the stages in multiplication and division

Final	**Initial**													
	Not assessed		Counting from one by imaging		Advanced counting		Early additive part-whole		Advanced additive part-whole		Advanced multiplicative part-whole		Advanced proportional part-whole	
Not assessed	40%	(496)	1%	(20)	1%	(28)	1%	(12)	0%	(2)	0%	(1)	5%	(1)
Counting from one by imaging	13%	(162)	17%	(389)	1%	(54)	0%	(4)	0%	(1)	0%	(0)	0%	(0)
Advanced counting	33%	(416)	54%	(1,203)	44%	(1,935)	3%	(57)	1%	(6)	0%	(0)	0%	(0)
Early additive part-whole	9%	(116)	20%	(443)	37%	(1,615)	46%	(954)	5%	(38)	1%	(3)	0%	(0)
Advanced additive part-whole	3%	(41)	7%	(162)	14%	(634)	37%	(769)	60%	(477)	3%	(9)	0%	(0)
Advanced multiplicative part-whole	1%	(13)	1%	(16)	3%	(134)	14%	(288)	32%	(257)	77%	(230)	16%	(3)
Advanced proportional part-whole	0%	(1)	0%	(1)	0%	(3)	1%	(13)	2%	(19)	18%	(54)	79%	(15)
Total	100%	(1,245)	100%	(2,234)	100%	(4,403)	100%	(2,097)	100%	(800)	100%	(297)	100%	(19)

Table 3-18: Year 5 patterns of improvement through the stages in proportions and ratios

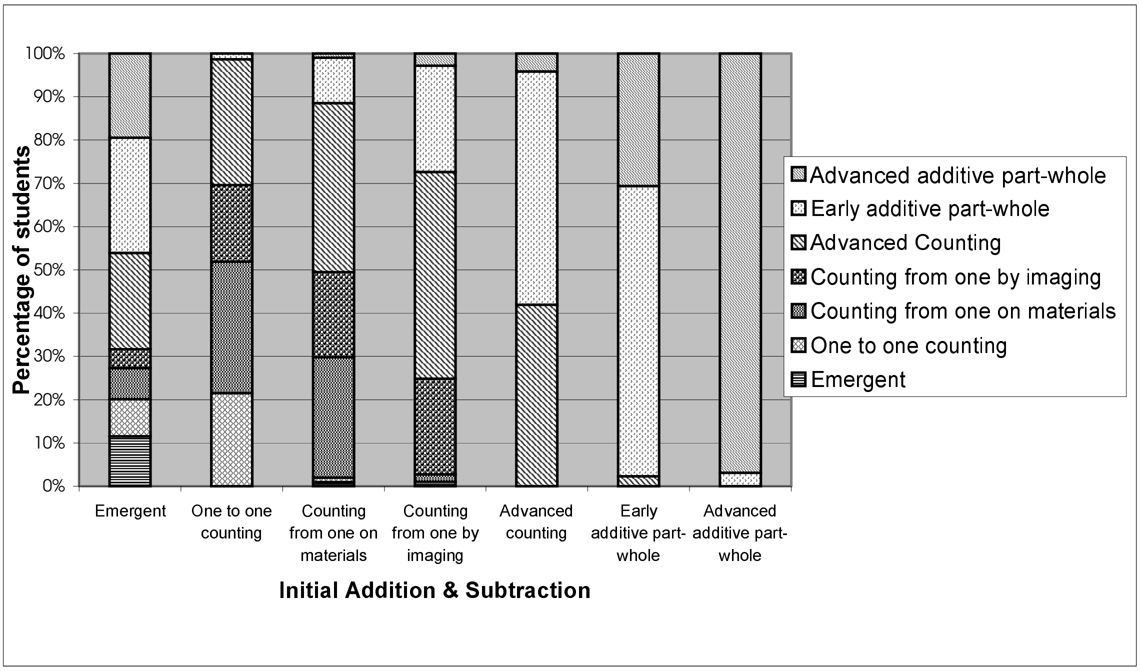


Figure 3-4: Year 5 patterns of improvement through the stages in addition and subtraction

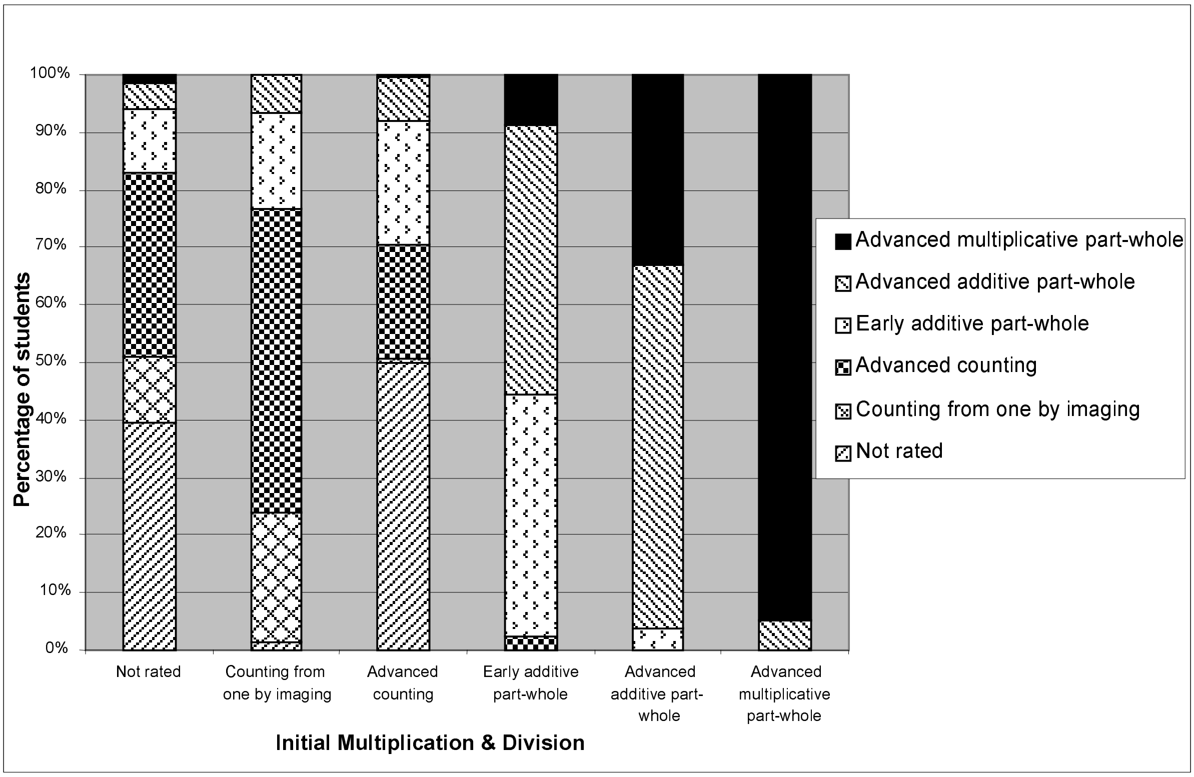


Figure 3-5: Year 5 patterns of improvement through the stages in multiplication and division

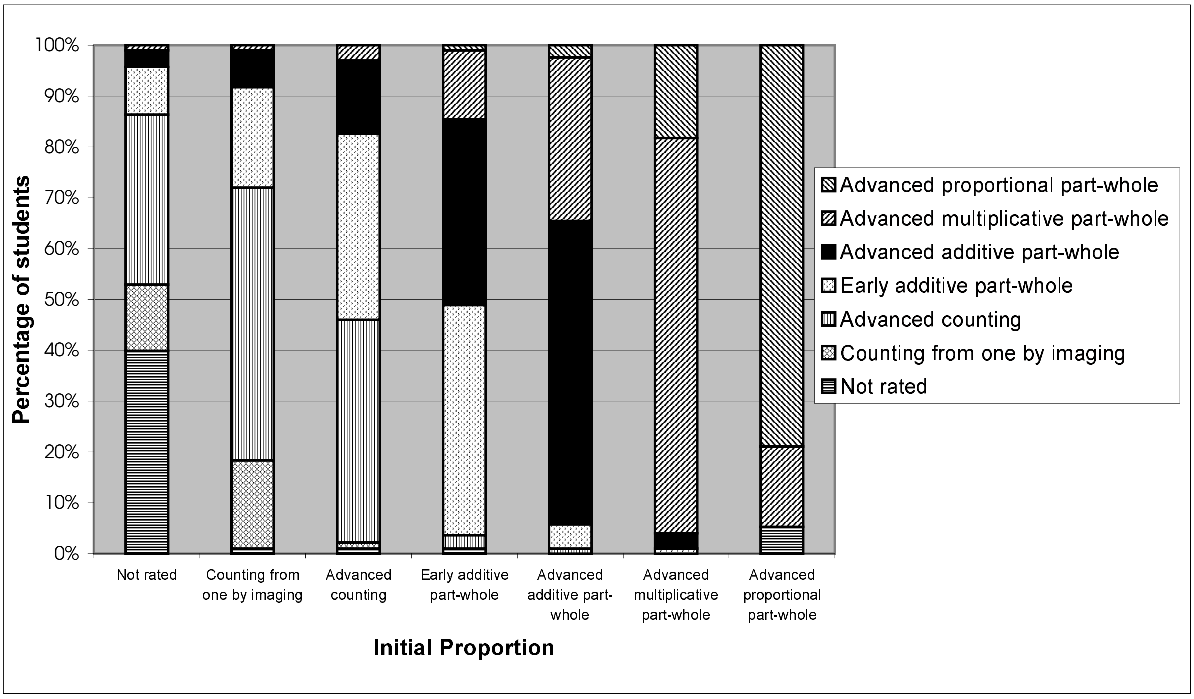


Figure 3-6: Year 5 patterns of improvement through the stages in proportions and ratios

In the knowledge aspects (see Tables 3-19 to 3-23), the pattern of achievement was similar to that for year 4 in terms of stages gained over the period of instruction – with the exception of number identification and decimals, where the majority of students were not assessed. In FNWS, the largest shift was for FNWS up to 1,000 and above. Fifty-five percent of students were at this stage initially, and 85% finally. These percentages are similar to those for students in year 4.

For BNWS, the pattern was similar, with the largest shift being around BNWS up to 1,000 and above, with 52% of students being at this stage prior to instruction and 82% after instruction.

In grouping, the movement of students was at the stages of grouping with tens and tens in 100, as with the year 4 students. Before instruction, 63% of students were at these stages but by the end of instruction this had increased to 69%. Twelve per cent of students reached tens and hundreds in whole numbers. For fractions, the movement of students was greatest for ordered unit fractions, with 15% at this stage prior to instruction and 36% after instruction.

Stage	Initial	Final
Emergent FNWS	2% (212)	2% (183)
Initial FNWS up to 10	0% (22)	0% (8)
FNWS up to 10	1% (71)	0% (16)
FNWS up to 20	2% (251)	1% (106)
FNWS up to 100	25% (2,767)	12% (1,288)
FNWS up to 1,000	56% (6,157)	50% (5,595)
FNWS up to 1,000,000	15% (1,615)	35% (3,898)
Total	100% (11,095)	100% (11,095)

Table 3-19: Year 5 FNWS – initial and final assessment

Stage	Initial	Final
Emergent BNWS	2% (271)	2% (229)
Initial BNWS up to 10	1% (52)	0% (20)
BNWS up to 10	1% (138)	0% (37)
BNWS up to 20	3% (331)	1% (137)
BNWS up to 100	27% (3,000)	14% (1,570)
BNWS up to 1,000	52% (5,792)	48% (5,337)
BNWS up to 1,000,000	14% (1,511)	34% (3,765)
Total	100% (11,095)	100% (11,095)

Table 3-20: Year 5 BNWS – initial and final assessment

Stage	Initial	Final
Not assessed	89% (9,827)	93% (10,323)
Emergent Number identification	1% (89)	1% (54)
Numerals to 10	0% (22)	0% (5)
Numerals to 20	0% (47)	0% (23)
Numerals to 100	2% (261)	1% (130)
Numerals to 1,000	8% (849)	5% (559)
Total	100% (11,095)	100% (11,095)

Table 3-21: Year 5 number identification – initial and final assessment

Stage	Initial	Final
Not assessed	2% (242)	2% (192)
Non-grouping with 5s and within 10	9% (956)	2% (252)
With 5s and within 10	21% (2,369)	8% (921)
With 10s	43% (4,743)	33% (3,676)
10s in 100	20% (2,221)	36% (3,972)
10s and 100s in whole numbers	4% (403)	12% (1,343)
10s, 100s, and 1,000s in whole numbers	1% (125)	5% (499)
10ths, 100ths, and 1,000ths in decimals	0% (36)	2% (240)
Total	100% (11,095)	100% (11,095)

Table 3-22: Year 5 grouping – initial and final assessment

Stage	Initial	Final
Not assessed	11% (1,266)	6% (701)
Non-fractions of regions	39% (4,346)	11% (1,260)
Assigned unit fractions	28% (3,151)	23% (2,572)
Ordered unit fractions	15% (1,684)	36% (4,018)
Co-ordinated numerators and denominators	5% (545)	18% (2,045)
Equivalent fractions	1% (72)	3% (330)
Ordered fractions	0% (31)	2% (169)
Total	100% (11,095)	100% (11,095)

Table 3-23: Year 5 fractions – initial and final assessment

Stage	Initial	Final
Not assessed	89% (9,887)	79% (8,746)
	0% (45)	1% (65)
Emergent decimal identification	6% (613)	6% (627)
Decimal identification	3% (372)	8% (937)
Ordered decimals	1% (118)	4% (457)
Rounded decimals	1% (50)	2% (202)
Decimal conversions	0% (10)	1% (60)
Total	100% (11,095)	100% (11,095)

Table 3-24: Year 5 decimals – initial and final assessment

Achievement of year 6 students

Most year 6 students were at the advanced counting stage or above in all operational domains before instruction. As with years 4 and 5, the results in addition and subtraction (see Table 3-25) were slightly better than the other operational domains. Just under half (46%) of the students were at the early additive stage after instruction and just over a third (36%) were at the advanced counting stage. The majority of students (60%) were using part-whole strategies before instruction. This rose to 83% after instruction, higher than for any other year group.

Tables 3-26 and 3-27 show that just over half the year 6 students were using part-whole thinking in multiplication and division prior to instruction and just under half were using part-whole thinking in proportions and ratios prior to instruction. Only around 6% of students were not assessed in either of these operational domains. The percentage using part-whole strategies increased after instruction to 77% for multiplication and division and 70% for proportions and ratios.

Stage	Initial		Final	
Emergent	1%	(99)	1%	(66)
One to one counting	0%	(35)	0%	(14)
Counting from one on materials	2%	(200)	1%	(66)
Counting from one by imaging	1%	(126)	1%	(111)
Advanced counting	36%	(4,128)	15%	(1,731)
Early additive part-whole	42%	(4,755)	46%	(5,292)
Advanced additive part-whole	18%	(2,112)	36%	(4,175)
Total	100%	(11,455)	100%	(11,455)

Table 3-25: Year 6 addition and subtraction – students' initial and final assessment

Stage	Initial		Final	
Not assessed	6%	(636)	3%	(291)
Counting from one by imaging	7%	(809)	2%	(257)
Advanced counting	33%	(3,783)	18%	(2,042)
Early additive part-whole	29%	(3,361)	28%	(3,182)
Advanced additive part-whole	19%	(2,145)	31%	(3,589)
Advanced multiplicative part-whole	6%	(721)	18%	(2,093)
Total	100%	(11,455)	100%	(11,455)

Table 3-26: Year 6 multiplication and division – students' initial and final assessment

Stage	Initial		Final	
Not assessed	6%	(698)	3%	(315)
Counting from one by imaging	15%	(1,755)	4%	(410)
Advanced counting	35%	(3,988)	24%	(2,695)
Early additive part-whole	24%	(2,714)	27%	(3,078)
Advanced additive part-whole	13%	(1,428)	23%	(2,609)
Advanced multiplicative part-whole	7%	(778)	17 %	(1,937)
Advanced proportional part-whole	1%	(94)	4%	(409)
Total	100%	(11,455)	100%	(11,455)

Table 3-27: Year 6 proportions and ratios – students' initial and final assessment

Patterns of improvement in the three operational domains at year 6 are similar to those for years 4 and 5. In addition and subtraction (see Table 3-28), 13% of the year 6 students who were at the emergent stage at the start of the project were still at the emergent stage at the end; 18% moved to the advanced counting stage and 27% moved to the early additive stage (a move of at least four stages); and about the same proportion (28%) moved to the advanced additive stage (five stages). As with the previous year levels, the number of students remaining at the same stage at the end of the project increased with each advancing stage. Slightly more students (56%) than at the year 5 level (53%) moved from the advanced counting to the early additive part-whole stage. Also following the previous pattern, fewer students (38%) moved between the early and advanced additive part-whole stages.

In multiplication and division (see table 3-29), the percentage of year 6 students who remained at the stage of counting from one by imaging after instruction increased from the previous year's 17% to 21%. The larger shifts in student achievement were made by the 48% who moved from counting from one by imaging to advanced counting by the end of the project (a move of one stage) and the 42% who moved from advanced counting to using early additive part-whole strategies. These percentages are similar to those for students in earlier years. Again, as with year 5 students, the percentages are higher for those shifting from the early additive part-whole stage (48%) to the advanced additive part-whole stage and from the advanced additive part-whole stage to the advanced multiplicative part-whole stage (41%).

The percentages of students shifting to more advanced stages of thinking about proportions and ratios (see Table 3-30) were similar to those for years 4 and 5, but were again slightly lower than for the other operational domains. In proportions and ratios, 16% of students remained at the counting from one by imaging stage at the end of the project. Again, the larger shifts were from counting from one to advanced counting (48%), from advanced counting to early additive part-whole (37%), and from early additive part-whole to advanced additive part-whole (37%). These percentages are very similar to those for years 4 and 5, but in some cases they are lower. The difference with the year 6 students was in the percentages who moved from the advanced additive part-whole stage to the advanced multiplicative part-whole stage (40%) and from the advanced multiplicative part-whole stage to the advanced proportional part-whole stage (24%). (These percentages were 32% and 18% for year 5 students.) Again, the pattern of improvement is consistent with that in the addition and subtraction and the multiplication and division operational domains, with fewer students shifting as the stages increase.

Final	Initial						
	Emergent	One to one counting	Counting from one on materials	Counting from one by imaging	Advanced counting	Early additive part-whole	Advanced additive part-whole
Emergent	13% (13)	0% (0)	1% (1)	0% (0)	1% (22)	0% (18)	1% (12)
One to one counting	6% (6)	17% (6)	1% (2)	0% (0)	0% (0)	0% (0)	0% (0)
Counting from one on materials	4% (4)	14% (5)	25% (49)	2% (3)	0% (4)	0% (1)	0% (0)
Counting from one by imaging	3% (3)	29% (10)	18% (36)	26% (33)	1% (26)	0% (3)	0% (0)
Advanced counting	18% (18)	31% (11)	46% (92)	43% (54)	35% (1,148)	2% (101)	0% (7)
Early additive part-whole	27% (27)	9% (3)	10% (20)	27% (34)	56% (2,330)	59% (2,817)	3% (61)
Advanced additive part-whole	28% (28)	0% (0)	0% (0)	2% (2)	7% (298)	38% (1,815)	96% (2,032)
Total	100% (99)	100% (35)	100% (200)	100% (126)	100% (4,128)	100% (4,755)	100% (2,112)

Table 3-28: Year 6 patterns of improvement through the stages in addition and subtraction

Final	Initial					
	Not assessed	Counting from one by imaging	Advanced counting	Early additive part-whole	Advanced additive part-whole	Advanced multiplicative part-whole
Not assessed	36% (231)	2% (13)	1% (19)	0% (11)	1% (11)	1% (6)
Counting from one by imaging	8% (53)	21% (168)	1% (33)	0% (3)	0% (0)	0% (0)
Advanced counting	29% (187)	48% (389)	37% (1,382)	2% (78)	0% (6)	0% (0)
Early additive part-whole	16% (99)	20% (159)	42% (1,591)	38% (1,265)	3% (66)	0% (2)
Advanced additive part-whole	8% (53)	9% (73)	17% (654)	48% (1,609)	55% (1,176)	3% (24)
Advanced multiplicative part-whole	2% (13)	1% (6)	3% (104)	12% (395)	41% (886)	96% (689)
Total	100% (636)	100% (809)	100% (3,783)	100% (3,361)	100% (2,145)	100% (721)

Table 3-29: Year 6 patterns of improvement through the stages in multiplication and division

Final	Initial						
	Not assessed	Counting from one by imaging	Advanced counting	Early additive part-whole	Advanced additive part-whole	Advanced multiplicative part-whole	Advanced proportional part-whole
Not assessed	37% (255)	1% (14)	1% (20)	0% (8)	1% (11)	1% (7)	0% (0)
Counting from one by imaging	11% (78)	16% (283)	1% (43)	0% (6)	0% (0)	0% (0)	0% (0)
Advanced counting	30% (207)	48% (841)	40% (1,584)	2% (56)	1% (7)	0% (0)	0% (0)
Early additive part-whole	14% (100)	23% (400)	37% (1,457)	40% (1,073)	3% (47)	0% (1)	0% (0)
Advanced additive part-whole	6% (42)	10% (167)	17% (669)	37% (1,012)	49% (698)	3% (20)	1% (1)
Advanced multiplicative part-whole	2% (15)	3% (47)	5% (208)	19% (515)	41% (581)	73% (565)	6% (6)
Advanced proportional part-whole	0% (1)	0% (3)	0% (6)	2% (43)	6% (84)	24% (185)	93% (87)
Total	100% (698)	100% (1,755)	100% (3,988)	100% (2,714)	100% (1,428)	100% (778)	100% (94)

Table 3-30: Year 6 patterns of improvement through the stages in proportions and ratios

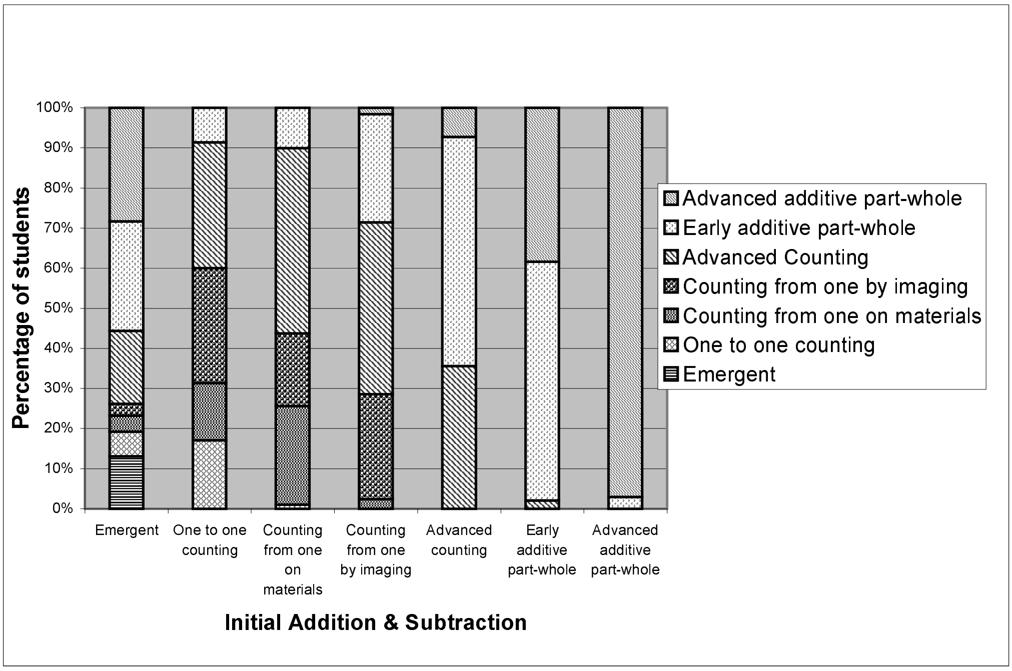


Figure 3-7: Year 6 patterns of improvement through the stages in addition and subtraction

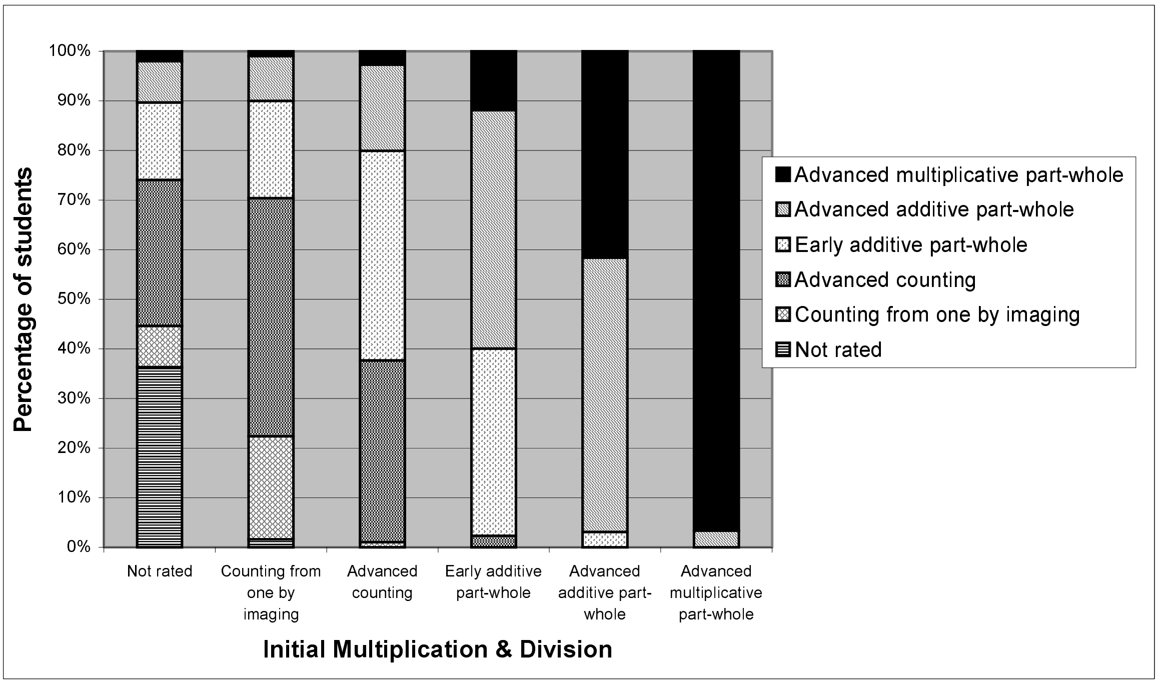


Figure 3-8: Year 6 patterns of improvement through the stages in multiplication and division

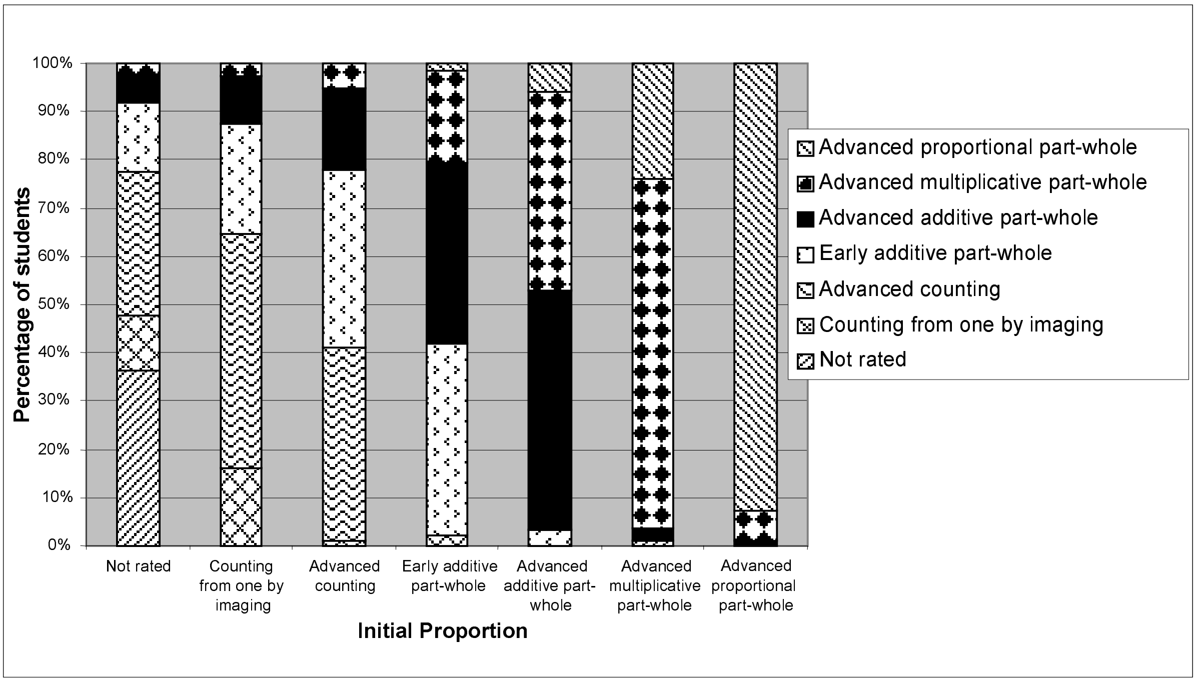


Figure 3-9: Year 6 patterns of improvement through the stages in proportions and ratios

Tables 3-31 to 3-36 illustrate that, in the knowledge aspects, the pattern of achievement for year 6 was similar to the patterns for years 4 and 5 in terms of stages gained over the period of instruction – again with the exception of number identification and decimals, where the majority of students were not assessed.

In FNWS the largest shift was in the percentage of students at FNWS up to 1,000 and above, which moved from 82% prior to instruction to 92% after instruction. (These percentages increased with each year group.) For BNWS the pattern was similar, with the largest shift being at BNWS up to 1,000 and above, with 78% prior to instruction and 88% after instruction.

In grouping, the movement of students was at the stages of grouping with tens and tens in 100 as with the other year groups. Prior to instruction, 66% of students were at these stages. By the end of instruction, this had increased to 60%. Sixteen percent of students reached tens and hundreds in whole numbers. For fractions, the movement of students was greatest for ordered unit fractions, with 22% at this stage prior to instruction and 35% after instruction.

Stage	Initial	Final
Emergent FNWS	1% (135)	1% (144)
Initial FNWS up to 10	0% (6)	0% (5)
FNWS up to 10	0% (35)	0% (9)
FNWS up to 20	1% (119)	0% (51)
FNWS up to 100	15% (1,729)	6% (734)
FNWS up to 1,000	56% (6,370)	42% (4,815)
FNWS up to 1,000,000	27% (3,061)	50% (5,696)
Total	100% (11,455)	100% (11,455)

Table 3-31: Year 6 FNWS – initial and final assessment

Stage	Initial	Final
Emergent BNWS	2% (212)	2% (234)
Initial BNWS up to 10	0% (17)	0% (9)
BNWS up to 10	1% (68)	0% (19)
BNWS up to 20	2% (185)	1% (78)
BNWS up to 100	18% (2,088)	9% (980)
BNWS up to 1,000	52% (5,981)	41% (4,699)
BNWS up to 1,000,000	25% (2,904)	47% (5,435)
Total	100% (11,455)	100% (11,455)

Table 3-32: Year 6 BNWS – initial and final assessment

Stage	Initial	Final
Not assessed	93% (10,702)	96% (10,938)
Emergent Number identification	1% (52)	0% (30)
Numerals to 10	0% (6)	0% (1)
Numerals to 20	0% (22)	0% (11)
Numerals to 100	1% (129)	1% (74)
Numerals to 1,000	5% (544)	4% (399)
Total	100% (11,455)	100% (11,455)

Table 3-33: Year 6 number identification – initial and final assessment

Stage	Initial	Final
Not assessed	1% (163)	1% (156)
Non-grouping with 5s and within 10	5% (524)	1% (123)
With 5s and within 10	17% (1,897)	6% (625)
With 10s	40% (4,578)	25% (2,819)
10s in 100	26% (3,008)	36% (4,111)
10s and 100s in whole numbers	7% (799)	16% (1,846)
10s, 100s, and 1,000s in whole numbers	3% (344)	9% (1,039)
10ths, 100ths, and 1,000ths in decimals	1% (142)	6% (734)
Total	100% (11,455)	100% (11,455)

Table 3-34: Year 6 grouping – initial and final assessment

Stage	Initial	Final
Not assessed	6% (693)	4% (407)
Non-fractions of regions	29% (3,297)	7% (849)
Assigned unit fractions	30% (3,380)	18% (2,101)
Ordered unit fractions	22% (2,560)	35% (4,006)
Co-ordinated numerators and denominators	10% (1,176)	24% (2,781)
Equivalent fractions	2% (226)	7% (753)
Ordered fractions	1% (123)	5% (556)
Total	100% (11,455)	100% (11,455)

Table 3-35: Year 6 fractions – initial and final assessment

Stage	Initial	Final
Not assessed	79% (9,095)	64% (7,364)
	0% (43)	1% (60)
Emergent decimal identification	7% (748)	6% (709)
Decimal identification	8% (904)	12% (1,387)
Ordered decimals	3% (392)	9% (971)
Rounded decimals	2% (205)	6% (626)
Decimal conversions	1% (68)	3% (336)
Total	100% (11,455)	100% (11,455)

Table 3-36: Year 6 decimals – initial and final assessment

Table 3-37 shows the pattern of improvement across the age groups. For instance, for year 4 students the greatest shift was between counting-based and part-whole stages. The number of students at the counting-based stages decreased from 63% to 37% (a reduction of 26%) over the course of the project, while the number of students at the early additive part-whole stage increased from 32% to 50% (an increase of 18%) and the number of students at the advanced additive part-whole stage increased from 5% to 13% (an increase of 8%).

The students in year 6 showed a similar decrease in the counting-based stages, from 40% to 17%. The gains were smaller than for year 4 at the early additive part-whole stage (41% to 46%) but higher at the advanced additive part-whole stage of (18% to 36%). As in 2001 and as reported for ENP students, the younger students showed slightly higher achievement levels after participation in the ANP than the older students. All students benefited to some extent from participation in the project.

Stage	Year 4 Initial	Year 4 Final	Year 5 Initial	Year 5 Final	Year 6 Initial	Year 6 Final
Emergent	1%	1%	1%	1%	1%	1%
One to one counting	2%	1%	1%	0%	0%	0%
Counting from one on materials	8%	3%	4%	1%	1%	1%
Counting from one by imaging	4%	4%	3%	2%	1%	1%
Advanced counting	49%	30%	44%	22%	36%	15%
Early additive part-whole	32%	50%	38%	50%	42%	46%
Advanced additive part-whole	5%	13%	10%	24%	18%	36%
Total	100%	100%	100%	100%	100%	100%

Table 3-37: Addition and subtraction – patterns of improvement by year

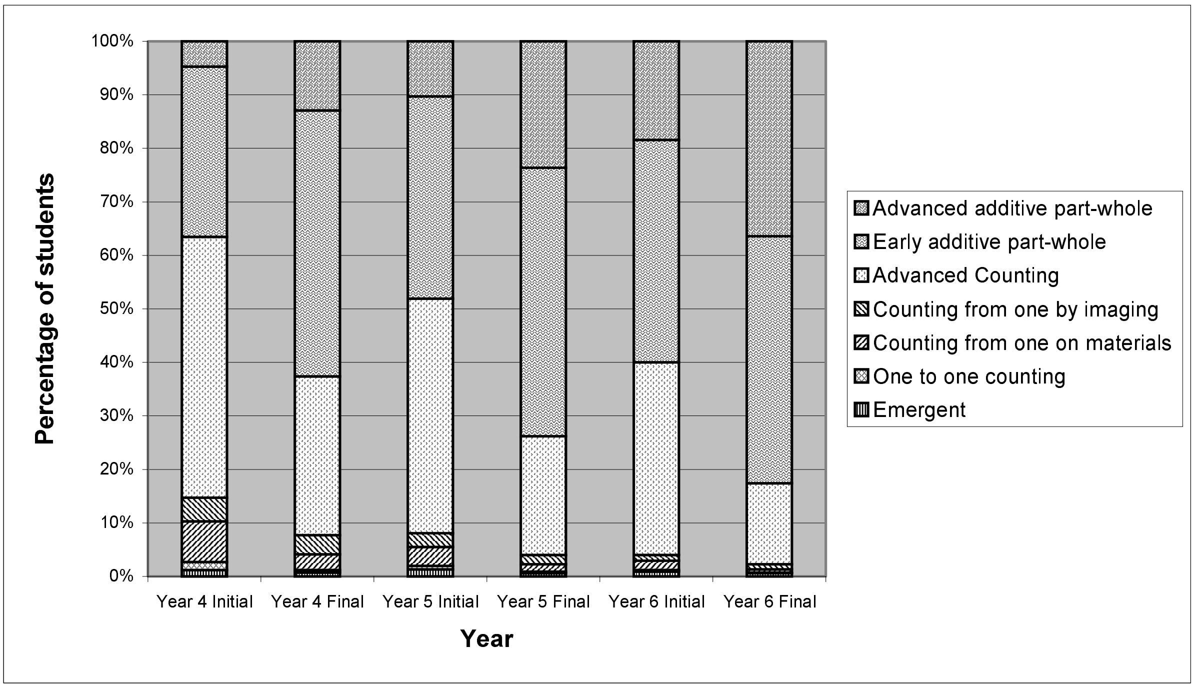


Figure 3-10: Addition and subtraction – patterns of improvement by year

Chapter Four: The Shift from Counting-based to Part-whole Strategies

Overview

Key to raising student achievement in number is the students' shift from counting-based to part-whole strategies. This shift signals a different and more sophisticated way of thinking about numbers, which enables students to solve more complex problems and is critical to later success in mathematics (Thomas and Ward, 2001, 2002; Wright, 1998; Young-Loveridge, 2001). Between years 4 and 6 it is important that as many students as possible make this shift, because otherwise they risk being "stuck on counting" and may "find it hard to make the transition to part-whole strategies" (Young-Loveridge, 2001, p. 73). Underpinning the shift to part-whole thinking is the understanding built up over many years of number-related activities both at school and in early childhood education. It is also important for students to have a strong knowledge base with which to develop more sophisticated thinking. The interdependence of strategy and knowledge is emphasised in the Number Framework (Ministry of Education, 2002a).

This chapter focuses specifically on those students who started with counting-based strategies. It traces the development of part-whole thinking by this set of students in each year level across the operational domains of addition and subtraction, multiplication and division, and ratio and proportions. It examines the effect of gender, ethnicity, decile, and region on the shift to part-whole thinking. It also compares the knowledge stages of those making the shift to part-whole thinking with those who retain counting-based strategies at the conclusion of the project. On the Number Framework, the cross-over from counting-based to part-whole thinking is placed between the advanced counting and early additive part-whole stages.

It is important to note here that the stages of understanding on the framework are not intended to represent even-sized steps. For the purposes of examining the shift in thinking, the results presented below treat all stages prior to, and including advanced counting as counting-based and hence not demonstrating the shift to part-whole thinking. The term "no change" has been used to indicate this group of students.

Tables 4-1, 4-2, and 4-3 and Figures 4-1 and 4-2 show that the percentage of students who initially used counting-based strategies and then shifted to part-whole strategies increased at each year level for each operational domain. The greatest shift was at the early additive stage, where the way numbers are split or joined allows more complex problems to be solved than at the advanced counting stage. Fewer students moved to the advanced additive and advanced multiplicative part-whole stages, which represent more sophisticated part-whole thinking. Very few students shifted to the advanced proportional part-whole stage. The proportion of those who shifted to these more advanced stages increased at each year level, with the exception that more students in year 5 reached the advanced proportional stage than did in year 6. The difference between year levels was more marked than that reported between the age groups in 2001 and the proportions of students making no change was also higher (Higgins, 2002). These differences in the results between 2001 and 2002 could be the effect of the project being extended to a greater number of students (from 8,095 students in 2001 to 33,209 in 2002). Schools involved in the project during 2001 were more likely to have participated in professional development as part of the numeracy project pilot in 2000. They had also benefited from working with a very experienced facilitator who were developing the project. Another reason for the differences is likely to be the higher proportion of students in 2002 who started with counting-based strategies for addition and subtraction (52% of the total sample in 2002, as compared to 41% in 2001).

Final	No change	Became early additive	Became advanced additive	Totals
Year 4	58% (3,890)	41% (2,752)	2% (119)	100% (6,761)
Year 5	48% (2,783)	48% (2,738)	4% (236)	100% (5,757)
Year 6	40% (1,846)	51% (2,414)	7% (328)	100% (4,588)
Totals	50% (8,519)	46% (7,904)	4% (683)	100% (17,106)

Table 4-1: Years 4, 5, and 6 addition and subtraction – shift from counting-based strategies to part-whole strategies

Final	No change	Became early additive	Became advanced additive	Became advanced multiplicative	Totals
Year 4	65% (5,371)	27% (2,222)	8% (626)	1% (47)	100% (3,266)
Year 5	55% (3,703)	32% (2,184)	12% (784)	1% (64)	100% (6,735)
Year 6	47% (2,475)	36% (1,849)	15% (780)	2% (123)	100% (5,227)
Totals	57% (11,549)	31% (6,255)	11% (2,190)	1% (234)	100% (20,229)

Table 4-2: Years 4, 5, and 6 multiplication and division – shift from counting-based strategies to part-whole strategies

Final	No change	Early additive part-whole	Advanced additive part-whole	Advanced multiplicative part-whole	Advanced proportional part-whole	Totals
Year 4	69% (6,335)	23% (2,136)	6% (559)	1% (96)	0% (0)	100% (9,146)
Year 5	60% (4,703)	28% (2,174)	11% (837)	2% (163)	6% (5)	100% (7,882)
Year 6	52% (3,325)	30% (1,957)	14% (878)	4% (270)	0% (10)	100% (6,440)
Totals	61% (14,383)	27% (6,267)	10% (2,274)	2% (529)	0% (15)	100% (23,468)

Table 4-3: Years 4, 5, and 6 proportions and ratios – shift from counting-based strategies to part-whole strategies

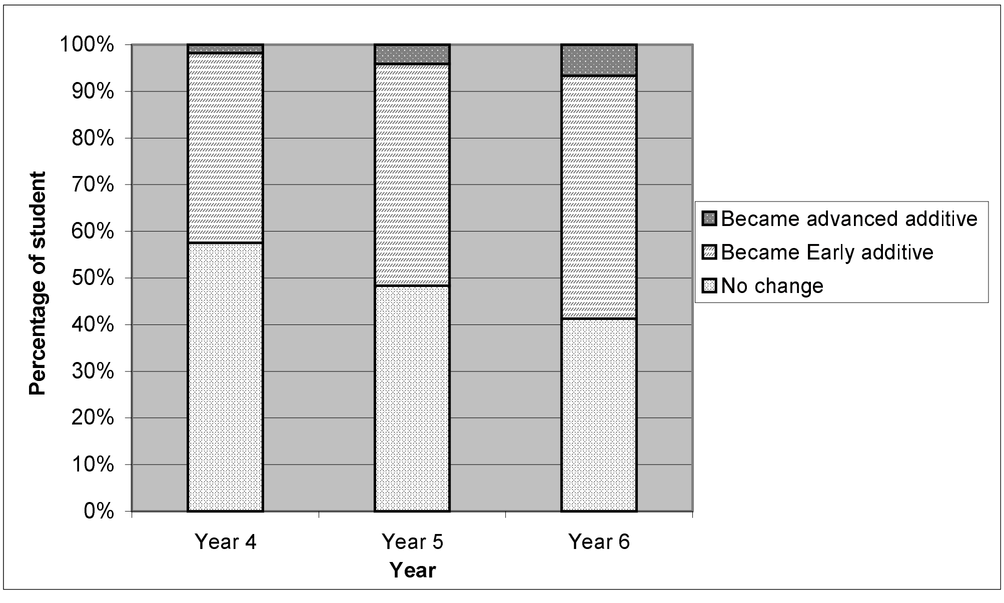


Figure 4-1: Years 4, 5, and 6 addition and subtraction – shift from counting-based strategies to part-whole strategies

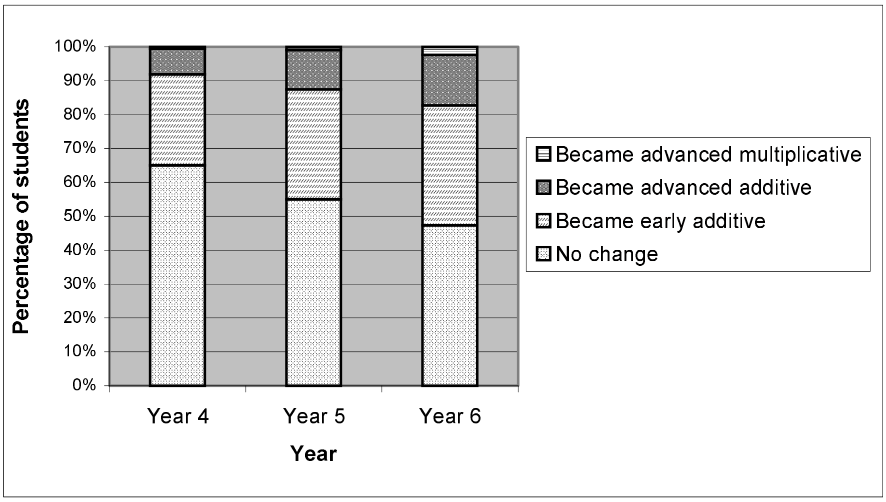


Figure 4-2: Years 4, 5, and 6 multiplication and division – shift from counting-based strategies to part-whole strategies

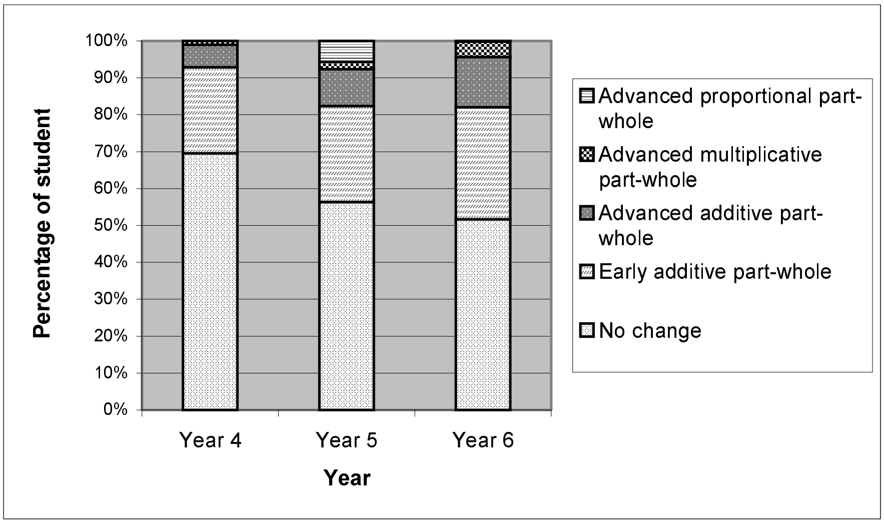


Figure 4-3: Years 4, 5, and 6 proportions and ratios – shift from counting-based strategies to part-whole strategies

The next section traces the effect of gender, ethnicity, decile, region, and knowledge on student achievement in addition and subtraction for year 4 students. The results for students in years 5 and 6 can be found in the appendices.

<u>Gender</u>

Gender (see Table 4-4) appears to have little impact on the proportion of students shifting to part-whole thinking. This is consistent with the 2001 results for the ANP (Higgins, 2002).

Gender	No change	Became early additive	Became advanced additive	Totals
Females	58% (2,062)	41% (1,439)	1% (51)	100% (3,552)
Males	57% (1,828)	41% (1,313)	2% (68)	100% (3,209)
Totals	58% (3,889)	41% (2,752)	2% (119)	100% (6,761)

Table 4-4: Year 4 addition and subtraction – final status by gender

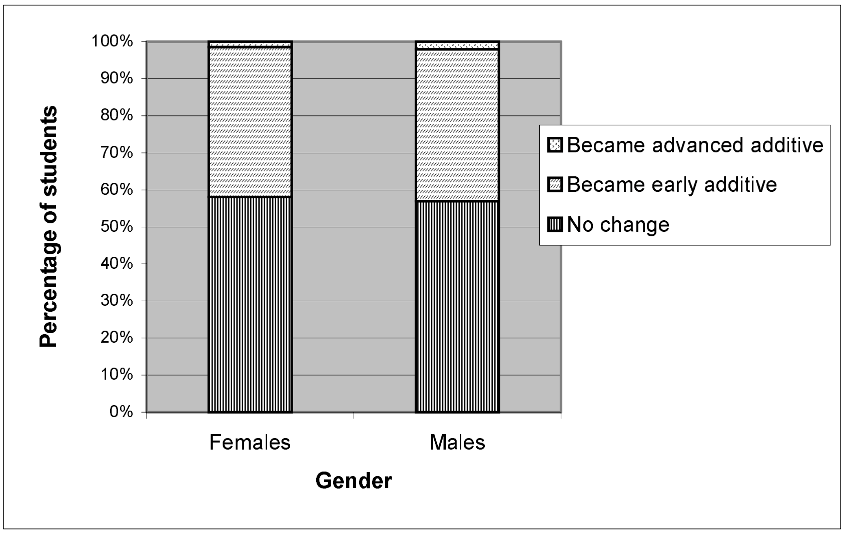


Figure 4-4: Year 4 addition and subtraction – final status by gender

Ethnicity

When this data is analysed against ethnicity, it is clear that about half those in each ethnic group who initially had counting-based strategies adopted part-whole strategies. Minor variations occur. Consistent with the findings for 2001, a lower proportion of Pasifika students (just over a third) make the shift to part-whole strategies.

Ethnicity	No change	Became early additive	Became advanced additive	Totals
New Zealand European	52% (1,832)	46% (1,631)	2% (74)	100% (3,537)
Māori	63% (1,198)	36% (696)	1% (20)	100% (1,914)
Pasifika	73% (570)	26% (201)	1% (8)	100% (779)
Asian	49% (126)	46% (118)	5% (14)	100% (258)
Other	60% (164)	39% (106)	1% (3)	100% (273)
Totals	58% (3,890)	41% (2,752)	2% (119)	100% (6,761)

Table 4-5: Year 4 addition and subtraction – final status by ethnicity

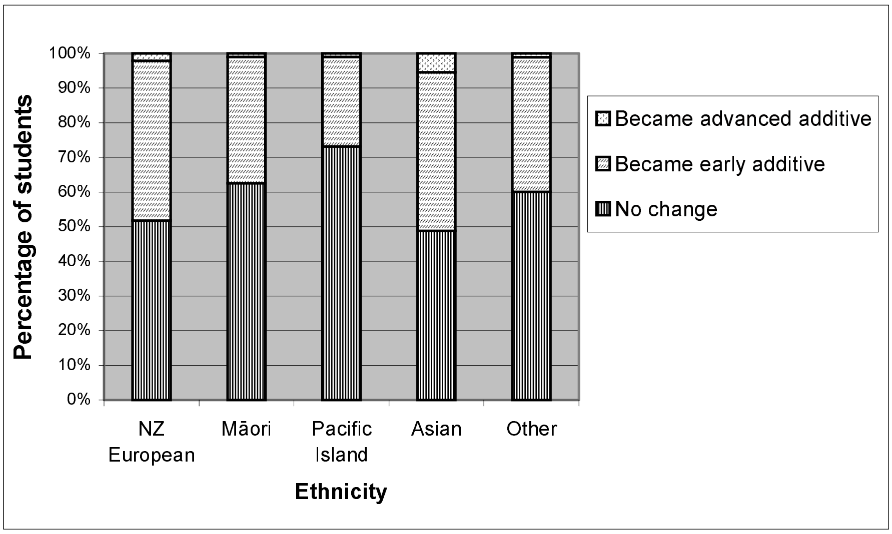


Figure 4-5: Year 4 addition and subtraction – final status by ethnicity

Decile

The pattern shown when achievement for year 4 is analysed by decile is graduated effect from the lowest to the highest decile. Just under a third of decile 1 students adopted part-whole strategies, while around half the students in the higher deciles did so.

Decile	No change		Early additive		Advanced additive		Totals	
1	69%	(726)	30%	(319)	1%	(10)	100%	(1,055)
2	63%	(428)	36%	(242)	1%	(7)	100%	(677)
3	60%	(560)	39%	(364)	2%	(17)	100%	(941)
4	61%	(507)	36%	(302)	3%	(23)	100%	(832)
5	54%	(458)	44%	(375)	2%	(13)	100%	(846)
6	51%	(163)	47%	(149)	2%	(6)	100%	(318)
7	50%	(219)	48%	(211)	3%	(7)	100%	(437)
8	48%	(267)	49%	(270)	3%	(19)	100%	(556)
9	45%	(168)	53%	(199)	2%	(7)	100%	(374)
10	46%	(191)	51%	(212)	2%	(10)	100%	(413)
Total	57%	(3,687)	41%	(2,643)	2%	(119)	100%	(6,449)

Table 4-6: Year 4 addition and subtraction – final status by decile

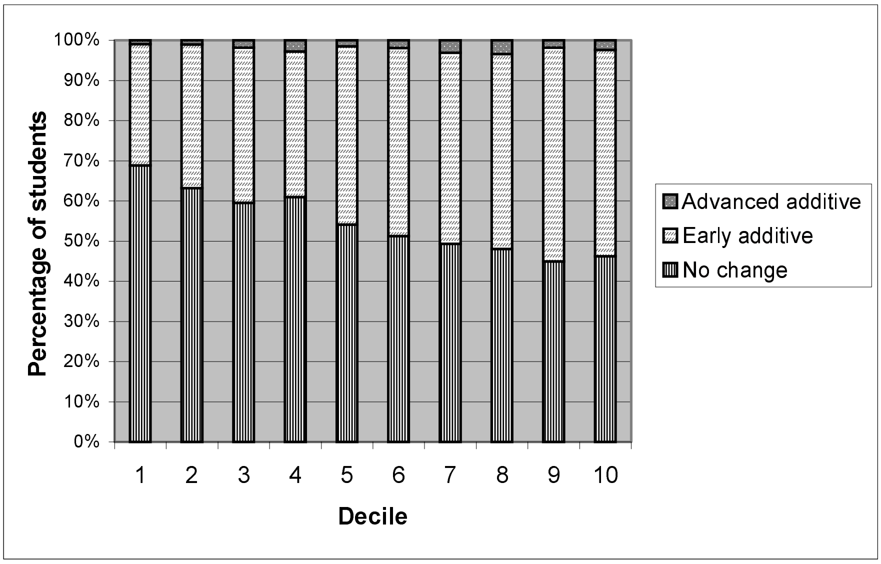


Figure 4-6: Year 4 addition and subtraction – final status by decile

Region

There was little regional difference in the numbers adopting part-whole strategies. Students in two regions, Northland and Otago, scored slightly better results than the other regions. One possible reason for this was that the overall experience of the facilitators in these regional teams was greater than for other regions, which in some cases had a higher proportion of less experienced facilitators.

Region	No change		Early additive		Advanced additive		Totals	
Auckland	61%	(1,300)	37%	(781)	2%	(34)	100%	(2,115)
Christchurch	57%	(380)	42%	(279)	0%	(2)	100%	(661)
Massey	55%	(421)	43%	(330)	2%	(14)	100%	(765)
Northland	52%	(256)	46%	(224)	2%	(11)	100%	(491)
Otago	43%	(122)	52%	(149)	6%	(16)	100%	(287)
Southland	56%	(178)	42%	(133)	2%	(6)	100%	(317)
Waikato	55%	(783)	43%	(611)	2%	(23)	100%	(1,417)
Wellington	63%	(416)	35%	(235)	2%	(13)	100%	(664)
Totals	57%	(3,856)	41%	(2,742)	2%	(119)	100%	(6,717)

Table 4-7: Year 4 addition and subtraction – final status by region

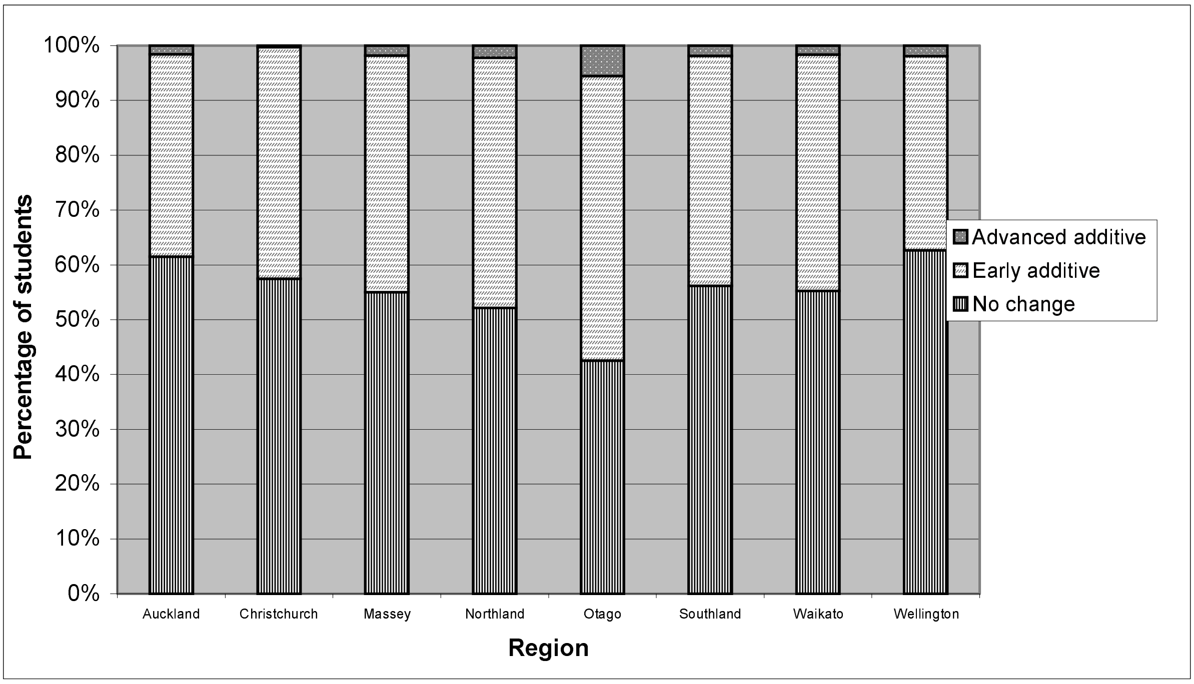


Figure 4-7: Year 4 addition and subtraction – final status by region

Comparison of Knowledge Base in Differing Strategies

The next set of tables compares the knowledge base of students retaining counting-based strategies with those who adopt part-whole strategies. For the knowledge domains of number identification and decimals and percentages this comparison was not possible, as the majority of students were not assessed. For the remaining knowledge domains the levels of knowledge of those with part-whole strategies was consistently higher than for those with counting-based strategies.

Strategies	Emergent	Initial up to 10	up to 10	up to 20	up to 100	up to 1,000	up to 1,000,000
FNWS							
Counting-based	2% (71)	0% (17)	1% (35)	4% (168)	42% (1,653)	45% (1,765)	5% (181)
Part-whole	0% (7)	0% (1)	0% (2)	0% (8)	11% (311)	67% (1,944)	21% (2,871)
BNWS							
Counting-based	2% (83)	1% (28)	3% (101)	7% (263)	43% (1,678)	41% (1,595)	4% (142)
Part-whole	0% (10)	0% (1)	0% (0)	0% (9)	15% (423)	66% (1,907)	18% (521)

Table 4-8: Year 4 FNWS and BNWS – knowledge for counting-based and part-whole

	Not assessed	Non-grouping with 5s and within 10	With 5s and within 10	With 10s	10s in 100	10s and 100s in whole numbers	10s, 100s, and 1,000s in whole numbers	10ths, 100ths, and 1,000ths in decimals
Counting-based	2% (72)	9% (361)	26%(1,027)	50%(1,935)	12%(470)	1% (23)	0% (1)	2% (1)
Part-whole	1% (27)	1% (14)	7% (194)	50%(1,441)	37%(1,065)	4% (117)	0% (11)	2% (1)

Table 4-9: Year 4 grouping – knowledge for counting-based and part-whole

	Ordered unit fractions	Co-ordinated numerators and denominators	Equivalent fractions	Ordered fractions	Co-ordinated numerators and denominators	Equivalent fractions	Ordered fractions
Counting-based	27% (1,032)	24% (916)	31% (1,187)	18% (699)	1% (56)	0% (0)	0% (0)
Part-whole	2% (42)	14% (394)	36% (1,028)	41% (1,181)	8% (221)	0% (4)	0% (4)

Table 4-10: Year 4 fractions – knowledge for counting-based and part-whole

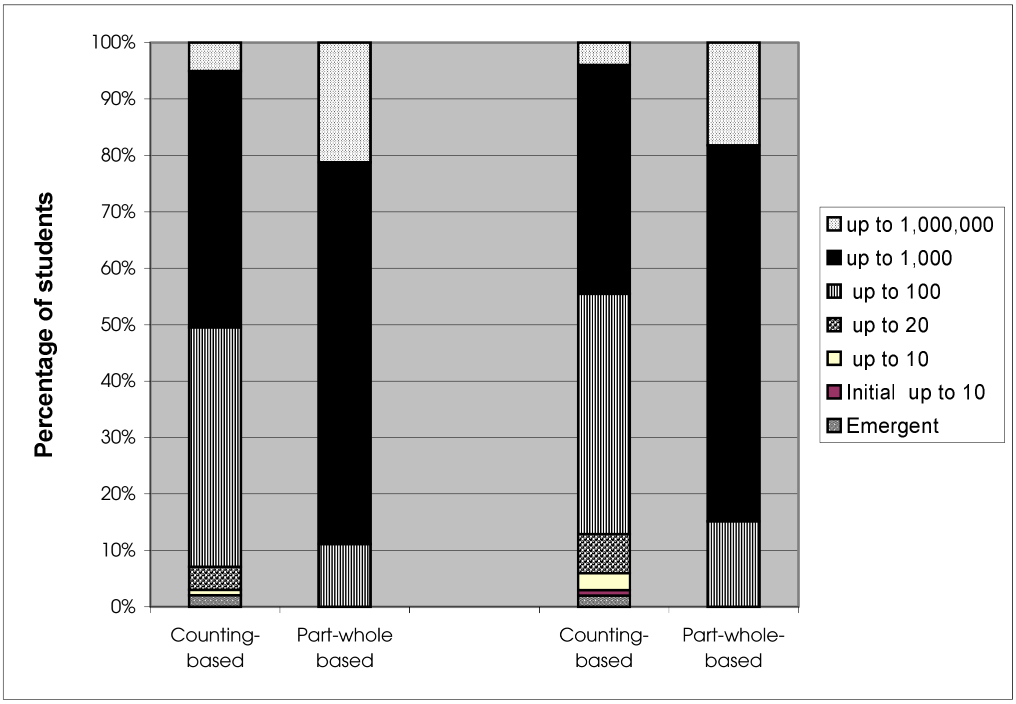


Figure 4-8: FNWS and BNWS – knowledge for counting-based and part-whole

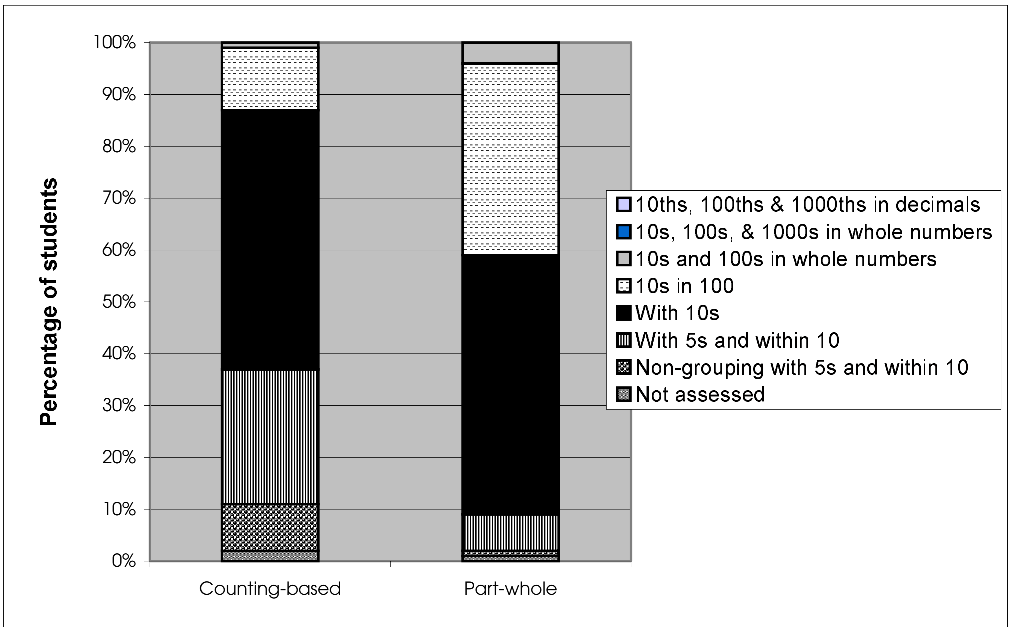


Figure 4-9: Grouping – knowledge for counting-based and part-whole

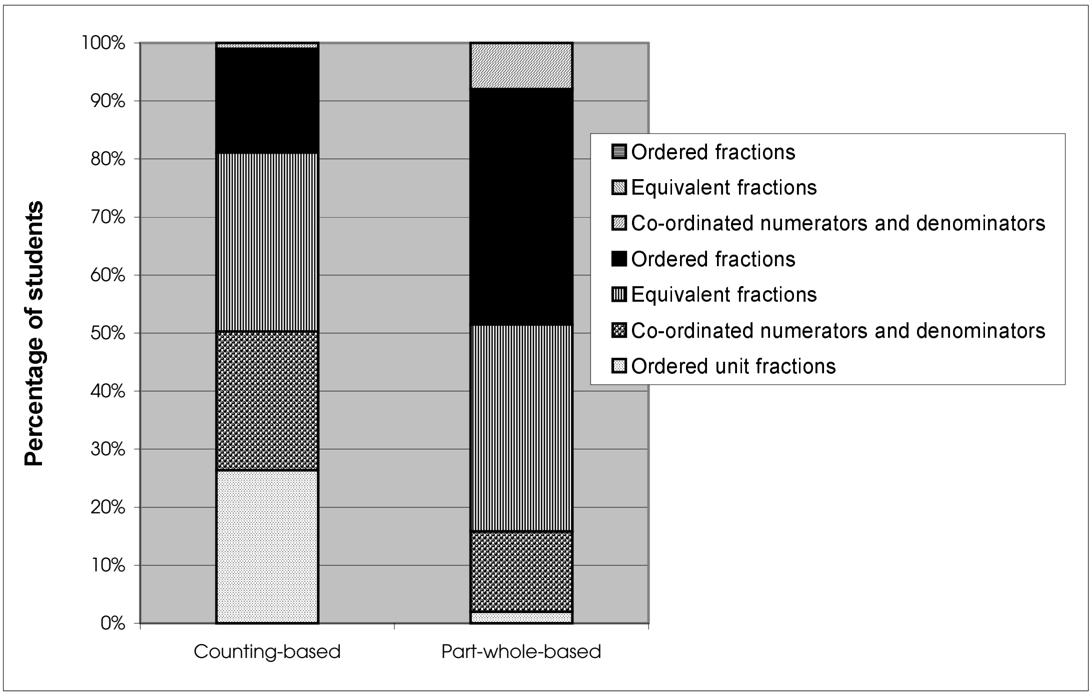


Figure 4-10: Fractions – knowledge for counting-based and part-whole

Comparison of 2001 and 2002 Shifts from Counting-based to Part-whole Strategies

The comparison of the 2002 and 2001 results shows a decline overall and for ethnicity and decile. Tables 4-11 to 4-14 show that more students in 2002 who initially had counting-based strategies made no shift to part-whole strategies by the time of the final assessment. These results are likely to be related to changes to the diagnostic interview in 2002, in which teachers were given a choice of forms to use based on students' strategy knowledge. This may have unintentionally created a ceiling effect through teachers inadvertently using the same form for the initial and final diagnostic interviews, when students should probably have been given a more advanced form.

	2001	2002
No change	37% (1,230)	50% (8,519)
Became early additive	52% (1,707)	46% (7,904)
Became advanced additive	11% (373)	4% (683)

Table 4-11: Addition and subtraction – 2001 and 2002 comparison

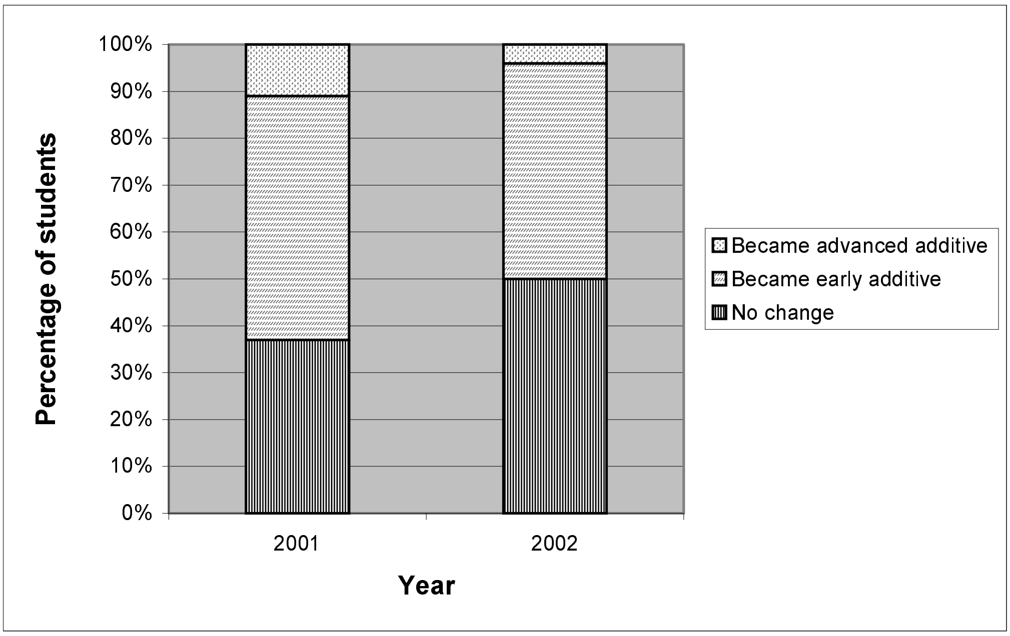


Figure 4-11: Addition and subtraction – 2001 and 2002 comparison

Ethnicity	No change		Became early additive		Became advanced additive	
New Zealand European 2001	34%	(669)	53%	(1,038)	13%	(244)
New Zealand European 2002	44%	(3,824)	51%	(4,397)	5%	(429)
Māori 2001	39%	(285)	50%	(366)	11%	(79)
Māori 2002	54%	(2,636)	43%	(2,100)	3%	(140)
Pasifika 2001	49%	(205)	46%	(190)	5%	(21)
Pasifika 2002	64%	(1,433)	34%	(777)	2%	(45)
Asian 2001	33%	(44)	52%	(70)	15%	(20)
Asian 2002	46%	(276)	47%	(284)	8%	(46)
Other 2001	34%	(27)	54%	(43)	11%	(9)
Other 2002	49%	(350)	48%	(346)	3%	(23)

Table 4-12: Addition and subtraction – 2001 and 2002 comparison by ethnicity

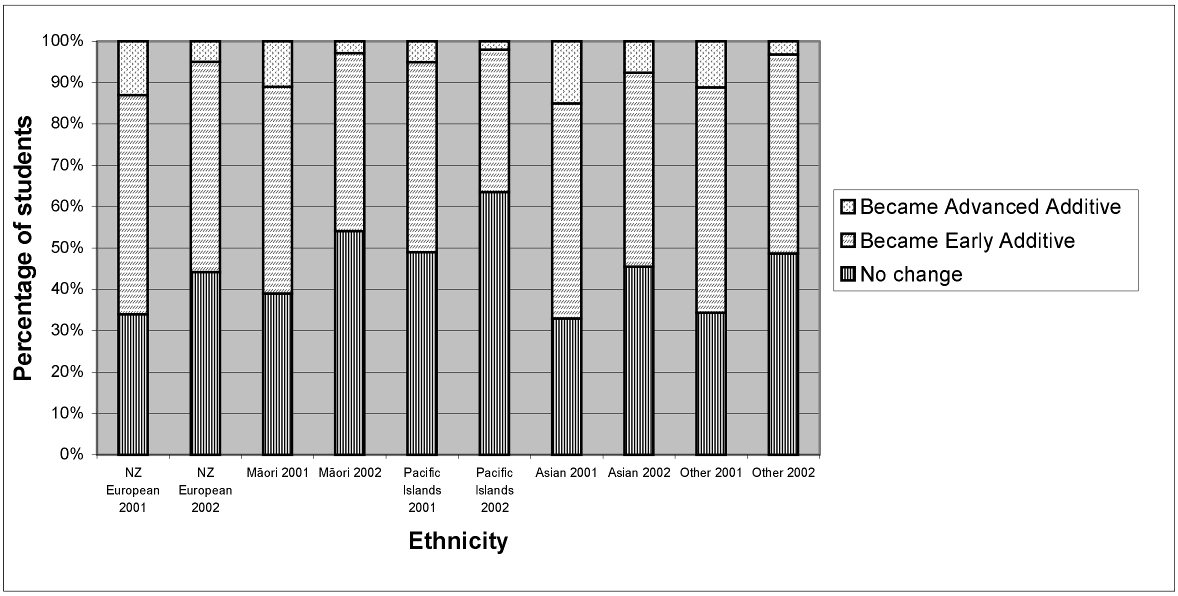


Figure 4-12: Addition and subtraction – 2001 and 2002 comparison by ethnicity

Decile	No change	Became early additive	Became advanced additive
Low (1-3) 2001	40% (592)	50% (731)	10% (146)
Low (1-3) 2002	64% (1,714)	35% (925)	2% (34)
Middle (4-7) 2001	36% (466)	52% (675)	12% (149)
Middle (4-7) 2002	55% (1,347)	43% (1,032)	2% (49)
High (8-10) 2001	31% (172)	55% (301)	14% (78)
High (8-10) 2002	47% (626)	51% (681)	3% (36)

Table 4-13: Addition and subtraction – 2001 and 2002 comparison by aggregated decile

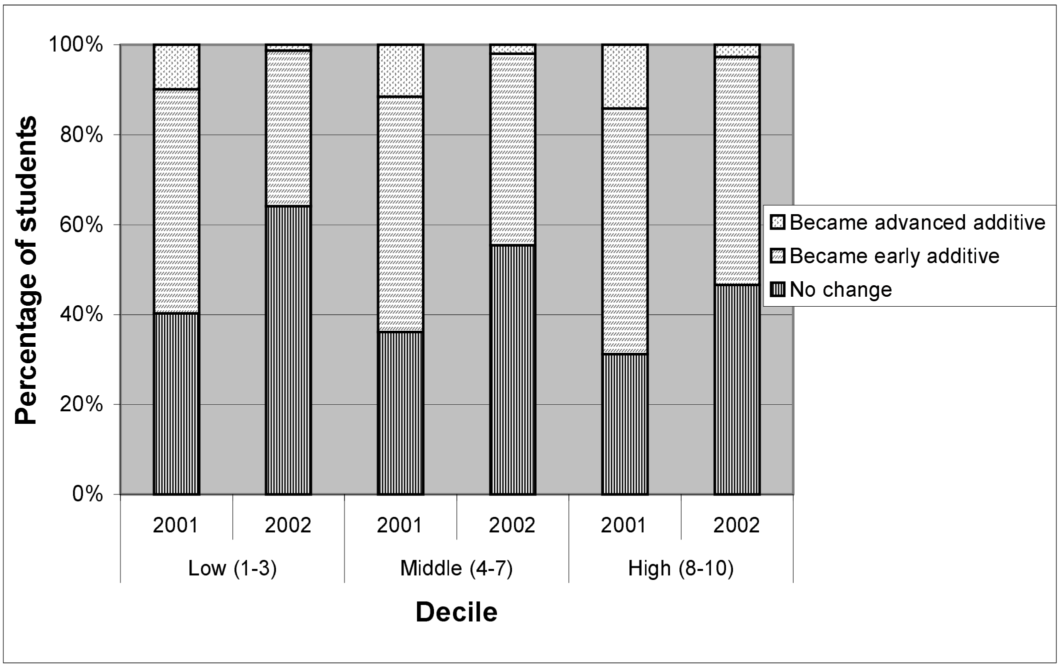


Figure 4-13: Addition and subtraction – 2001 and 2002 comparison by aggregated decile

Decile	No change		Early additive		Advanced additive	
1 2001	62%	(136)	34%	(74)	4%	(8)
1 2002	60%	(1,734)	38%	(1,111)	2%	(66)
2 2001	31%	(183)	54%	(321)	15%	(86)
2 2002	54%	(976)	43%	(753)	3%	(54)
3 2001	41%	(273)	51%	(336)	8%	(52)
3 2002	53%	(1,180)	44%	(968)	3%	(70)
4 2001	36%	(242)	52%	(355)	12%	(82)
4 2002	51%	(1,179)	45%	(1,029)	4%	(98)
5 2001	35%	(35)	47%	(47)	17%	(17)
5 2002	45%	(898)	50%	(1,000)	4%	(83)
6 2001	32%	(87)	57%	(154)	11%	(29)
6 2002	47%	(382)	50%	(404)	4%	(30)
7 2001	42%	(102)	49%	(119)	9%	(21)
7 2002	46%	(451)	51%	(506)	3%	(34)
8 2001	28%	(67)	52%	(124)	19%	(46)
8 2002	41%	(541)	54%	(716)	6%	(74)
9 2001	33%	(12)	58%	(21)	8%	(3)
9 2002	37%	(362)	55%	(541)	9%	(88)
10 2001	33%	(93)	56%	(156)	10%	(29)
10 2002	37%	(370)	57%	(573)	6%	(65)

Table 4-14: Addition and subtraction – 2001 and 2002 comparison by decile

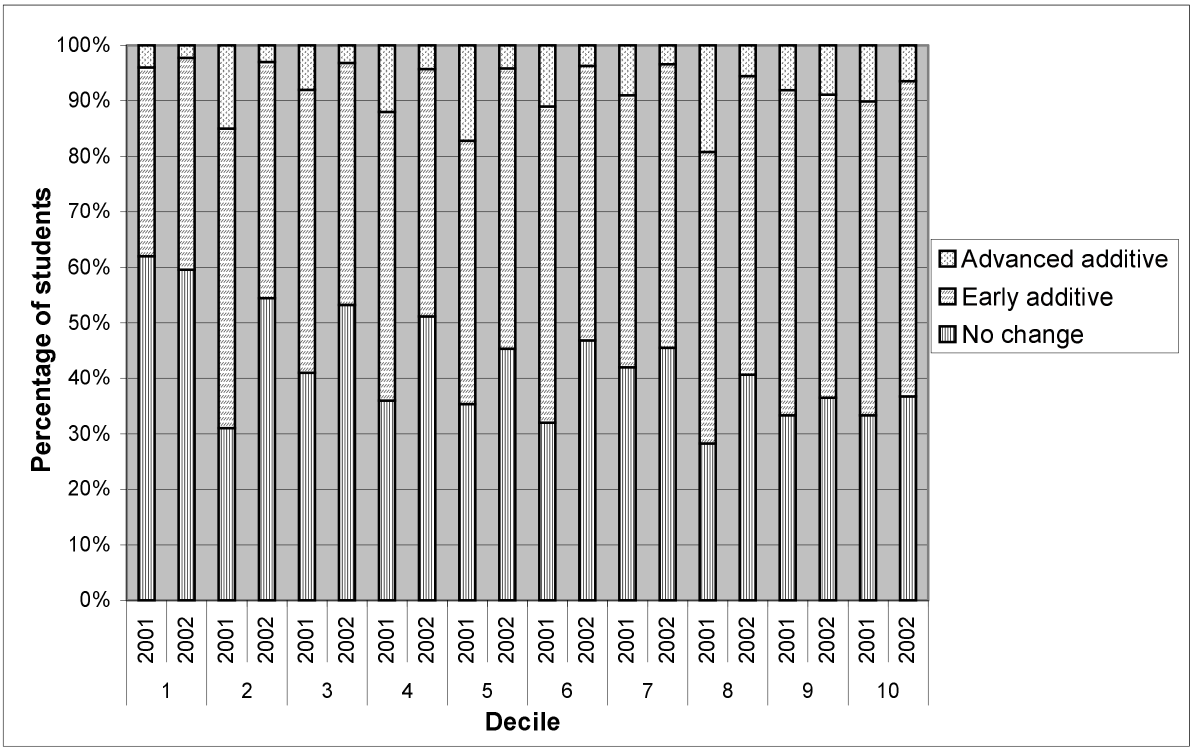


Figure 4-14: Addition and subtraction – 2001 and 2002 comparison by decile

Chapter Five: Perspectives of Adult Participants

The ANP aims to raise student achievement in mathematics by improving teacher capability. Key aspects of the project are the Number Framework, which provides an explanation of key stages in the development of mathematical thinking and the workshops and in-class support for teachers. The questionnaires used to find out the perspectives of adult participants sought information about the impact of the project on teachers' professional knowledge as well as about the future sustainability of the project in schools.

This chapter reports on the questionnaire responses from three groups of participants in the project: teachers, principals, and facilitators. The questionnaires are similar to those answered by participants in the Early Numeracy Project (Thomas and Ward, 2001, 2002) and participants in the 2001 ANP (Higgins, 2002). The questionnaires are based on those by Bobis (1999) in her evaluation of Count Me In Too in New South Wales.

Common themes can be tracked across multiple data sources (such as responses from different groups of participants) as well as across responses to different questions within a questionnaire. Where possible, comparisons will be drawn between responses to similar questions by participants in the New Zealand Numeracy Projects in previous years. The views of teachers and principals in lower decile schools are compared with the views of the whole teacher and principal groups. This focus on the perspectives and practice of those in lower decile schools is continued in Chapter Seven.

Of the 680 questionnaires sent out, 337 were returned, resulting in a 55% response rate. Of these 337, there were four incomplete returns and one that arrived too late for analysis. The following analysis is based on the remaining 332. Among the 332 questionnaires analysed were 133 from teachers in decile 1, 2, or 3 schools. The return rate for this sub-group of teachers was 54%. Of the 157 questionnaires sent to principals, 99 were returned, a response rate of 63%. Within this group of 99, 33 were answered by principals from decile 1, 2, or 3 schools. The return rate of this sub-group of principals was 60%. Thirty-one of the 51 questionnaires sent to facilitators were returned, a 61% response rate.

Category	Details	Percentage (n = 338)
Size of school*	<100	6
	101–200	24
	201+	67
Region*	Auckland	30
	Waikato	13
	Northland	6
	Massey	16
	Wellington	6
	Canterbury	15
	Otago	8
	Southland	6
Decile*	1–3	43
	4–7	34
	8–10	23
Age*	20–25	8
	26–35	29
	36–45	29
	46–55	26
	56+	8
Gender*	Female	83
	Male	17
Years of teaching experience*	1–5	29
	6–10	21
	11–15	13
	16–20	13
	21+	24
Level currently being taught*	2/3	2
	3/4	14
	4	9
	4/5	14
	5	6
	5/6	28
	6	9
	6/7	18
Years of teaching experience at years 4–6*	1–5	58
	6–10	23
	11–15	9
	16–20	4
	21+	6
Length of time at current school*	1–5 years	64
	6–10 years	21
	11–15 years	8
	16–20 years	4
	21+ years	3
Highest level teaching qualifications*	Diploma of Teaching	43
	Postgraduate Diploma	8
	Bachelor’s Degree	46
	Master’s Degree	3
Undertaking further study*	Yes	34
	No	66

Table 5-1: Demographic data for teacher respondents

*Not all respondents completed this question

Category	Details	Percentage (n = 99)
Size of school*	<100	24
	101–200	31
	201+	45
Region*	Auckland	29
	Northland	8
	Waikato	8
	Massey	13
	Wellington	8
	Christchurch	15
	Otago	12
	Southland	9
Decile*	1–3	26
	4–7	39
	8–10	25

Table 5-2: Demographic data for principal respondents

Category	Details	Percentage (n =31)
Years of teaching experience	1–5	3
	6–10	3
	11–15	23
	16–20	15
	21+	54
Years of advisory experience	1–5	20
	6–10	7
	11–15	1
	16+	3

Table 5-3: Demographic data for facilitator respondents

Impact on Teachers' Professional Knowledge

Teachers' professional knowledge includes knowledge of the discipline of mathematics, knowledge of how to teach mathematics, and knowledge of how students learn mathematics. This section will report on each of these aspects in turn.

Teachers' content knowledge of mathematics

Following the trend of previous years, the majority of teachers (71%) felt that their content knowledge of mathematics had been improved by their participation in the ANP. The positive responses from those in low-decile schools (68%) were consistent with this overall trend when considered separately. Typical comments were:

> *Yes, very much so. The project has shown me how to teach maths concepts in a way the students will understand. The project has given my teaching more structure.*

A small percentage of teachers (13%) felt that the project had not had any impact on their content knowledge. Some teachers (10%) did not comment on this question, while others felt their knowledge had not changed (10%) or were unsure as to whether it had (1%). Three per cent felt that this question was not applicable to them. There was little difference in the rates of these responses between all teachers and those in lower decile schools.

Of those 71% who reported an improvement in their content knowledge, the aspects that had changed were numerous and varied, but the key areas identified were the same as for 2001 (Higgins, 2002). One area (21%) was that of mental strategies. This was mentioned by a much greater percentage of teachers in the previous year. Typical comments included:

> *Absolutely – the strategies such as doubling and halving, rounding and calculating in head horizontally has been great. Wonderful to learn with students (and admit it!)*
>
> *Yes – better understanding of how to manipulate numbers.*
>
> *Yes. I am able to teach addition and subtraction using strategies other than the conventional algorithm method.*
>
> *Yes my own knowledge of number has also increased a few stages. I am using a lot more part/whole strategies myself and know better how to teach these strategies.*
>
> *Yes difference between knowledge and strategies and how much 'process/experience' students need to develop an understanding of the concept.*

Teachers (19%) also commented that their knowledge of number had deepened. Again, this theme came through much more strongly in the 2001 questionnaires.

> *Yes I have had to put aside some of the rigid rules I have taught in the past. The content knowledge is not the problem, rather the freedom of ideas which I find really exciting.*

A few teachers (5%) felt their knowledge of fractions and decimals had increased, one commenting, "I have more content knowledge of fractions and place value".

	Mental strategies	More in-depth knowledge	Fractions, decimals, and percentages
Teachers	21%	19%	5%
Facilitators	29%	39%	57%

Table 5-4: Percentages of respondents who commented on particular factors enhancing teachers' content knowledge

Facilitators were also asked to comment on teachers' content knowledge. Most facilitators (81%) felt that teachers' content knowledge had improved. Their responses identified the same areas as teachers did. The most frequently identified area (57%) was that of fractions and decimals. Comments included:

> *The greatest impact was on fractions/decimals/percentages content knowledge – specifically: they gained new knowledge or strengthened long forgotten knowledge of working with simple pieces of equipment, e.g. bead strings to 100 in tens, to show and compare fractions, decimals, and percentages.*
>
> *Yes – most apparent in fractions/decimals/proportions but also in general part-whole thinking and varieties of ways to solve problems. Also on the real need to develop understanding of place value.*
>
> *Yes, without a doubt. Some ANP teachers commented that they were not part-whole thinkers themselves and were excited to become so. Fractions knowledge was an area which also enhanced through the project.*

Facilitators (39%) also identified a general deepening of teacher content knowledge.

> *On a scale of 1–6 with 1 representing "surface understanding" and 6 representing "deep understanding" teachers from one cluster (12 teachers) averaged 4.*

A greater proportion of facilitators (29%) commented on the development of mental strategies. Typical comments included:

> *The majority of the teachers I worked with are now using more mental strategies themselves. For example one teacher said, "Moved away from straight algorithms. I was introduced to strategies I didn't know". Another said, "I know more about mental strategies that can be used for different problems".*

Teachers' understanding of how students learn about number

The majority of teachers (80%) and all facilitators felt that teachers' understanding of how students learn maths had improved. Only 7% of teachers felt that it had not. Teachers identified three main areas. These were strategies (20%), the framework (19%), and their ability to assess students' learning needs (20%). Facilitators identified one area of teaching approaches.

> *Yes, while materials were used in the past we quickly moved to using number properties. Using imaging was rarely used.* (Teacher)

> *Yes! The students are beginning to see the importance of the process or what goes on in their head to get to the answer. The students are now quick to explain what they do! Students can identify what process they use.* (Teacher)

> *Definitely. I realised, once we started the ANP, that I had been missing out the imaging stage to a large extent. I recognise its importance now.* (Teacher)

> *Yes, graded steps and progression through these stages/levels as growth occurs. Testing at start was very good to pinpoint exactly where students were at, the gaps in their knowledge, and how they were using processing strategies.* (Teacher)

	Mental strategies	Developmental Number Framework	Teaching and assessment approaches
Teachers	20%	19%	20%
Facilitators	0%	0%	61%

Table 5-5: Percentages of respondents who commented on particular factors enhancing teachers' pedagogical knowledge

> *I feel happier accepting a range of strategies and because I am questioning more effectively it is easier to understand the students' thought processes. I understand the stages better.* (Teacher)

> *Yes, greater understanding of the use of equipment, increased emphasis on students sharing strategies, good use of questions to develop a community of enquiry.* (Facilitator).

> *Yes – readings and discussions of summaries from research. Most are aware of Pirie Kieran model and many use materials, imaging, number properties in their classroom teaching.* (Facilitator)

Approaches to teaching mathematics

Most respondents felt that there had been changes to the way in which teachers taught mathematics as a result of the project. Over half the teachers (57%) felt they had changed their approach. Only 3% felt that they had not. Most principals (90%) commented that teachers had changed their approach. All facilitators felt that teachers had changed their approach. Teachers' responses fell into three key areas: small-group work (30% of comments), the use of equipment (37% of comments), and the teaching of strategies (23% of comments). Principals (31%) felt that the key area of change was in teacher knowledge.

Facilitators felt that the key change to teachers' approaches was in the implementation of small-group work in mathematics. Typical comments included:

> *The different strategies used for teaching number in the school, the time for numeracy (number), the curriculum, and the emphasis on it in the maths programme.* (Principal)
>
> *Yes, I spend more time asking the students how they worked it out. I try to let the students know which strategy they are learning/will learn next.* (Teacher)
>
> *Yes – greater focus (strategies and knowledge); grouping according to strategies – regular movement between groups. More activities to support teaching focus.* (Teacher)
>
> *Yes. I use materials a lot more. I'm aware of imaging. I plan for more groups than I used to.* (Teacher)
>
> *Yes, I use far more equipment, particularly with my slow and middle groups. I have definitely moved away from an over emphasis on algorithms and explore a variety of mental strategies and delight in the divergent approaches more able students use. I learn all the time.* (Teacher)

	Inclusion of strategies	Equipment	Group work	Teacher knowledge
Teachers	23%	37%	30%	0%
Principals	0%	0%	0%	31%
Facilitators	0%	0%	100%	0%

Table 5-6: Percentages of respondents who commented on particular factors enhancing teachers' approach to teaching mathematics

> *Yes. More focused. Engage students more in explaining their calculation strategies. Accepting and demanding a greater range of calculation strategies from students. More group oral and practical work and less recording of algorithms.* (Teacher)
>
> *I've always enjoyed the 'oral', so this aspect is kind of "legitimised". The resources and 'snappy' activity titles, handles are a big addition.* (Teacher)
>
> *Yes. I am making far more effective use of materials and reinforcing imaging. By the time students work independently they have a sound foundation of strategies.* (Teacher)
>
> *Definitely. Many of them did not follow a using materials → using imaging → using number properties model prior to being part of the ANP.* (Facilitator)
>
> *Yes, greater understanding of the use of equipment, increased emphasis on students sharing strategies, good use of questions to develop a community of enquiry.* (Facilitator)
>
> *Yes, Grouping for teaching, use of materials modelling for the students. Greater preparation. Teaching to students' needs. Broadened understanding of how students learn.* (Facilitator)

Impact on teachers' attitudes

Teachers, principals, and facilitators commented on teachers' attitudes. The majority of facilitators (61%) felt teachers' attitudes had changed. This was in contrast to principals, of whom about half (46%) considered there to have been a change. About two-thirds of teachers (67%) felt their attitude had changed. A smaller group of teachers (21%) and facilitators (29%) felt that there had been no change. A small proportion of each group of respondents were unsure if there had been a change.

The areas identified were similar to those in last year's questionnaire. The key reason for change suggested by principals was teachers' enthusiasm. Sample comments were:

> *All (both) have been highly motivated and excited by the programme and the results being achieved in students' thinking and knowledge of number.* (Principal)

> *Very positive – they have felt the teaching/modelling model has changed their teaching and knowledge and students are enjoying maths more.* (Principal)

> *It varied. Some were enthusiastic from the outset, others were cautious and it took some time to get all "on board". One person remains unconvinced.* (Principal)

> *Improved self confidence, expanding the students' knowledge and strategies. Movement away from the use of algorithm.* (Principal)

> *Enthusiastic. Teachers could see that concentrating on processes showed huge benefits for nearly all students.* (Principal)

Teachers identified two main areas as facilitating a change in teacher attitude: mental strategies and greater enthusiasm and confidence.

> *Yes, I am a lot more confident about my own ability to teach effectively and now enjoy teaching maths.* (Teacher)

> *Yes! – Love teaching it – spend lots of time on maths – Confidence level much higher – now confident of what stage students are at and where they need to go.* (Teacher)

> *Yes, I have questioned the students with more thought and awaited their responses for a longer period. I now consider all methods of acquiring the answer.* (Teacher)

> *There is greater focus and direction in our teaching, i.e., more aware of teaching towards individual students mental thinking stages.* (Teacher)

> *My attitude has always been that maths is awesome and it only goes to emphasise all the things I've ever taught – the easiest way!* (Teacher)

> *Yes, I'm more inclined to use groups more and pursue higher expectations of learning for the students.* (Teacher)

	Mental strategies	Teacher confidence and enthusiasm	Teacher knowledge
Teachers	20%	23%	8%
Principals	0%	28%	0%

Table 5-7: Percentages of respondents who commented on particular factors impacting on teachers' attitudes

There were no strong themes emerging from the facilitators' responses that suggested the reasons for a change of attitude. A selection of comments follow:

> *Positive – initially very keen, ranging a little as they tried to get their heads around the teaching of strategy lessons, and in some cases grouping for mathematics. Positive with final results and in thinking ahead to implementing it in 2003 when they feel they will have a better understanding.* (Facilitator)

> *Mixed. A certain amount of suspicion. Many believed that they were excellent maths teachers and therefore did not have much to learn. Attitudes of year 6 teachers not always positive – were locked into the vertical algorithm and saw no need to change.* (Facilitator)

At this time of the year – enthusiastic and positive. Teachers have generally undergone the predictable [PD path] – with initial excitement → enthusiasm → struggle/big effort → beginning to see the light. Now appreciate the enormity of the PD change and just how far they've come. Also how far yet to go! Looking forward to doing it better next year. (Facilitator)

Some initial resistance to change which ceased once I began modelling in the classroom. End-of-year evaluations and exit interviews indicate great success. (Facilitator)

Very positive, but they had concerns about their content knowledge. Group organisation and planning time could also impact on their attitude, but by the end they could all see the benefits. (Facilitator)

Impact on Students

The majority of respondents (over 80% in all cases) commented that the impact on students was overwhelmingly positive. A small percentage from each group (around 10%) thought that it was varied. The comments from principals, teachers, and facilitators included references to students' new-found enthusiasm for maths, their use of a wider range of problem-solving strategies, and the rise in their achievement.

Maths has been revitalised. Students are enthusiastic. Ordinary language skills have developed – lots more maths talk. Students are using a wider range of strategies to solve problems. (Principal)

Very positive. Students revelling in their (often) new-found ability to manipulate numbers and solve problems mentally in a variety of ways. Great for self-esteem. (Principal).

Totally empowering – particularly our Māori students and our at-risk kids have a really positive attitude to maths and belief in their understandings. (Principal)

Made them aware of other problem solving strategies, thereby giving them more confidence to participate in activities and the ability to solve other problems. (Teacher)

The students really enjoy manipulating equipment when solving problems. Maths has become more accessible to them through strategy development. They have a greater understanding of concepts and can apply these strategies to solve a range of problems. (Teacher)

Respondents	Positive impact of project on students
Teachers	81%
Principals	87%
Facilitators	84%

Table 5-8: Percentages of respondents who commented on the positive impact of the project on students

Skill level has increased and students 'feel' their success. They enjoy maths and moan when we miss maths. They click on more and relate things together better. (Teacher)

I am amazed at the improvement in students' articulation of strategies! They can say exactly what they're doing but at the beginning they were very poor at this. They can analyse and compare strategies of others. (Teacher)

Greater intrinsic motivation by most students shown. Improved mental calculations and number knowledge. Has given low achievers a real boost, filling some gaps quickly and making them see some success in maths. (Teacher)

All students seem to really enjoy mathematics. The use of materials part of their lessons, along with maths games and whole class knowledge activities, reinforces a renewed enthusiasm for the subject. This fact is reinforced by teacher comments. (Facilitator*)*

Students' enthusiasm for maths. A huge improvement in both their knowledge and content exemplified by the increasing use of vocabulary and vocalisation of their processes. (Facilitator)

It has opened up doors to enjoyment, understanding, discussion, co-operative learning, higher attainment in numeracy, language, and articulation and overall skills development. (Facilitator)

Sustainability of the Project

The development of school policies and the use of resources

Teachers commented on the development of school policies and resources. School systems are important to maintaining the impact of the project. The key area that the teachers commented on was the improvement in resources (including equipment and books) that resulted from their school's involvement in the project. Over half (56%) commented on this. Areas that a smaller proportion of teachers (15%) mentioned included more money being tagged for mathematics, changes to reporting policies, reviews of the long-term curriculum plan, and more time spent on number.

Reasons for participating in the ANP

The main reason that principals (66%) gave for participating in the project was previous involvement in ENP. Some principals (18%) termed this "continuity after ENP", while for others (17%), the reason for participating was to improve the mathematics programmes in the school.

Parent reaction

Teachers and principals were asked to comment on parental reaction to the project. Nearly half the teachers (41%) and principals (46%) reported a positive parental reaction, while a small percentage of teachers (6%) and principals (13%) thought it had been mixed. Some teachers (23%) and principals (7%) felt that there had been very little interest shown.

Least helpful aspects of the project to teachers

The least helpful aspect of the project identified was related to the provision of resources. Many of these comments were about their late arrival. This issue is unlikely to arise again as it was part of the initial stages of a project.

Most helpful aspects of the project to teachers

The most helpful part of the project was variously stated to be the facilitator (34%), resources (29%), and the opportunity to gain knowledge (17%). Suggestions for improvements from all respondents centered on human and physical resources, with a key comment being "more time with the facilitator".

Concluding Comments

The sense derived from the questionnaire responses is that the project has been very successful in bringing about changes in classroom practice and school procedures and that these are reflected in better student achievement in mathematics. The following comments from principals and teachers are only a selection from many in a similar vein.

> *Only wish my child had the opportunity to be part of it*. (Principal)
>
> *Keep it going. All schools/teachers should have access to the ANP and ENP.* (Principal)
>
> *The method concepts are excellent. I love how even the slow students are getting to come out with a much higher level of understanding and how the bright students will know no limits. Fantastic.* (Principal)
>
> *An excellent project and one that has been very necessary for a long time.* (Principal)
>
> *Very worthwhile for pupils, staff, and BOT. Gives good data.* (Principal)
>
> *It was a big learning curve for staff – but all have agreed that the project has been a valuable one.* (Principal)
>
> *This is the most effective project I have been involved in since I began teaching. I am thoroughly impressed with it.* (Teacher)
>
> *Excellent project. Eye-opener for teachers which shows the impact it made on students learning. The huge progress children made at the final assessment. I was amazed at the way students were retaining the number strategies and working a problem out.* (Teacher)

Chapter Six: A Case Study of Teacher-led Group Discussion – Issues and Challenges for Low-decile Classrooms

Introduction

The goal of actively engaging students in mathematical learning is probably motivated by claims that exploring mathematical ideas using equipment and talking about mathematical ideas promote learning. Both these claims have a long history in curriculum documents in English-speaking Western countries. For example, the Nuffield Mathematics Project in the 1960s is regarded as having started the shift towards basing teaching approaches on practical experience (Cockcroft, 1982; Walkerdine, 1984). Since that time, emphases on working with equipment and engaging students in discussion have appeared in documents as distinct, as well as intertwined, themes.

A key claim about the value of discussion can be traced back to the Cockcroft Report (1982). Cockcroft (1982) recommends that "Mathematics teaching at all levels should include opportunities … for discussion between teacher and pupils and between pupils themselves" (p. 71). This has often been cited to justify small-group discussion. This early emphasis on discussion has continued in US documents such as *Curriculum and Evaluation Standards* (NCTM, 1989), which states that "Communicating helps students to clarify their thinking and sharpen their understandings" (p. 26). *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) also claims that "Learning to communicate about and through mathematics is part of learning to become a mathematical problem solver and learning to think mathematically" (p. 11). Parallel claims have been made about the value of working with equipment. The Cockcroft Report (1982) says, "Practical work is essential throughout the primary years … For most students practical work provides the most effective means by which the understanding of mathematics can develop" (p. 84). *Curriculum and Evaluation Standards* (NCTM, 1989) stresses that " … active involvement in, and expression of, physical manipulations encourages students to reflect on their actions and to construct their own number meanings" (p. 38). The New Zealand mathematics curriculum (Ministry of Education, 1992) similarly states that "Teachers know that the students are capable of solving quite difficult problems when they are free to use concrete apparatus to help them think the problems through". While this is suggested for younger students, the curriculum adds that "such an approach is equally valid for older students and should be used wherever possible" (p. 13).

Given these dual but possibly conflicting emphases in curriculum documents over three decades, it is not surprising that in the New Zealand teaching community there are differing views about what active engagement means. These views range from engagement in which physical action is emphasised (often referred to as "hands-on"), to engagement emphasising discussion, whether or not accompanied by physical activity. A "hands-on" approach has often been seen by teachers particularly as a means of catering for the learning needs of students from non-English-speaking backgrounds or of Māori and Pasifika ethnicity (Higgins, 2001). Visser and Bennie (1996) noted this narrow interpretation of "active learning" (in which discussion is omitted) in their evaluation of the *Beginning School Mathematics* resource. At the time of the Year 3 Mathematics Project in 1998, this was still a commonly held interpretation (Higgins, 2001). For some teachers, the activity became more important than the mental or intellectual engagement which the activity was originally said to promote.

Observations of classroom practice and interviews with and questionnaires answered by teachers suggest that the ANP has compelled teachers to confront this duality (as well as their own understanding of engagement). It has led many of them to reconceptualise their definition of active learning or "hands-on" to incorporate discussion. A model of teaching designed to provide opportunities for interaction is presented as part of the ANP. It involves the initial use of materials, the use of imaging, and finally the use of mathematical properties (see Chapter One for a fuller description of this approach). The role of the teacher is key to ensuring opportunities for students to explain their thinking. Students need to both listen to and give explanations of their problem-solving strategies if an inquiry-based approach such as the ANP is to be successful. Explanations are often aided or mediated by the use of equipment and written recording of the strategy. Sometimes the manipulation of materials and recording of the action is done by the teacher, sometimes it is done by the students, and sometimes it is done together.

The analysis of teacher-led mathematics sessions draws on the sociocultural theory of action. Lessons are viewed as a series of actions with an event used as the unit of analysis. An event is defined as a complete sequence of actions (within a teacher-led session) that has a beginning and an end. Typically within any one teacher-led session there could be up to nine events. An event commonly begins with the teacher posing a problem as an opportunity for students to refine their use of a particular strategy such as "groups of ten" for solving it. For example:

> Teacher*: We're going to say that these are some fish that have been caught. Now, groups of 10. Now we've got Tina here and she's caught six fish. Okay. We've got Mary and she caught seven fish on her fishing trip, and we've got Liam who caught only four fish. What do you think would be an easy way that we could work out how many they've got altogether? How many fish were caught?*

The notion of an event is useful for analysis as it enables examination of completed sequences of action with a particular focus on comparing interaction patterns across individual teacher episodes and between the six teachers in the case study. The utterances in each event have been numbered for convenience of referral. The analysis applies (to the teacher-led sessions in this case study) tools such as the interactivity flowchart from Sfard and Kieran's (2001) investigation of the interactions of students working independently of the teacher. The idea of the flowchart is to identify patterns of interaction that might otherwise remain hidden. Through applying this tool it may be possible to see the extent to which the teacher is listening and responding to the students' utterances or is simply "hearing nobody but herself" (Sfard and Kieran, p. 71). The analysis also incorporates the notion of norms of interaction (Yackel and Cobb, 1996) and the teacher strategies of *eliciting*, *supporting*, and *extending* from Fraivillig, Murphy, and Fuson's (1999) framework for advancing students' thinking.

The activity in any classroom event can be characterised by an intention and it is important that students make sense of the purpose of the teacher-led group setting if they are to get the most benefit from it. Learning can be viewed as the extent to which those watching the action understand its intention. Language used by the students is often the most obvious indication of this understanding, but it is important to consider all aspects of the activity, including the actions taken by the participants. This is particularly relevant in the case of the teacher-led mathematics lessons, which all used equipment and written recording. The micro-analysis of each event considers what was said as well as what was done to try to gauge the extent to which the student explanations were mediated through the teacher, the other students, and the materials. It also includes the action before and after sequences of speech as a means of establishing student understanding of the event. Patterns of teacher and student talk and action analysed across episodes are used to check for consistency in the interpretation of students' understanding. The analysis provides multiple perspectives from the inside views of the teacher and students and from the outside views of the researcher.

This chapter draws on interviews and classroom observations in two lower decile schools in South Auckland that were involved in the ANP. The case study focused on investigating effective ways of working with Māori and Pasifika students as well as those with English as a second language. Teacher/student interactions were recorded during three sessions in each classroom. The chapter also describes particular issues for low-decile schools.

Findings

The mathematics sessions in the six classrooms observed followed a similar sequence. The sessions were typically three-quarters of an hour to an hour long and took place on every day of the week. In all cases, the teacher started with an introductory sequence with the whole class and then divided the class into three or four instructional groups. While the teacher worked with one of these groups, the other groups worked independently on assigned tasks.

<u>Patterns of interaction</u>

In all six classrooms the sequence of interactions in the teacher-led setting was of teacher-student-teacher-student turn-taking. It was rare that students initiated an interaction or that they interacted with other students unless directed to do so by the teacher. On the surface, these teacher-led sessions could be thought of as simply being teacher directed, as in a transmission model of teaching. However, when more closely analysed it was the use that the teacher made of the student responses that proves to be an important difference between this and the traditional approach. An extract from an event in one of the classrooms illustrates the way in which a teacher can orchestrate a group discussion.

The event begins with the teacher working with five year 4 boys (Bruce, John, Martin, Terry, and Alan) who are using early part-whole strategies to solve number problems. They have been asked to solve the problem "23 and how many more make 78?", which the teacher has posed, giving them the information that "Freddo the frog's going to live at number 23 and he's going to move all the way up to number 78". The boys each have empty numbers lines (having no digits at all on them) on a sheet of paper to work on.

Once the boys have marked Freddo's move on their number line they explain their answers as follows:

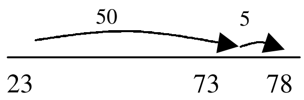
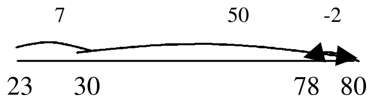
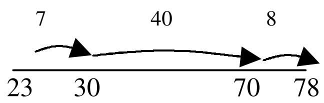
What is done	What is said	
	(21) Teacher:	Okay. John, are you going to tell us what you did? We're listening to John now. Okay, John's going to tell us what he did. See if you got the same answer.
	(22) John:	Um jump all the way to … aw, you started at 23, jump all the way to 73 is 50 jumps, then jump 5 more.
	(23) Teacher:	Okay. Good. And what was your answer?
	(24) John:	55 jumps.
	(25) Teacher:	Okay good. Bruce, do you want to tell us what you did?
	(26) Bruce:	Start at 23, take 7 jumps, then take 50 more jumps, which would take us to 80, then jump 2 back.
	(27) Teacher:	Okay. Good. Martin, what about you? Ooh, we've got lots of different ways of doing it today.
	(28) Martin:	Take 7 steps takes you to 30, and takes 40 steps that takes you to 80, and take, oh, 70 and take 8 more to 78?
	(29) Teacher:	Have you got the same? Alan what did you do?
	(30) Alan:	The same.
	(31) Teacher:	The same again. Aw, good. Is it easier or harder without those numbers?
	(32) Boys:	Easier, easy, Miss.
	(33) Teacher:	You reckon it's easier?
	(34) Martin:	Harder, Miss.
	(35) Teacher:	It means you have to do a lot of counting in tens.

Figure 6-1: Event 1 – protocol

The teacher has previously introduced the number lines without the tens marked on them. The extract begins immediately after the teacher has given the students time working individually or in pairs on their solution. The teacher asked John to explain the way he solved the problem of "23 and how many more make 78?". She reiterates the classroom norm that everyone listens to John's response (utterance 21). She prompts John to complete his explanation (the first for this problem) by asking how many jumps it was altogether (utterance 23). By establishing that the answer is 55 early on in the sequence of interaction, the focus of the discussion can then shift to different ways of solving the problem. Her prompt, "See if you have got the same answer" (utterance 21) sets up the expectation that the students may have different ways of solving this problem. Her question at the end (utterance 31), despite the different answers (32 and 34), enables her to state that the reason for the empty number line was to force the students to count in tens (utterance 35).

The following figure shows the results of using Sfard and Kieran's flowchart. The figure shows that the teacher is managing the flow of explanations offered by individuals in the group. This is most clearly seen in the gradient of the line as well as the direction of the arrow. A circle indicates the initiator of the interaction and the arrowhead the recipient. This can be seen from the gradient of the line as well as the direction of the arrow. Different gradients indicate different interaction flows, for instance a downward sloping arrow indicates that the interaction is from teacher to student; conversely, an upward sloping arrow indicates the interaction was from student to teacher; and a horizontal arrow indicates the interaction is between the teacher and the whole group, or vice versa.

In this part of the event, the pattern of teacher interactions (apart from in the initial interaction with John) is in sequences of single teacher utterances with each child. This is in contrast to other events in about half the classrooms observed, in which teacher-student sequences were typically longer, having somewhere between three and five interactions in a teacher-student sequence. The teacher utterances to the whole group, labelled "joint" on the flowchart (see Figure 5-2), illustrate the typical pattern for all the teachers observed, by occurring at the beginning or end of events (utterances 31, 33, 35). In the extract presented, all the students except Terry get a turn to present their solution[1]. The student's own drawing of Freddo the frog on the empty number line (see the "what is done" column of the figure above), as well as the teacher's drawing of their solution on a whiteboard as they speak mediates the presentation of the explanations.

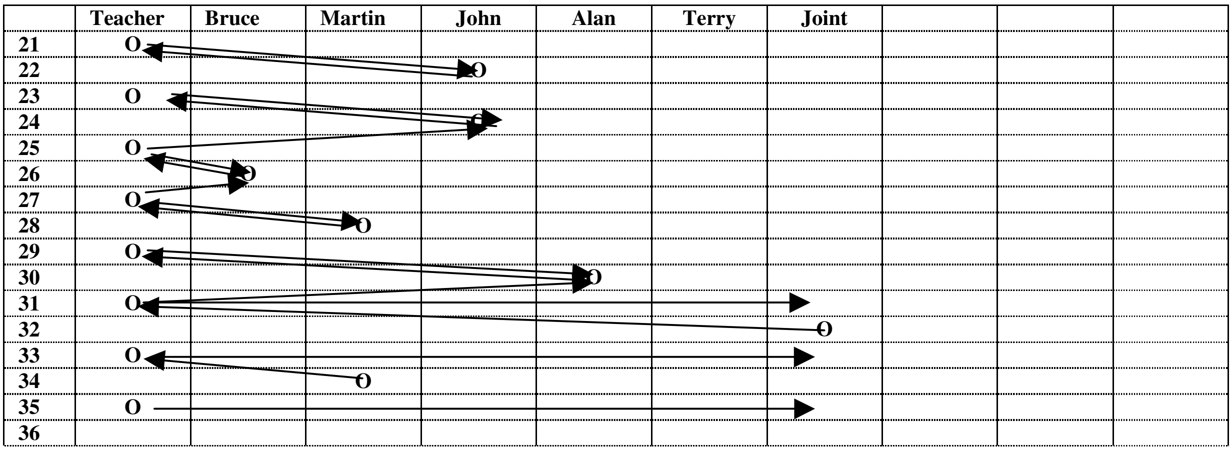


Figure 6-2: Interactivity flowchart of part of Event 1

The use of artifacts, in this case the empty number line drawings, as a reference point shapes the explanations given by individuals (Werstch, Del Rio, and Alvarez, 1995). The drawings also have the potential to shape the way in which members of the group, including the teacher, understand the explanations. It is, perhaps, the teaching strategy of orchestrating small-group discussion (Fraivillig et al., 1999) that has enabled teachers to reconceptualise their understanding of "active learning" to include discussion of ideas. This discussion has allowed teachers to see how students' explanations of their strategies can be transformed through the use of equipment, including the stage of mental imaging. This changed use of equipment has been described by Gravemeijer (1994) as a "thinking" (as opposed to a "working" model) of mathematical ideas. However, the point that Werstch et al. (1995) raised about mediation being an active process needs to be kept in mind.

> While the cultural tools or artifacts involved in mediation certainly play an essential role in *shaping* action, they do not *determine* or *cause* action in some kind of static, mechanistic way. Indeed, in and of themselves, such cultural tools are powerless to do anything. They can have their impact only when individuals use them. … Even the most sophisticated analysis of these tools cannot itself tell us how they are taken up and used by individuals to carry out action. Instead, mediation is best thought of as a *process* involving the potential of cultural tools to shape action, on the one hand, and the unique use of these tools, on the other. (p. 22)

[1] Terry does present his explanation later in this teacher-led session.

The rest of this section consists of an examination of the use made of artifacts in terms of the students' and teacher's understanding of the activity setting – in this case the small teacher-led group. While we are aware that the typical pattern of interactions is teacher-student-teacher-student, we need to know more about the forms that the teacher-student interaction takes. This will be the principal focus of the analysis of the events that follow.

Different forms of teacher-student interactions

The forms of teacher-student interaction varied with different groups of students. Newman, Griffin, and Cole (1989) note that this variation depends as much on the student approach as on the teacher's action. Some of this variation arises from the extent to which the teacher and members of the group can interpret a student's response. Newman et al. (1989) suggest that this can lead to "important differences in the form of the interactions across groups" (p. 108). In particular and pertinent to the observations in this case study, it was the teacher-student interactions around errors that best illustrate these differences. Newman et al. (1989) note that "it was the teacher's and students' efforts to resolve these errors which produced variations in the form of the problem-solving interactions across groups" (p. 109).

In the cases where the student gives the teacher very little feedback, it leaves the teacher with nothing to work with. Newman et al. identified two key points worth considering. The first was that, "Many of the seemingly incidental aspects of a setting can affect the dynamics of an interaction leading to significant differences in cognitive outcomes" (p. 111). The second was that where the student's actions were uninterpretable "the teacher had nothing to appropriate and had to revert to something that had happened previously "to re-establish a basis for interaction", and "without interpretable productions by the student, the teacher can only guess at the appropriate next move" (p. 112). They suggest that the differences in instructional interactions lead to differences in outcomes, but that these are "not a direct result of the students' entering abilities" as "there can be many reasons for a student's failure to provide adequate feedback to the teacher" (p. 112).

A later event in the teacher-led sequence with the group of boys just discussed is notable for the element of competition that had by this time built up among the group of boys. Bruce, in the intervening event, suggested an incorrect answer to the problem 18 + ? = 59. His explanation of how he arrived at this incorrect answer was "You go all the way in there, all the way round there and into there, which goes into that" (he was reading off his number line as he said this). This explanation contrasts with his explanation in the event just discussed (see Figure 6-1, utterance 26).

This example appears to illustrate several of the points above. Firstly, it illustrates the different ways in which individuals use the artifact (the number line) by shaping rather than determining or causing the action taken (Werstch et al., 1995). In this case the number line has not helped the teacher or other members of the group to interpret Bruce's explanation. This appears to have been a case where the teacher was left with "nothing to appropriate", so she moved onto another example (discussed below) to "re-establish a basis for interaction" (Newman et al., 1989).

The reasons for this change in Bruce's behaviour can only be surmised, but the teacher suggested that it was perhaps the competitive nature of this all-boy group that had led to a the development of a certain level of bravado – trying to be quickest at arriving at an explanation that contained "the biggest jump". This perhaps illustrates Newman et al.'s other point about the effect of incidental aspects of a setting on the dynamics of interaction.

The event discussed below provides further evidence that this group of boys had negotiated the teacher-led group as a setting of competition between them.

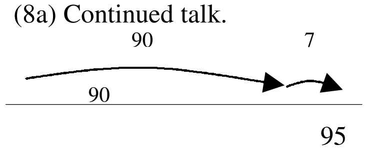
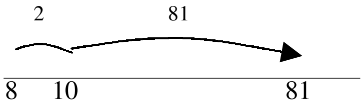
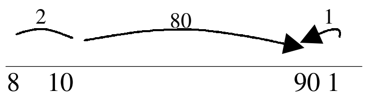
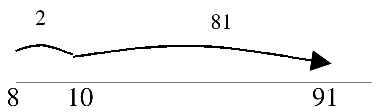
What is done	What is said	
	(1) Teacher:	Okay. I'm going to give you another one together then I'm going to give you some to do by yourselves or with a buddy. Alright. At the front again. Aw, what'll be a good number to start it this time. A really little one – number 8. And let's go all the way up to …
	(2) Martin:	90 something.
	(3) Teacher:	90 … 1.
(4) Students talking and working it out.	(4) Alan:	Miss, I did a massive step.
(5a) More talk as they work it out.	(5) Teacher:	You've done a massive step. I know that's a big jump for Freddo the frog. He's pretty clever this frog.
(6a) Talking continues	(6) Teacher:	It's okay, John? Work it out. We got plenty of time.
	(7) Terry:	I took a massive step. I took 81 steps.
(8a) Continued talk.	(8) Teacher:	You could take it to 80 and then one more …
	(9) Teacher:	Okay. Are we all finished? Bruce, what about this time? Oh, he's gone all shy on us. Right, Alan's going to tell us. Okay, Alan.
	(10) Alan:	You start off from 8, you take 90 steps all the way to 98 and go back 7 which takes you to 91 and altogether is 83.
	(11) Teacher:	Okay. Good. Aaw. Terry, are you going to tell us what you did?
	(12) Terry:	I took 2 steps to make 10 and I took a big 81 steps to 91 and that equals 3 steps altogether.
	(13) Teacher:	Okay. Sort of similar to what … Martin, you tell us what you did.
	(14) Martin:	I took 2 steps, that took you to10, and took 80 steps, takes you to 90 and took one more, equals 83.
	(15) Teacher:	You can see how Terry and Martin did it a little bit different. What do you think Terry did?
	(16) John:	I didn't hear it.
	(17) Teacher:	What did Terry do? You tell us again Terry in a loud voice so John can hear.
	(18) Terry:	I took 2 steps to make 10 and it took one to 81.
	(19) Teacher:	Okay. You took 2 steps and how many steps?
	(20) Boy:	81
	(21) Teacher:	81 steps. What did Martin do that it was a little bit different?
	(22) Boys:	2 steps then 80 steps … he cut the 81.
	(23) Teacher:	He broke the 81. Down into a big step of 80 then a little step of 1. Okay. Whereas Terry did it in 1 …
	(24) Terry:	Big one.
	(25) Teacher:	… jump.

Figure 6-3: Event 2 – protocol

It seems that the boys set the stage for their competition by choosing 91 as the end point to Freddo's jump starting at 8. Alan's initial comment (utterance 4) can be seen as a rally call for the others to compete with more massive steps. While all the students appear to have an understanding of the activity setting, Bruce and John (maybe affected by Bruce's demeanour) do not participate in the interactions in this event (Newman et al., 1989). The three different solutions are supported by the boys' drawings. The teacher draws the group's attention to the differences between Terry’s and Martin’s solutions (see utterance 15 on). The teacher is unsuccessful at getting Bruce to interact in this event, highlighting the point made earlier that the teacher is to some extent reliant on the student’s willingness to contribute and ability to explain their solution strategy clearly.

In this sequence, the form of the teacher-student interactions appeared to be governed by the competitive atmosphere of this group. As with the previous example, this resulted in short teacher-student interactions. This can be seen on the interactivity flowchart (see Figure 6-4).

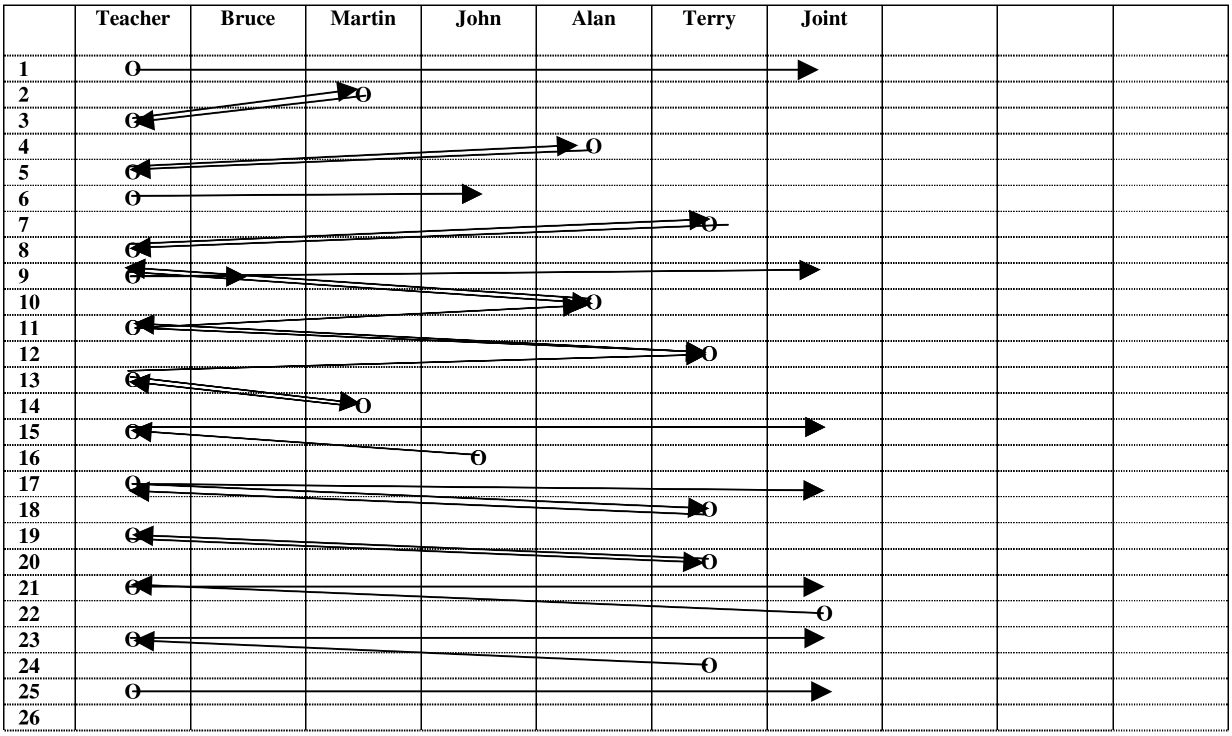
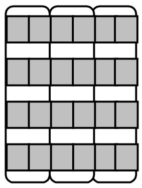
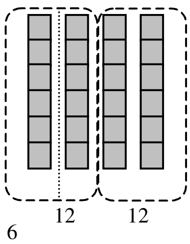


Figure 6-4: Interactivity flowchart of Event 2

Teacher-student interactions took a different form in an event in another classroom. These students were a year or two older than those in the previous example; they were at a more advanced strategy stage and were working in a different operational domain. The greater difference between the groups, however, may have been the second group’s orientation towards co-operation rather than competition. This could be seen as incidental to the mathematical task in hand (Newman et al., 1989). The students have just finished an example using 16 interlocking cubes, hence the teacher’s instruction (utterance 1).

The students worked in pairs. There were three pairs in all, one pair of two boys, one with two girls, and one mixed pair. Each pair of students made one model, rather than a model for each student. This decision by the teacher may have led to the co-operative stance taken by the students. In this event, the teacher establishes a pattern of leaving the students to work together with the equipment, and then, after a few minutes, inviting explanations from each pair.

What is done	What is said
(1a) They count as they're doing it … just add 2 on each one … oh, that'd an easy way … just take what you've got and add 2 on.	(1) Teacher: Okay, I want you to take altogether 24 cubes. So you've got 16 already. Get some more to make 24.
(2a) They talk … its like 3 quarters of 24 … oh, like you can just separate it into quarters then count 3 quarters … or else you can just 1 third of 4.	(2) Teacher: Okay, what I want you to do now. Now that you've got your little grid thing going, I want you to find … how are you going to find three quarters of 24? You've got 24 cubes there. How are you going to find 3 quarters of that? Have a think about it and do it.
	(3) Jack: Oh, half the 24 and then, which is 12. Half it, which is 6 and then add 3 of them together. (4) Teacher: Have you finished? (5) Teacher: Okay. 3 quarters of 24. What did you do to find that out? George. (6) George: You have 6 rows of 4 and you just divide it by 2 rows. Like you 2 rows and 2 rows and 2 rows. So that'll be 2 sets of 4 (in a row). (7) Teacher: We want, when you look at that number, how many parts are you dividing it into? (8) George: 4. (9) Teacher: 4. How many parts have you divided it into? (10) Mary and George: 3.
	(11) Teacher: Do you want to have another go at doing it? (12) Mary: To get 3 fourths we, um, like we just like know that there's 24 here … (13) Teacher: Okay, while you're trying to remember pass it in to Vee and Jack. What did you do? (14) Vee: First of all, we halved it. (15) Teacher: Actually you can all just quickly listen. Just put your things down. (16) Vee: First of all we halved it to make 12. And then we halved it again. (17) Teacher: Okay, half of 24 equals? (18) Vee: 12. (19) Teacher: And then what did you do? (20) Vee: We halved it again to make, um 6. (21) Teacher: And why did you halve it again? (22) Vee: So we could put (Jack says "3 other halves") 3 of them together to make 3 quarters. (23) Teacher: By going half and half you were putting it into …

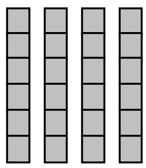
	(24) Vee:	Yeah.
	(25) Teacher:	So, how much is 3 quarters?
	(26) Vee:	18 – Jerry agrees.
	(27) Teacher:	Did you all understand what Vee and Jack did? Carrie and Jay. What did you do?
	(28) Carrie:	We just did like, 3 fourths, 3 fourths of 24 (she laughs a bit).
	(29) Jay:	And we used the fraction sign as the takeaway sign.
	(30) Teacher:	Takeaway?
	(31) Jay:	3 takeaway …
	(32) Carrie:	4 takeaway 3 (thinking 4-3=1)
	(33) Teacher:	Equals?
	(34) Carrie:	1.
	(35) Teacher:	To leave you what? What does that 1 represent?
	(36) Carrie:	6. A one 6.
	(37) Teacher:	Is it representing ??? So you've gone 4 takeaway??? You've used a fraction to do a subtraction.
	(38) Carrie:	Oh that rhymes!
4 rows of 6	(39) Teacher:	So 4 takeaway 3 equals 1. That's what you said. Right? What does that 1 mean?
	(40) Carrie:	One whole thingy. Cos it's like, separate now. So this is the whole bit.
	(41) Teacher:	Okay, can you tell me the fraction for that one?
	(42) Carrie: 1 sixth. This thing.	
	(43) Teacher:	But we were finding out 3 quarters.
	(44) Carrie:	See there's 4 quarters in a whole and that makes … now I remember!! We had 4 quarters and then we took 3 quarters away.
	(45) Teacher:	So what's that one that's there?
	(46) Carrie:	1 quarter.
	(47) Teacher:	1 quarter! Good girl.
(48a) They all laugh.	(48) Carrie:	Oh, clicked!

Figure 6-5: Event 3 – protocol

The students and the teacher all appeared to have a common understanding of the activity setting, which was to offer explanations of their actions to the group. George and Mary, in the first explanation offered to the group, suggested a way of partitioning the 24 into sets of 8 made up of 2 rows of 4 (see utterance 6). The teacher referred them back to the fraction (see utterance 7) to establish how many parts in the whole (see utterances 8 to 10). The teacher's expectation that they might need more time and another go at solving it had already been established as a socio-mathematical norm with this group of students (Yackel and Cobb, 1996).

Vee and Jack's easily interpretable explanation provided the teacher with material to appropriate in working with the two pairs who were still unsure of how to solve the problem.

The most remarkable sequence in this event is with Carrie and the teacher (see utterances 28 to 48), culminating in a memorable "aha" moment –- treasured in teaching (see utterance 48).

Carrie reflected on her achievement both immediately after the event:

> *Click (laughs). … Remembering and like how I started getting the fractions in my head. Aha. … It was like the three quarters and then I clicked when I heard there were quarters so separated into quarters and then took three quarters away and then clicked it was one quarter. … when I like, go mmm in my head three* quarters *instead of just like doing what I did before.*

And again the next day.

> *Well I found useful the way that it's said, because if you think it, like yesterday how we had three quarters and I just thought cos the way I saw it was three fourths but then as it was said it was three quarters so that made it useful knowing that there's like four quarters in a whole and it started making me get it more and making me remember and, like know how to do it properly. … And another way of it being useful to me is, like it helps me understand so I can learn more about it and learn better things about it instead of just staying stuck on thinking it's three fourths.*
>
> *Unifix cubes are really helpful because if, like you can't do quick equations on your head, you can add them up. Like how we did that three quarters. I did all together and then make it into quarters separate, so one quarter, one quarter, one quarter, one quarter. And then I counted three and then took three quarters away. That's how I knew it was three quarters.*

Vee whose explanation to the group was the catalyst for Carrie's understanding had a similar interpretation of the event.

> *It helped Carrie and Jalie because they didn't know what to do and it helped them understand what you had to do. So, er, like, if we didn't explain it they would have still been stuck trying to figure out how they could put it into four groups and it would've tooken them a lot longer to figure it out.*

The interaction flowchart (see Figure 6-6) illustrates the pattern of sustained interactions between individuals (or pairs) and the teacher. Apart from some interactions between the teacher and the group as a whole at the start of this event (utterances 1–5), there were two long and one slightly shorter group of interactions in the rest of the event. The form of these teacher-student interactions varies greatly from those presented earlier.

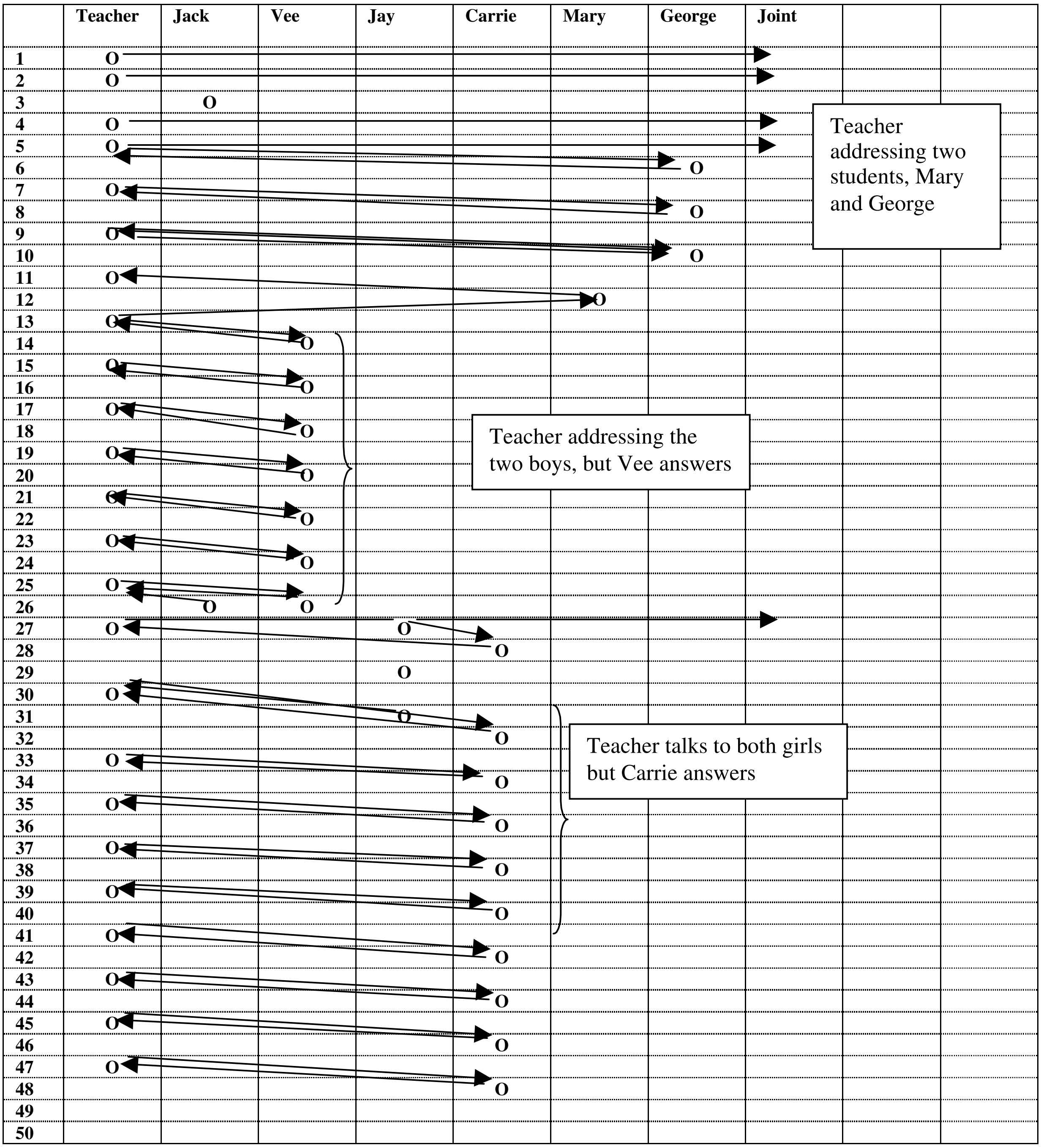


Figure 6-6: Interactivity flowchart of Event 3

So what were the factors that allowed the students (and Carrie in particular) to interact with the teacher over a sustained period? Some factors appeared to be to do with the students' effective goal-directed behaviour (Newman et al., 1989). The students' comments in interviews the day after this event provided further evidence of this. Carrie's comments suggest that not only does she have goal-directed behaviour, but that she also understands how the teacher needs student interactions as a basis for helping them to understand the mathematics.

> *Talking can help you get the answers because you might, like, misunderstand. Like with three quarters, you might think it says three fourths like me. So listening is a good thing to do and talking with the teacher so if you understand it, say something so you can get*

help on, like, what you're doing. And watching. Like watching what to do with the little things that you get. Yeah, mmm.

Vee also understood the teacher's role:

Well, if the teacher just did it, it wouldn't be teaching, because it'd be like the teacher would already have been telling you, but from the other, from the kids, telling you, like, you get more of an explanation because they don't actually know it, they have to figure it out.

He also made insightful comments about working co-operatively:

Well, if you're working in a groups you have to co-operate with your group and not be by yourself and you have to share your ideas. You have to give your ideas to someone else – don't keep it to yourself. So if you're working in a group you have to co-operate as one group not as individuals.

Concluding Comments

The teacher's role is to manipulate the situation so that students can engage intellectually with the key mathematical ideas. This will almost always involve discussion, frequently with other students, in a controlled, non-competitive way. It may also require the teacher to use equipment or other devices to ensure that she or he can comprehend the students' understanding (or misunderstanding) of the ideas.

Chapter Seven: Views of Active Learning in Low-decile Schools

Many of the teachers participating in the Numeracy Project commented on the importance of "hands-on" or active learning. In the case study, it was apparent that active learning had been reframed to include a greater emphasis on discussion as well as work with materials. The teaching model and the framework promoted in the project underpinned this shift. Perhaps the greatest change has been in the way in which the teachers conduct the teacher-led group. Teachers, particularly in the middle school (years 4 to 6), have often viewed group work as an opportunity for them to explain ideas to those students at a particular stage of learning so that the students can go away and individually complete written assignments in their books. A member of the senior management at one of the case-study schools explains how she sees this change:

> *So I think it is that change into group situations, group teaching and using equipment, and the whole change in the thinking of how maths is taught. … There was some resistance to it at the beginning, and I think once they started trying it out in the classroom, and they saw how it worked, I think that change in their attitude and as a result the changes we saw in the students' attitude towards maths and their enjoyment, because in senior school it was more bookwork than using equipment previously. … I think that the students enjoy it much more now when they can manipulate the equipment and learning that way. They learn easier and their knowledge is increasing quicker and their strategies as well. … now they are going into these detailed explanations of how they worked out the problems.*

The teachers talked about their intention of getting students to share their ideas. They described this as one of the key changes they made to their teaching as a result of the project. Mary commented that her emphasis on discussion had intensified with the project:

> *… getting them to share and discuss and share with each other and discuss with each other what they have learnt and how they have done it and then share with the group … I probably do it a lot more often now than what I did before. … I think that it's really important … because they bounce ideas off each other and, I mean, that's how I learn as well. …*

Esse felt that one of the most successful aspects of the project for her had been getting students to discuss mathematical ideas:

> *I think the most important thing I found out about this project is the students. They are very good at explaining what they are thinking. I give them a problem: for example, I say that I had ten apples and then I get another fifteen apples, how many apples do I have altogether? And then I give them time to think about that, and they put their hands up if they know what to do with it and then explain it.*

For Anne, it was important to have a balance between working individually and working with others:

> *… You give [all kids] time to think about it on their own and then time to work together.*

Factors Inhibiting Change

Participants in the case study identified several key issues associated with changing the characteristics of the teacher-led sessions. These included the confidence of the teachers in terms of their understanding of the mathematics being taught, the confidence of the students in explaining their mathematical thinking in a small-group situation, and the expectations of small-group learning that were held by both students and parents.

Teacher knowledge and confidence

Insufficient teacher confidence in their own mathematical knowledge appears likely to be a reason teachers have not emphasised discussion as much in the past. Mary described how this shift occurred for her:

> *It's made me more aware of how I actually do things in a small-group situation. … it's that whole hands on … doing it … talking … discussing … … Whereas before it was probably … because it was a confidence thing and I just want them to stay on track and do that and I don't want to get side-tracked and things like that …*

Chris also commented how the framework had helped her to focus in the small group sessions:

> *Having a framework, doing the initial testing and having groups on the mat where you interact with the students in a smaller group rather than the whole class or you interact in a group that they are having common boundaries, common areas, they make common mistakes … then you understand that, hey, they need another approach. They need a different strategy basically. That is what has helped me and I realise that oh, let's not ask them to do level three stuff if they cannot do level two aspects. If they haven't got the foundation they can't operate, they can't perform more complex operations in mathematics.*

Students' confidence

Students' confidence in speaking in front of a group is an important issue to address if this model of teaching is to work effectively. All participants in the case study thought that peer discussion within the teacher-led group session was a useful way to build up students' confidence. Senior management at one of the case-study schools commented, "Well, I think it works very well, because it's just one-on-one and they just listen to one person and they learn from that one, and the value of the peer teaching … is really, really good". Esse, one of the teachers from the same school, explained how the students' involvement in the project generally had helped them to overcome shyness in speaking out:

> *The other core thing about this project is that there are students who were too shy to talk, and now they are able to explain, express themselves. … building up their confidence to talk in front of other students. There is one boy in my class that … he's very smart, he's smart but he is shy, he's so shy to talk but now he talks. Yeah, yes. And sometimes there are things that I didn't think of and the students brought them up.*

One way that Hettie and other teachers in the case-study schools found of managing the risk was to give students the opportunity of working with a buddy on the problem first and then move to the larger group discussion:

> *… if they don't want to tell me, maybe they feel comfortable talking to a buddy. I suppose it's getting over that thing of taking a risk and not being wrong in front of everybody. They can talk with their buddy that they feel comfortable with, and then maybe if it's wrong then their buddy will assist them, and I guess in a comfortable*

situation, rather than me saying try again. But I think definitely with those kids that are perhaps not as confident … sharing with their buddy is a safer option for them I think than … talking with the teacher.

Anne explained how she did this:

I also think it just takes the pressure off the kids a bit when they can tell somebody else, and if one kid's a bit lost the other kid tries to explain to them, so that it benefits their learning because they have to verbalise what they think … so it's not just the person who is stuck getting help … it helps the other one too. Pasifika kids … you should do much more co-operative work … culturally that is what they are used to. … Yeah, and I just think that they are used to talking to each other and quite like solving things together, and they can be helpful to each other.

Chris also commented that she used other students to explain concepts:

… maybe students sometimes do not understand what you have said. They would probably better understand their peer better than they would you … To teach students of 10 years old you have to sometimes think and talk like them, and so they understand you better. Whilst we have … got to be good role models … I have partners talking to one another because sometimes they have a better understanding and they think through together better. … They have a lot to offer, they support one another, they are not afraid to take chances because they have somebody alongside them.

Lynette talked about the importance of the relationship that a teacher builds with their students:

It's just the understanding and the relationship that you build with your students. Like if they feel confident enough with you then … yeah, definitely you are going to see lots of things happen, but if you are one of those teachers that don't show you care and that sort of thing … kids pick up on that. And if you are the grumpy and the grouch … they are like "Well, if I say something, you are just going to growl at me anyway". You have got to know your individual students and you have got to know how to cater for them. You know … culturally … self-esteem and all that sort of thing. It all ties in, definitely. With my students, they need a lot of the positive praise because a lot of our students don't get it. They hardly hear it and when you praise them all the time you just see their eyes look up and they think "Wow I am going to try harder and do that". Even is if it is just a little praise. … So it comes back to individual students and knowing them as well.

She said it was hard getting students to explain themselves at first, "But the more practice I had and the better I got at it and so did the students … and the more comfortable they felt in sharing". About shy students, she said, "Yeah you have just got to build them up slowly, basically. Eventually they do share". One strategy she thought helped was: "When a student explains itself I always try to write it down and then we refer back to it. … And then they have got them there for referrals and they can read back and go, "Oh yeah I explained that and that's how I worked that out". … I learnt that if you put their name down to it they take a bit more responsibility.

Hettie talked about establishing the norm (Yackel and Cobb, 1996) that it is okay to make a mistake and that the most important thing is to have a go at explaining your thinking.

> *Just thinking about those kids who are unsure, if they are on track and unsure if they really understand the concept … often won't take that risk first. … So it's probably, I guess, somehow encouraging them to speak up. Have a go and setting up the way that it is okay to make a mistake. … I mean they know that Jack or Aaron … have probably got a pretty good idea. … I think they are working it out in their head but they will often hang back to hear what the others say and then think, "Oh yeah, I am on the right track and so now I will say what I was going to say", rather than going first perhaps.*

<u>Students' views about teacher-led group work</u>

Being able to justify their answers is important.

> *… there could be hundreds of ways of learning things and there is no wrong way and I don't want my kids to think that there is a wrong way. I think it's really important that they are open to whatever ways that there are and whatever way they do it … as long as they can justify it and all that sort of thing …* (Mary)

> *… the challenges for the kids … the year 5 and 6 where they have had four or five years of teaching in a different method and that is quite difficult for them to get out of that way of thinking. Just to get them to explain how they are thinking is a real challenge for them …* (Anne)

> *[The biggest teaching challenge is] getting the kids to really think about what they are doing and getting them to say what they are doing … getting them to think about. … it's not really the answer that I am interested in. Because often they will just want to give you the answer, but actually to change their thinking and that … although I am interested in the answer I really want to know how you got it and that's probably the hardest.* (Hettie)

Lynette explained in some detail how it took time and "prepping" to establish new expectations of students that they explain how they came to their answer:

> *When a student comes up with an answer, basically you try and say "Well, how did you work that out?" and put it back onto them. Like before we even started the project we used to do that and they found it quite hard trying to explain … like "Well, what do you mean? I know the answer. What do you mean? Explain it." But with us doing it, and a lot of reinforcing of saying, "How did you work that out?" "Oh that's really neat." "How did you get that answer?" And it's just prepping the student and building the student up to be able to share the information, even though some of the information that they share isn't quite correct. … With some students it was easy to explain how they got the answer, but some find it quite difficult to do because they can do it in a written form but for them to explain it was quite hard for some of my students to do. So it's a lot of prepping them and giving them a lot of things to do so it's building them up to be able to explain those things and it comes from modelling as well. When you say to some students, "Explain what you mean", they sort of look at you as if to say "Well, what do you mean? I have just given you the answer?" I have found during all those sessions that a lot more students are willing to share, whereas before they would just sit back and "I am not going to share".*

It was quite good as well with students, sharing their ideas, that you got the different ways of how they did work it out. So it was quite interesting. Back in those days, you used to say to students, "Oh yeah, what's the answer?" They would give you the answer and that was it. You didn't ask them "How did you get that?" or "How did you work that out?" I found it really quite good for myself to find out how the kids are working it out in their mind. … You try to build them up slowly. You don't force them because that is the worst thing, as they will just go "Aaahhh, I don't want to share". … It's hard sometimes, because you really want that kid to share, but they might be able to do it in written form and show you in written form. (Lynette)

Hettie explained how some students need a lot of support to formulate explanations:

… okay tell me what you are doing and lots of feeding and what did you do next. … Yes and I suppose in a way I am spoon-feeding what I want them to say as well, because they won't probably come up with it by themselves. Whereas the top groups have sort of clicked onto the idea about explaining their explanations. … I guess modelling is what you call it … "This is what I did; what did you do?" And using the language that I want them to use, "I took three jumps and then I … ". So … I guess things like writing it down when they are telling me, or doing it when they are doing it … "So you did this" and rephrasing what they have said to put it perhaps in a better explanation, like taking what they have said but putting it into the language that [makes sense]. … Writing up the different ways, because I think it helps them to see it as well as hear it and put it into a maths form. Even they were saying it takes three jumps … it's plus three or whatever. … I think by me doing it [with equipment] as well I kind of get a better understanding of what they are trying to say to me, but I am also checking it as well and, I guess, modelling to the kids that what they are saying is valued as well. … I am actually interested in what they are saying. [With other groups] … because they are pretty switched on kids that, I suppose, we go a lot faster with that group, and so you don't spend perhaps as much time going and rephrasing and all those things, because they are all sort of on the same wavelength and you can go a bit faster with the. … Whereas with the lower level kids, it's perhaps "Show me what you are doing and then come back".

Chris also commented that language was a challenge for some students:

… having a section like fractions and decimals, which is very heavily dense and weighted on language, unlike other areas, like in multiplication we have a lot of symbols. In fractions, we have a lot of language, and some of our students might not have the skills to understand that language, or the way that it is asked, or the way it is explained, and so you need extra help with that.

Establishing a new set of classroom social norms around questioning (Yackel and Cobb, 1996) is a really important part of establishing effective discussion. Lynette explained how she did this:

… that's when you prep them and that's when you teach them that it is okay to question, it's okay to ask questions if you are not sure, or if you want to share an idea it is okay. We are all going to listen, and we all respect one another's opinions, and so you have to build your kids upon that as well.

<u>Re-educating parents about teacher-led group work</u>

The maths book is a common way for parents to gauge how their child is doing in maths. Increasing the emphasis on discussion both in teacher-led and independent groups often means that there is less in the students' maths books. Lynette talked about the need for parent education about this change in emphasis:

> *So the best success for me is to see that the students have actually learnt something from it. That is my biggest success. … I have changed my style of teaching maths. I found that quite good. … I think it's making me more aware of what I should be doing. Just trying to bring out more language from them whereas before it was a lot of bookwork, but now I found that we hardly do a lot of things in the books. That's not good for parents, because the parents like to see a lot of bookwork. You have got to send their books home at the end of the year … but, hey … where is all the work? I think parents need to be made aware of how the programme goes as well. I found it good but I found it challenging and I am still learning.*

How the Teachers Changed Their Views and Practice of Active Learning

<u>The teaching model</u>

The teaching model provided a structure for active learning that enabled teachers to reformulate their views of active learning. They saw that they needed to have another step beyond getting the students to physically manipulate the materials. Anne explained how she came to see imaging as an important step in developing students' understandings of the properties of number.

> *Yeah, and that's something that the ANP has come through with. Kids have to physically manipulate and understand a concept first, because they can go to the number properties, and I think the big difference with the ANP … is the imaging part. That's like the crucial part now for kids – actually understanding the concept, and I think that most people have gone from, "Okay, you can use the materials and then you do number properties" [to] using the materials and then … say to [the kids] look at that picture in your head when you are trying to solve something. … That is what I am trying to do.*
>
> *I had never thought of the middle section [imaging] before … I think I always understood that kids had to manipulate materials first … to get the concept and then when you have done that part and you can do that problem. … [What is different now is] the imaging part. I think that you then use what you have taken from the materials, and then you take that a step forward. So you can shield the materials but they are still there if they need them … but encourage [the kids] to get the picture of that concept in their heads or the number in their head that they can then use.*

Linking the work with equipment by using written recording is important. Lynette explained how she did this:

> *We are doing fractions, and we had to use Unifix blocks, and then they had to share them out between four people … we had biscuits. We used the equipment, and then we said, "Well how can we record this?" and that's when you go through your written form and show them how to do that. So you interchange between the written recording and working with equipment.*

Chris uses the equipment to help students think through the mathematical processes and as a basis for explaining to others what they are doing. In this sense she's using the equipment as a *thinking* rather than a *working* model to teach the concepts.

> *Yes the ANP project has helped [kids] understand some of the processes simply because kids were able to manipulate equipment that helps them learn better. I find that it helps them through the mathematical process simply because they were able to work through the process in terms of using the equipment. … Like those two boys I had, … they were able to get those parts of the number … and by them doing it and speaking out and talking about what they did, others were able to catch on to the process.*

<u>In-class support</u>

For many of the teachers in the case-study schools, the facilitator provided useful modelling of questioning styles to elicite explanations of mathematical thinking from the students. Hettie explained how this happened for her around different types of equipment:

> *… how to question those kids to get … the explanations and I suppose that that was probably the most useful, and just coming to grips with giving the tens frames out and how you initiate that conversation with those students about what's happening with those. It was quite hard to get them away from … "Don't worry about what the answer is, just tell me what you are going to do to get there."*

Esse commented that she learnt from the facilitator's modelling with her own class. For Lynette it was the nature of the facilitator's feedback that was useful:

> *Coming in to do modelling: I have found that to be really beneficial … and getting her to observe me as well and give tips to me. … She gave quite a bit of feedback. She gave the positive feedback and then she gave suggestions … "How about trying this or trying this" so I found that quite good. … The positives were how I had the students set up and in the activities so that I could work with the group. … while you were working with a group [it was important] that you ensured that your students were doing meaningful activities but activities that they could do independently without bothering you. … It definitely helps, because once you have kids coming up to you in your group that just cuts your teaching time down definitely. … Also she said about getting feedback from students … she said that there was a lot of drawing out and getting them to explain so that was quite good. Also, the recording she said was good, but she said I should have had it up … because I like to work on the floor with my students and so she said I should have had it up and displayed it so that students could refer to it … and I said "Well, that is good for me to know". I thought that she gave quite a bit of feedback.*

The senior management at Esse's, Lynette's, and Hettie's school commented similarly:

> *I think it's been the in-class modelling, and showing teachers exactly what to, what questions to ask and how to ask, how to get the knowledge out of the student.*

Chris also commented that "the in-class modelling obviously broke that down for me in terms of how to perhaps use it as part of the classroom practice". For Mary, it was the facilitator's modelling that helped her to do this. She explained:

> *… the kids would share with each other and then share with bigger groups and lots of discussion and it just worked so nicely watching it and being able to observe that with my own kids. … because you are kind of scared of the unknown and what is going to fall apart, and I mean that's all part of teaching you have got to try things to see if they work and … not being the actual person doing it really helped …*

Members of the senior management team at Mary's school underscored the importance of the relationship between the school and the facilitator in the following comments:

> *… the thing that's been the most successful about it is that the facilitator is there, and if you need the support you can just ring her any time. They are in the school every couple of weeks following through, giving support, modelling lessons, doing observations, answering questions. I think that that's been the most beneficial …*
>
> *In my view, yes, it is the real strength of it and it has the potential therefore to be its greatest weakness because if that facilitator is the wrong person … for however you want to define wrong … and the relationship between the facilitator and the school breaks down, then the programme and its implementation is in jeopardy.*

Use of other curriculum areas

Some teachers use other curriculum areas, such as language, as an opportunity to practise explanations. Lynette outlined how she did this:

> *Well, it's through questioning, like during your oral-language time and … you try to integrate it into all the curriculum areas, and then the more practice they get, the more understanding they are when it comes to doing maths concepts. … At the beginning of the year, we used to just get answers and then when we asked students "Well how did you work that out?", if a kid couldn't do that then you would pick on a student that you knew could explain, … who had a bit more confidence to share, [and ask] "Okay, so this is how I work it out?"*

Use of specific classroom approaches

One approach specifically mentioned by Mary was that of reciprocal teaching in use in her reading programme:

> *It reminds me of reciprocal teaching, actually, in a lot of ways, because I do reciprocal teaching in my reading … with my readers, and I love that, and we tend to go off track, but we discuss so many things, and I love that discussion and the way that they are learning, and I feel that that has come into my maths now.*

Added Challenges in Lower Decile Schools of an Emphasis on Discussion

Participants identified a number of challenges specific to low-decile schools. These included teacher expectations and beliefs about students in low-decile schools, English as a second language, and the transience of students.

Teacher expectations for and beliefs about students in low-decile schools

The principal of one of the schools sees teacher ability and teacher expectation as a critical factor in the existence of the large achievement gap noted in the several international studies across New Zealand schools, commenting that "Kids will rise or fall according to the level of teacher expectation". She elaborated further:

> *In some cases in low-decile schools, either teacher expectation is too low or, coupled with that, some teachers are prepared to use the low-decile nature of the school as an excuse: "Oh well what do you expect from these kids?" I am prepared to accept that it makes the job harder, and therefore it governs our response to those needs. … [For instance] at the 6-year-old level, 7% of our kids were reading at or above the chronological age. 93% were below. I have no reason to believe that their numeracy level would be significantly any different. So that is a huge learning deficit to be coming*

to school with, and it went further. If you look at the next year and the next year after that the impact that we made was there but it was not huge. It wasn't until year 4, 5, 6 that we turned 7% into something like 60%.

Well, what it says to me is that not only are these kids behind at new entrant or 6 year old level, they are way behind. So we are pouring all the money and all the resources and teaching into the junior level that we can, and for a year we made not much difference, and for two years we made not much difference, because, yes, they are catching up, but they are still behind. It takes three years before they get past, because I have said to parents "Bear in mind that for someone to take a year to catch up it means you have to do more than a year. If you are just going to progress a year in a year you are actually standing still because you are getting older". So if you are going to reduce a deficit you have got be going at two or three to one.

Many teachers comment on a perceived need to emphasis "hands-on".

<u>English as a second language</u>

The poor language ability in English of many of the students was raised by most of those interviewed as an added challenge for those teaching in low-decile schools. The senior management team at one of the schools commented on this:

… students don't usually make a big leap with their reading improvement … but as they get older, they are catching up because their English gets better and their understanding of what's happening around them gets better. They have more experiences of school activities …

A lot of the ANP is explaining how to solve a problem and is really closely tied to their level of English as well, because if they are not very strong English-wise, that will reflect on how they are doing in maths.

It would also to be interesting to see (instead of splitting up into decile categories) if there was some way of measuring the percentage of kids on the roll who were ESOL. Because if you have got a programme, be it numeracy, literacy, or anything else that has a significant language base … So in other words you don't really understand what this maths is about until you actually understand the words that we are using to talk about this maths, and you are sitting there straight from India or wherever, and you don't understand English, you are at a disadvantage.

The senior management team pointed out that there was a difference between the Ministry of Education funding category for ESOL and the school's instructional category:

We would say that between 40 and 50% are ESOL and 25% of them are officially recognised through ESOL Ministry of Education funding. The surprising thing when you look through is the percentage … born in New Zealand. … Yeah, you'd think that it was a problem that would be at its most severe with the recent immigrants … and to an extent that is true, but it is surprising how long it takes for that to have grown out of the system … . And then you get parents or grandparents that don't speak Samoan very well any more and they don't speak English very well either, and they are just like something in between. … And that gets back to the language that they are expected to use in the maths programmes … just the whole sentence structure … .

The teachers at the school also saw facility with English as an issue:

> *English as a second language is a big challenge. I don't actually have that this year a lot because for a lot of my kids, English is their first language, and so that's been heaps easier this year. I have really noticed it and … most of them speak English as a first language and all that kind of thing and because of that they get boosted up almost. But definitely the ESOL factor, and with the ESOL factor as well comes in the home reading, the homework, and all that kind of thing. If the parents can't help then [what]? … like I said, the English thing and all that kind of thing, especially the English … .more so than the financial kind of side of it … but … it makes it easier for them to understand what is going on and what we are learning about and things like that …* (Mary)

> *I think that sometimes the language base isn't there. Which for numbers not so much but for other strands … measurement … knowing what to call something … shapes and not knowing what the name of that shape is. I think the challenge here is always to be practical, yet, in saying that, some of my students like that abstract problem solving.* (Anne)

> *Well, sometimes I find the language being a problem, and that is why I try as much as possible to identify the language within the unit. Like now with the less able ones, the language is like quarters and halves and so they are able to identify the language. There is language within maths that we have to identify. Especially a section like [fractions] which is heavily weighted on language.* (Chris)

A member of senior management at the other school made similar comments when asked about the challenges in getting students to explain their ideas:

> *I think the main problem is that the students lack that oral language ability and they lack the vocabulary that they need to do these explanations, and for a lot of them it is easy to just say "I just knew it" "I just did it" and it's harder to go into what they were thinking, what they were saying in their mind and how did they get to that answer, and especially … with the ESOL students it's very hard for them to verbalise that kind of thing and it requires lots and lots of teacher modelling and lots and lots of peer modelling as well, before you can get them started off, so they need, the main thing that we've been telling teachers is they need to encourage students to listen to each other, and if they don't know how to explain it and there is no one in the group who can really explain it then the teacher can say "Well, this is how I did it". And so they now have now a model of "Oh yes, oh this is another way in which it could have done it (if they have explained one or two ways). I could have done it this way as well", or this way so that we are just feeding in the language to them.*

Hettie, one of the teachers at the same school commented:

> *… for some of them it's not having the vocab to say it in the first place … with ESOL. It depends on who their group is as well … like if there are other kids who are really confident in their group, they will often just sit back because they know that someone else will do it.*

Transience

Both schools acknowledged the transient nature of some of their students. The principal of one school explained the high levels of transience.

> *… last year we had eighty per cent transience in the school. I mean, that's a huge turnover of students. To get that eighty per cent, some of the families come and go two or three times in the year, all mostly tied into economics. They go away and stay with Nana and Papa until they save the money to come back and pay the power bill. Those sorts of things, or get behind in rent. But, yes, we still have a very high transient population and putting the rents down to a percentage of what they're earning has not yet seen stabilisation. We did see it … we did see transience rise after 1991 when the rents went up, but we haven't yet seen it stabilise since … for this year, you know, since the rental system has been a bit more reliant on what they earn. So it shows that they're still struggling, they're trying to get over those years of burden really. [The students] miss huge amounts of very early important grounding in their schooling. … I think it's that really important – continuous tuition in one place.*

The principal first discusses the difficulty of measuring the level of transience, and then surmises as to the probable causes and effects:

> *There is a very complex debate about how you measure transience and who has a higher level of transience than other [schools], and so on. But there are some characteristics that I think you can draw. When you are in an area that has a significant percentage of our housing being tenanted, when you have people who are in casual employment, then the level of families moving … as in transients, … .is likely to be higher. In my view, kids moving school is seldom to do with the school, it is to do with employment issues and housing issues that affect the family as a whole. There is a percentage of our kids who are in that category … more likely to be Māori or Pasifika people … and kids who are transient are more likely to come with more severe learning deficits.*

Two of the teachers at this school confirm the effect transience appears to have on the students is achievement:

> *I have got the extension class and the reason there is the extension kids is because they've got stable … like they have lived in the same house for so long and have always been at this school and English is their first language. But previously [in other classes I've taught] … you would just look at the kids and think "No wonder you don't know anything, because you have been to eight schools in two years."* (Mary)
>
> *But you do get kids … you see the profiles and stuff … the kids who have been to eight different schools. There has been no consistency in their learning, and then just sometimes it's their interest in what they are doing sometimes as well, because they probably think "Well, I will probably be moving soon".* (Anne)

Capacity of parents to provide extra help

In lower decile schools the capacity of the parents to provide extra tuition is limited. The senior management team one school commented on this.

> *But the other thing that is different between high- and low-decile schools is that high-decile schools for students who are not achieving, the parents ensure that they have extra classes outside school hours. Both in reading and language and English and maths, and every second person you talk to in one of those schools, "Oh my child is*

having maths lessons" … whereas in decile 1 we have got to do it all. You don't get that happening in your community. … Even the ESOL students in high-decile schools, they are all going out to English classes after school hours as well.

<u>Expectations of parents</u>

The expectations of parents in a decile 1 school is often more conservative than those in higher deciles. The principal at one school commented.

> *The other point which you haven't touched on which I think needs to be highlighted somewhere … the parental expectation of the school is essentially a conservative set of expectations. They want their kids to be taught to read, write, add up, and behave and to do it in an environment that is safe. That is what they expect from the school. And when I say read and write and add up I am talking primarily in English and the skills of numeracy that are being able to do the full operations and so on that … they see that as the core focus of the school. That is a conservative set of expectations. It's not a liberal set … it's probably more akin to the set of expectations that my parents had when I went to school donkeys years ago. Those expectations haven't changed really. So the programme like the ANP which has a sharper focus, is one that they are comfortable with. What I am not sure that they are comfortable with, going from that parents' evening that we had, is some of the strategies. Not perhaps comfortable but familiar, … with some of the strategies that the kids use because they have a traditional idea of how you do multiplication and how you do division.*

The principal talked about educating parents about the importance of primary schooling:

> *… we particularly notice it, from when we do baseline study at the beginning of the year and then baseline again at the end of the year and you see that, we've lost half the students just about in some classes. It's quite disheartening really. … I mean we're getting students onto special programmes and next thing they're gone again. … We know … what would help but … it's got to be something that's done nationally. So that in the faces of people watching T.V. come these ads that say, "Make sure your child goes to school every day and make sure you don't have many changes in the early years". … Parents in … the lower economic area they think that because their child is five, six, or seven … it doesn't really matter, they can have lots of change of school. [They think that it is not] until they get to nine or ten [that] schooling is really important. So that kind of parent education has to change, and who can change it? Only some body like the Ministry nationally by doing these huge ad campaigns, like the Feed the Mind campaign. That has to be turned now into "get your child to school every day" and "years 1 to 3 are the crucial learning times of schooling".*

Specific Strategies Used in Bringing about Change in Low-decile School Classrooms

<u>Emphasis on numeracy and literacy</u>

At one school, the senior management team recognised the importance of emphasising numeracy and literacy and sought school policy changes to ensure this happened. They explained how they would review the appropriateness of trying to deliver a balanced curriculum for their school by revising their school scheme in terms of the amount of time spent on each strand of the mathematics curriculum as well as increasing the emphasis on numeracy and literacy. In prioritising numeracy and literacy, they planned to ask questions such as "How much time do we need for those?" and "What time of the day should we deliver those at?" in thinking about delivering the curriculum. They had clear reasons for doing this, as explained by the principal:

> *The link between socio-economic status and learning deficits is international, irrefutable and … universally accepted. The areas of literacy and numeracy are at the heart of the learning deficits that these kids have, so, therefore, the onus of the school to put all its resources and its time and attention that it possibly can into addressing literacy and numeracy gaps. … The lower the decile, the greater the emphasis. This is decile 1, so you don't come any greater and it's also 800 kids … .*
>
> *So, therefore, to me this is heart and soul core business for this school, and if we are not addressing issues like this, then I would have to question why the hell we are here at all. What this programme has done has taken that key area of numeracy and focus on the particular numeracy component of what we use to call maths, and, I mean, I think it's been widely accepted that the numeracy skills have been the one who have been particularly … and they are the ones that have been particularly critical. So I am struggling to be able to say how the alignment could be any greater. … And it's perfectly proper, right, and sensible and most of all educationally sound that the government did give priority to these low-decile schools for a programme like this, and so they should. That is where the need is greatest.*

She explained why they concentrate on the basic subjects:

> *And also the other thing is that in the decile 1 schools they are mostly Māori and Pasifika students, I would envisage, and sometimes programmes need to be really worked and re-worked and re-worked to make sure we're doing what is comfortable for learning for those kids. And because they don't get the home support … as much, things have to be different in schools. Much more concentrated effort on the basic subjects, much more people time put into those kids, and a lot of the stuff where … that you might normally send home for finishing off can't be done. It's got to be completed at school. That means some things have to go to make way for the important subjects. And so that's why bringing this … year 1 to 4 with concentration in literacy and numeracy was so great, because now we've got the non-expectancy of having to cover all those other subjects so deeply, 'cause we've got more time now to concentrate on what has to happen at school.*

Additional staff

Both schools employed additional staff to work with their students. These included teacher aides and specialist teachers. At one school, there is a special programmes team that has been formed to provide additional instruction to transients and students with poor language skills. The principal explained when and how the school set up the special programmes team.

> *Yeah … twelve years … that's where we pour our decile-based funding, TFEA funding, and decile-based registered for bulk-funding money, and what have you. Everything that we … can lay our hands on. That gets poured into … well, most of it goes into special programmes. The ESOL money goes in there as well. … We've also got two teacher aides at the moment [and] three classroom teachers that we employ above entitlement and that enables us to have slightly smaller classes across the board.*

At the other school, a consideration when employing extra staff was that staff should reflect the ethnic mix of the school, with particular attention given to providing extra help for students from non-English-speaking backgrounds.

But I think the key is that they need more adult input. They need more group support, and they need time to go over and go over and go over and go over because they're only doing it in the school time and not really getting it at the home time.

We put a lot of money in, into people. We're a decile 1A, our students, at home they don't have the adult support that they could have, so we've made a decision many years ago that a lot of our money will be put into people. So we employ four teachers over our staffing budget, staffing schedule, but we also employ twenty-four teacher aides. So that's a huge number, but they're all put into special programmes where it's one-on-one or one-to-two daily programmes so students can get the adult assistance or adult care that they don't get at home. … If we're wanting assistance with our English second language students and they happen to be Iraqi, then we've employed an Iraqi teacher aide, which is what we've done. We've employed a couple of Sāmoan teacher aides because that's been an area of concern. We've got an Indian teacher aide. So we've got a variety of cultures in the school, somebody from Kiribas, Tonga, you know, spread … whatever – just about whatever culture we've got in the school, we've either got a teacher or a teacher aide in that area. So, yes, that's a very big consideration when we appoint staff. … Well, I guess the big step is that we've recognised the need to make an environment which they are comfortable within. … We all do staff development, have staff development on cultural needs in the big cultures of our school and that, within that staff development comes understanding of the needs and how students in those cultures succeed probably, so a lot to do with staff development.

Concluding Comments

There are a number of issues that impact on teaching and learning in low-decile schools. Factors that have the potential to inhibit change to classroom practice are teacher knowledge and associated confidence and students' confidence in discussing ideas. For both students and parents, the reasons for the emphasis on discussion in mathematics may not be immediately apparent. This emphasis on discussion is further complicated by teacher expectations and beliefs about students in low-decile schools, the large proportion of students who have English as their second language, and the transcience of students. The more conservative expectations of parents about schooling and their often limited capacity to provide extra help in the home are added factors.

Establishing an emphasis on numeracy and literacy and having additional staff (trained and untrained) in school are specific strategies that have been found to be successful in changing school practice in low-decile schools.

Chapter Eight: Summary

Student performance at years 4 to 6 improved across the six aspects of number monitored during the ANP in 2002. As in 2001, this growth was irrespective of students' age, gender, ethnicity, school region, and decile. Again, the gains made by students were variable, with students of Asian and European descent making greater gains at the more advanced levels in all six aspects of number.

An important part of raising student achievement is the shift from counting-based to part-whole thinking. Of those who initially had counting-based strategies the greatest shift for each operational domain was to the early additive stage. Fewer moved to the advanced additive and advanced multiplicative part-whole stages and fewer still to the advanced proportional part-whole stage. Of the knowledge domains assessed, the levels of knowledge for those with part-whole strategies was consistently higher than for those with counting-based strategies.

Comparison between the 2002 and 2001 results shows a decline for age, ethnicity, and decile. More students in 2002 who initially had counting-based strategies made no change to part-whole strategies by the final assessment. This may be the result of changes to the diagnostic interview in 2002, in which teachers were given a choice of forms to use based on students' strategy knowledge. A ceiling effect may have been created through teachers inadvertently using the same form for the initial and final diagnostic interviews, when students should probably have been given a more advanced form.

Teachers' professional knowledge and practice was enhanced by participation in the ANP. Teachers' practice became more tightly focused on number, with a particular emphasis on the teaching of mental strategies to solve number problems. The change to small-group instruction, in particular, provided greater opportunities for students to explain their problem-solving strategies. Teachers developed a more sophisticated view of active learning, one that included discussion, unlike the previous understanding, which was narrowly defined around equipment.

An important factor in helping teachers to change their practice was in the provision of in-class support by facilitators, who were able to act as a bridge between teachers' existing and new practices and work with schools to align school-wide systems to the aims of the project. Similar findings were reported in 2001.

As in 2001, improvements in teacher attitudes towards mathematics and the teaching of it were made through the knowledge gained from participation in the project. Again, student attitudes towards learning mathematics (as judged by teachers, principals, and facilitators) were sustained in 2002 by their teachers' continued participation in the project.

Factors identified as inhibiting change in low-decile schools include teacher expectation and beliefs about students in low-decile schools, the proportion of students from non-English-speaking backgrounds, the capacity of parents to provide additional assistance to students who are struggling, and the transience of many students. Successful strategies for enhancing achievement in low-decile schools include focuses on literacy and numeracy and the provision of additional staff (both trained and untrained) in school.

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Appendix A: The Number Framework

Stage Zero: Emergent
Students at this stage are unable to consistently count a given number of objects because they lack knowledge of counting sequences and/or the ability to match things in one-to-one correspondence.

Stage One: One-to-one Counting
This stage is characterised by students who can count and form a set of objects up to ten but cannot solve simple problems that involve joining and separating sets, like 4 + 3.

Stage Two: Counting from One on Materials
Given a joining or separating of sets problem, students at this stage rely on counting physical materials, like their fingers. They count all the objects in both sets to find an answer, as in "Five lollies and three more lollies. How many lollies is that altogether?"

Stage Three: Counting from One by Imaging
This stage is also characterised by students counting all of the objects in simple joining and separating problems. Students at this stage are able to image visual patterns of the objects in their mind and count them.

Stage Four: Advanced Counting (Counting-On)
Students at this stage understand that the end number in a counting sequence measures the whole set and can relate the addition or subtraction of objects to the forward and backward number sequences by ones, tens, etc. For example, instead of counting all objects to solve 6 + 5, the student recognises that "6" represents all six objects and counts on from there: "7, 8, 9, 10, 11."

Students at this stage also have the ability to co-ordinate equivalent counts, such as "10, 20, 30, 40, 50," to get $50 in $10 notes. This is the beginning of grouping to solve multiplication and division problems.

Stage Five: Early Additive Part-whole
At this stage, students have begun to recognise that numbers are abstract units that can be treated simultaneously as wholes or can be partitioned and combined. This is called *part-whole thinking*. A characteristic of this stage is the derivation of results from related known facts, such as finding addition answers by using doubles or teen numbers.

Stage Six: Advanced Additive Part-whole
Students at the advanced additive stage are learning to choose appropriately from a repertoire of part-whole strategies to estimate answers and solve addition and subtraction problems. They see numbers as whole units in themselves but also understand that "nested" within these units is a range of possibilities for subdivision and recombining. Simultaneously, the efficiency of these students in addition and subtraction is reflected in their ability to derive multiplication answers from known facts. These students can also solve fraction problems using a combination of multiplication and addition-based reasoning. For example, 6 x 6 as (5 x 6) + 6.

Stage Seven: Advanced Multiplicative Part-whole
Students at the Advanced Multiplicative stage are learning to choose appropriately from a

range of part-whole strategies to estimate answers and solve problems involving multiplication and division. Some writers describe this stage as “operating on the operator”. This means than one or more of the numbers involved in a multiplication or division is partitioned and then recombined.

For example, to solve 27 x 6, 27 might be split into 20 + 7 and these parts multiplied then recombined, as in 20 x 6 = 120, 7 x 6 = 42, 120 + 42 = 162. This strategy uses the distributive property.

A critical development at this stage is the use of reversibility, in particular, solving division problems using multiplication. Advanced Multiplicative Part-whole students are also able to estimate answers and solve problems with fractions using multiplication and division.

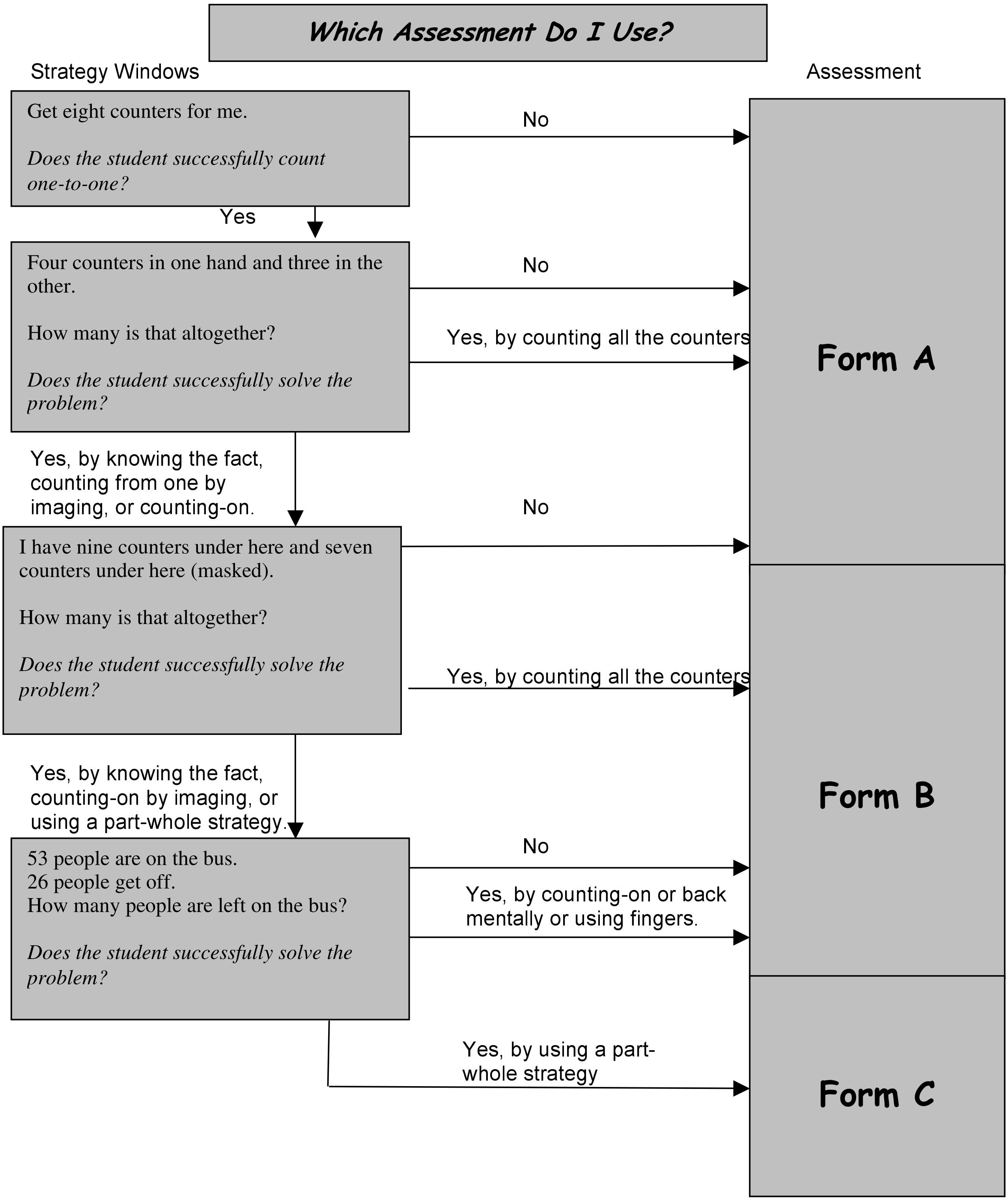
Stage Eight: Advanced Proportional Part-whole

Students at the Advanced Proportional stage are learning to select from a repertoire of part-whole strategies to estimate answers and solve problems involving fractions, proportions and ratios. This includes strategies for the multiplication of decimals and the calculation of percentages.

These students are able to find the multiplicative relationship between quantities of two different measures. This can be thought of as a mapping. For example, consider this problem: “You can make 21 glasses of lemonade from 28 lemons. How many glasses can you make using 8 lemons?”

To solve the problem, students need to find a relationship between the number of lemons and the number of glasses. This involves the creation of a new measure, glasses per lemon. The relationship is that the number of glasses is three-quarters the number of lemons. This could be recorded as: 21:28 as 3:4, 21 is three-quarters of 28, three-quarters of 8 is 6.

Appendix B: Advanced Numeracy Project Assessment



Appendix C: Summary of Questionnaires

PRINCIPAL QUESTIONNAIRE

1. What factors most influenced your decision to apply to participate in the Advanced Numeracy Project?
2. From your perspective, what has been the **attitude** toward the project of your participating teachers? Please elaborate on your response.
3. In your view, has the project had any **positive** impact on the delivery of the mathematics programme of the participating teachers? Please elaborate on your response.
4. In your view, has the project had any **negative** impact on the delivery of the mathematics programme of the participating teachers? Please elaborate on your response.
5. In your view has the project had any impact on the teachers "**as a person**" either in or outside the classroom? (eg confidence, enthusiasm, tiredness)
6. From your perspective, has the project had any impact on the **other teachers** in your school who were not participating in the project?
7. In your view, what impact has the project had on the **children** participating in the project? (Consider attitudes, skills, understandings etc)
8. What has been the general reaction of **parents** to the Advanced Numeracy Project?
9. What **policy/resource changes** has your school made as a result of participating in the ANP project? Please elaborate on your response.
10. What aspects of this project could be **improved**? Why?
11. Are there any other comments you would like to make about the project?

BACKGROUND INFORMATION
Region:
Size of School:
Decile:

QUESTIONNAIRE FOR TEACHERS

1. Do you think that your **attitude** towards maths has changed as a result of your participation in the project? Please elaborate on your response.
2. Has your **content knowledge of maths** been developed in any way as a result of your participation in the project? Please elaborate on your response.
3. Has your understanding of **how children learn number** changed as a result of your participation in the project? Please elaborate on your response.
4. Has the **way you teach number** changed as a result of your participation in the project? Please elaborate on your response.
5. In your opinion, what aspects of the project **helped you most**? How and why?
6. What aspects of the project were **least helpful or confusing**? Why?
7. What has been the general **reaction of parents** to ANP?
8. In your view, what impact has the project had on the **children** participating in the project? (Consider attitudes, skills, understandings etc)?
9. What aspects of this project could be **improved**? Why?
10. What **policy/resource changes** has your school made as a result of participating in the ANP project? Please elaborate on your response.
11. Are there any other comments you would like to make about the project?

BACKGROUND INFORMATION

Region:
Size of school:
Decile:
Your age: ☐ 20 - 25 ☐ 26 - 35 ☐ 36 - 45 ☐ 46 - 55 ☐ 56+
Gender:
Years of teaching experience (including this year):
What year level are you currently teaching?
How many years experience have you been teaching in years 4-6?
How long have you taught at this school?
Highest level of teaching qualifications completed:
Are you currently engaged in furthering your qualifications?

QUESTIONNAIRE FOR FACILITATORS

1. From your perspective, what has been the **attitude** toward the project of your participating teachers? Please elaborate on your response.
2. In regard to the teachers that you worked with, do you think that ANP had an impact on their **maths content knowledge**? If so, please give specific examples in relation to the teachers you were working with.
3. In regard to the teachers that you worked with, do you think that ANP had an impact on their **pedagogical knowledge**? If so, how?
4. In your view, has the project had any **positive** impact on the delivery of the mathematics programme of the participating teachers? Please elaborate on your response.
5. In your view, has the project had any **negative** impact on the delivery of the mathematics programme of the participating teachers? Please elaborate on your response.
6. In your view, has the project had any impact on the teachers "**as a person**" either in or outside the classroom? (eg confidence, enthusiasm, tiredness)
7. From your perspective, has the project had any impact on the **other teachers** in your school who were not participating in the project?
8. In your view, what impact has the project had on the **children** participating in the project? (consider attitudes, skills, understandings etc)
9. What aspects of this project could be **improved**? Why?
10. Was the ongoing facilitator training helpful to your work as a facilitator? Please elaborate.
11. On reflection was the initial facilitator training for this year helpful to your work as a facilitator? Please elaborate.
12. Are there any other comments you would like to make about the project?

BACKGROUND INFORMATION

Years of teaching experience:
Years of advisory/facilitator experience (including this year):

Appendix D: Year 5 – Final Status of Students Who Were Advanced Counters

Gender	No change	Became early additive	Became advanced additive
Females	47.71%	48.57%	3.71%
Males	49.04%	46.42%	4.53%

Table D-1: Year 5 addition and subtraction – final status by gender

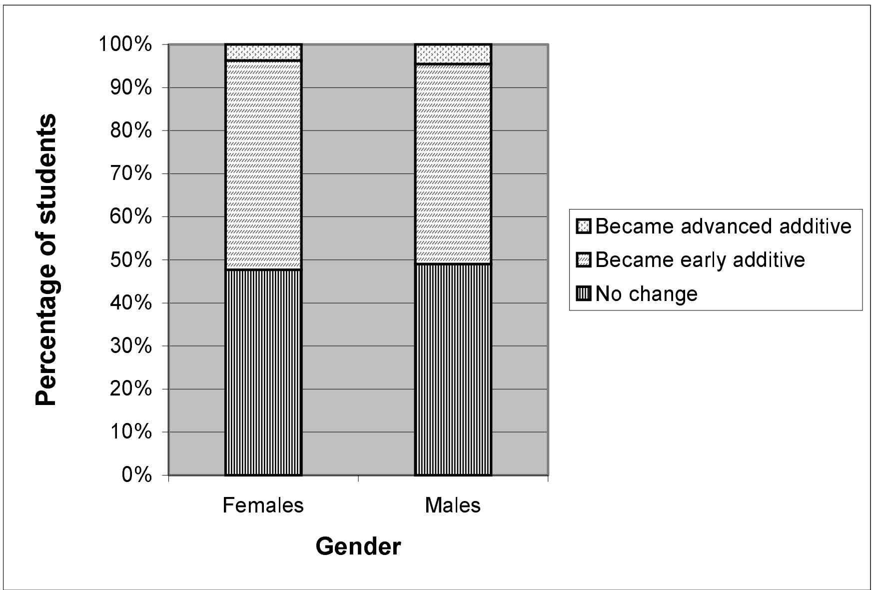


Figure D-1: Year 5 addition and subtraction – final status by gender

Ethnicity	No change	Became early additive	Became advanced additive
New Zealand European	42.19%	50.03%	5.58%
Māori	52.60%	44.84%	2.54%
Pasifika	62.77%	35.68%	1.54%
Asian	48.06%	45.30%	6.62%
Other	46.44%	50.18%	3.37%

Table D-2: Year 5 addition and subtraction – final status by ethnicity

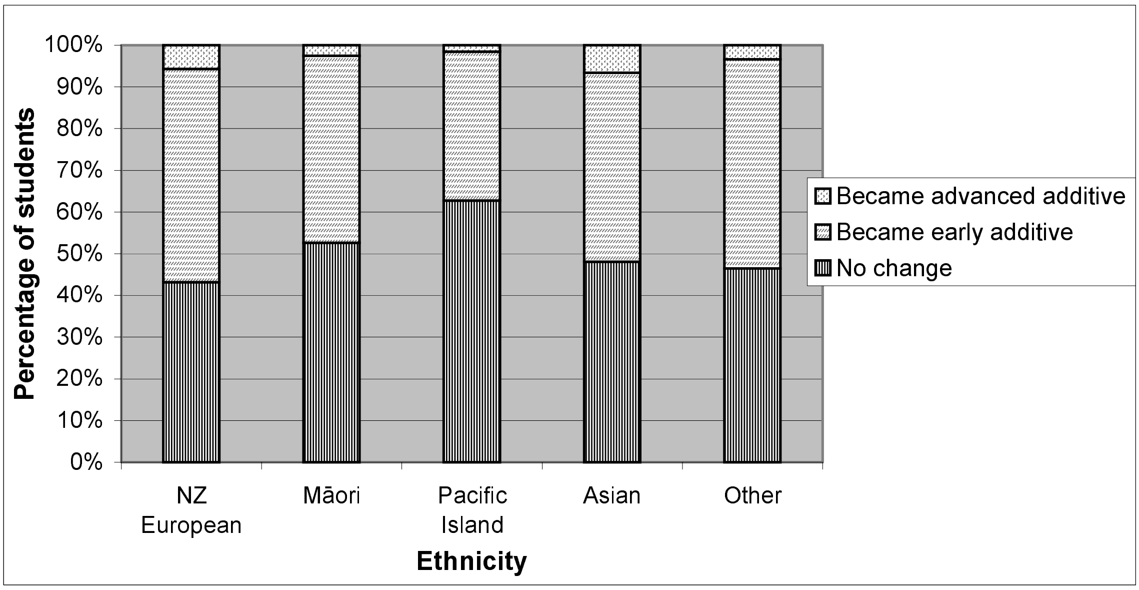


Figure D-2: Year 5 addition and subtraction – final status by ethnicity

Decile	No change	Early additive	Advanced additive
1	56.10%	41.90%	2.00%
2	54.24%	43.30%	2.45%
3	52.80%	44.51%	5.06%
4	53.59%	42.67%	3.72%
5	42.08%	53.92%	4.00%
6	49.34%	46.38%	4.27%
7	40.49%	55.76%	3.73%
8	37.75%	55.14%	7.09%
9	32.84%	56.39%	10.75%
10	34.41%	57.64%	7.94%

Table D-3: Year 5 addition and subtraction – final status by decile

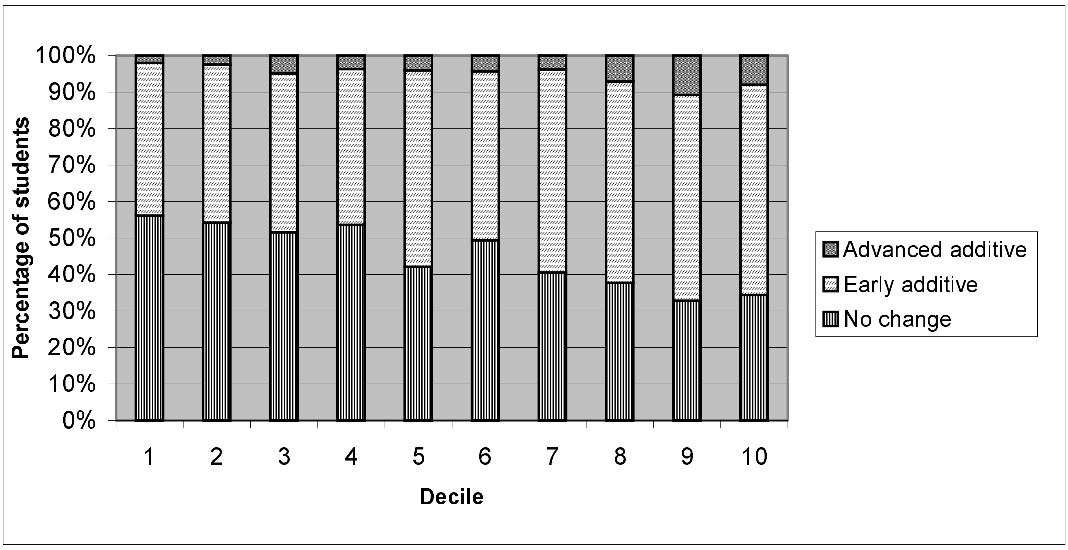


Figure D-3: Year 5 addition and subtraction – final status by decile

Region	No change	Early additive	Advanced additive
Auckland	53.29%	43.71%	2.98%
Christchurch	51.01%	47.32%	1.65%
Massey	52.00%	43.98%	4.01%
Northland	38.61%	56.83%	4.55%
Otago	38.14%	51.89%	9.96%
Southland	32.95%	54.92%	12.12%
Waikato	43.06%	52.13%	4.62%
Wellington	52.17%	46.04%	1.77%

Table D-4: Year 5 addition and subtraction – final status by region

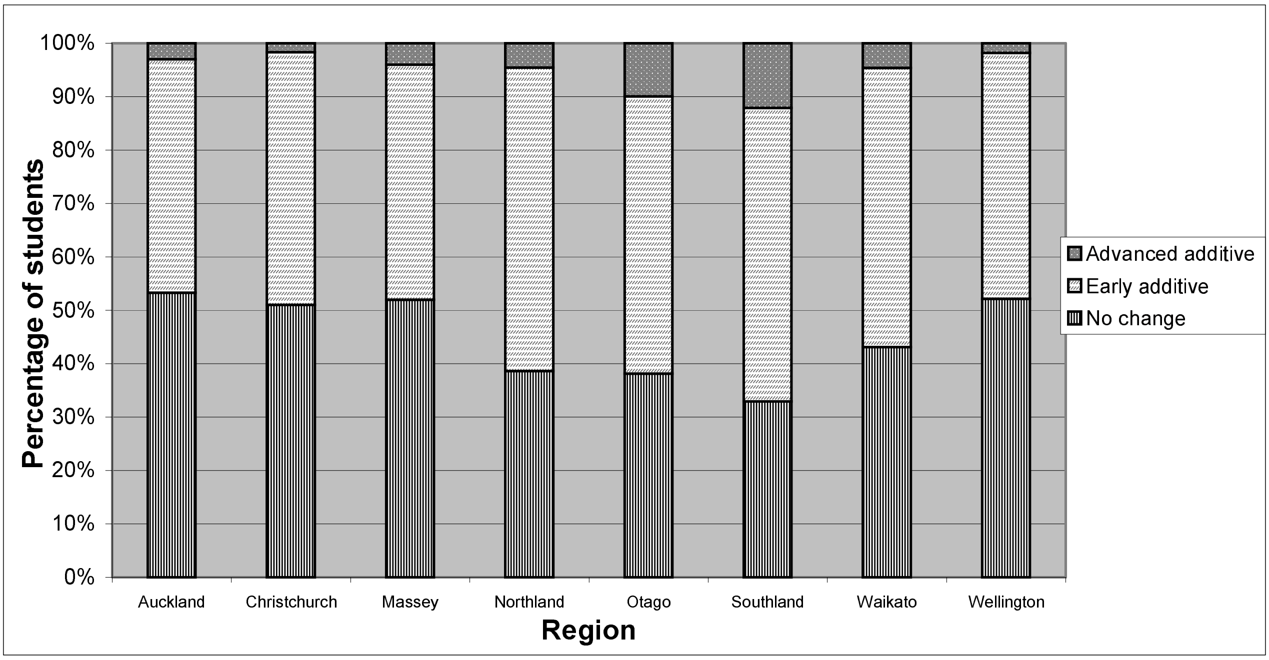


Figure D-4: Year 5 addition and subtraction – final status by region

Appendix E: Year 5 – Knowledge Base of Students Who Were Advanced Counters

	Emergent	Initial up to 10	up to 10	up to 20	up to 100	up to 1,000	up to 1,000,000
Counting-based	3%	0%	1%	3%	34%	50%	8%
Part-whole	1%	0%	0%	0%	7%	54%	29%
Counting-based	3%	1%	1%	5%	38%	45%	7%
Part-whole	1%	0%	0%	0%	11%	60%	28%

Table E-1: Year 5 FNWS and BNWS – knowledge for counting-based and part-whole

	Not assessed	Non-grouping with 5s and within 10	With 5s and within 10	With 10s	10s in 100	10s and 100s in whole numbers	10s, 100s, and 1,000s in whole numbers	10ths, 100ths, and 1,000ths in decimals
Counting-based	3%	8%	22%	50%	16%	1%	0%	0%
Part-whole	1%	1%	6%	41%	42%	8%	1%	0%

Table E-2: Year 5 grouping – knowledge for counting-based and part-whole

	Ordered unit fractions	Co-ordinated numerators and denominators	Equivalent fractions	Ordered fractions	Co-ordinated numerators and denominators	Equivalent fractions	Ordered fractions
Counting-based	20%	24%	31%	22%	3%	0%	0%
Part-whole	1%	11%	29%	44%	14%	1%	0%

Table E-3: Year 5 fractions – knowledge for counting-based and part-whole

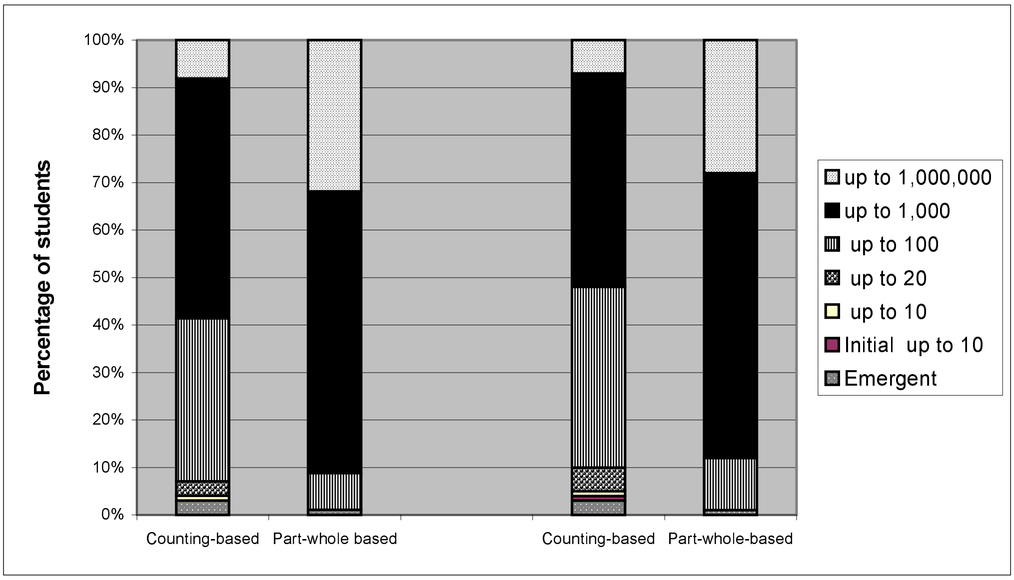


Figure E-1: Year 5 FNWS and BNWS – knowledge for counting-based and part-whole

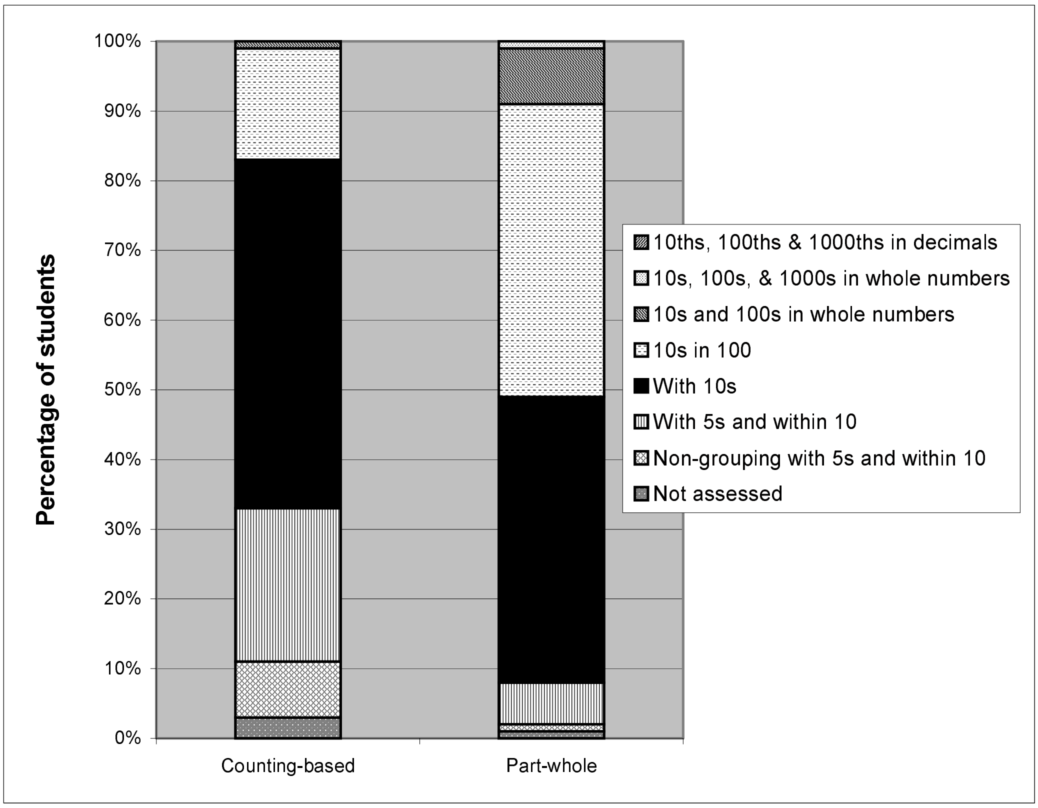


Figure E-2: Year 5 grouping – knowledge for counting-based and part-whole

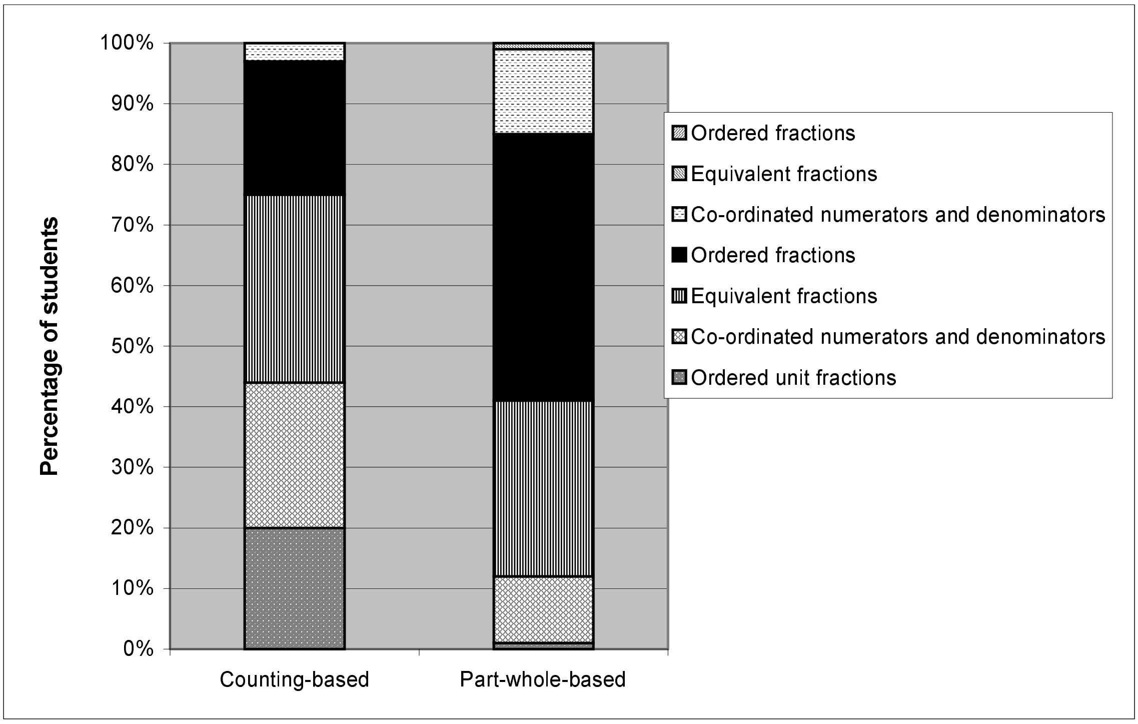


Figure E-3: Year 5 fractions – knowledge for counting-based and part-whole

Appendix F: Year 6 – Final Status of Students Who Were Advanced Counters

Gender	No change	Became early additive	Became advanced additive
Females	39.99%	52.54%	7.46%
Males	40.51%	52.69%	6.79%

Table F-1: Year 6 addition and subtraction – final status by gender

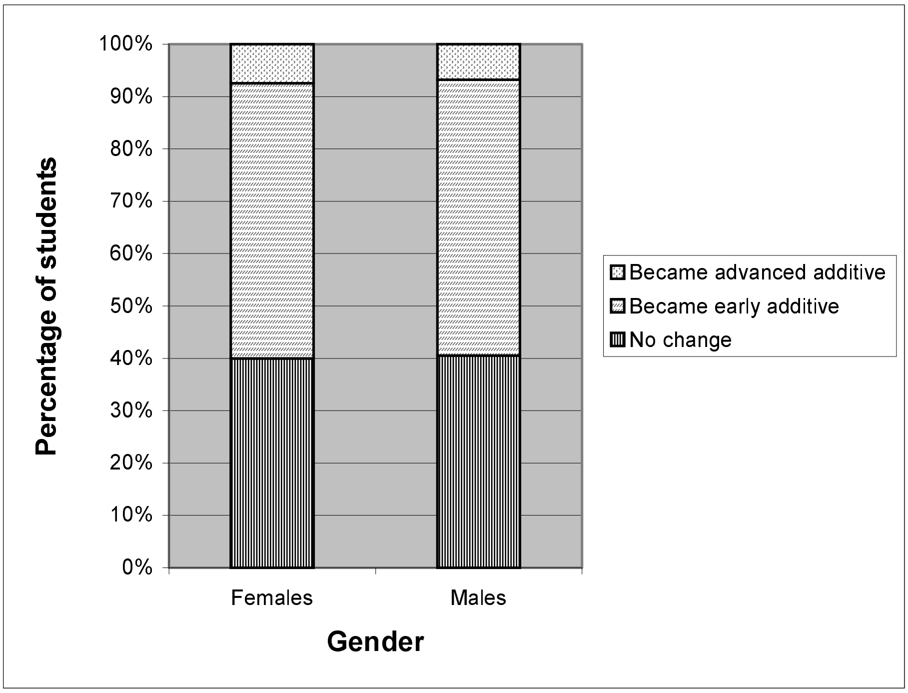


Figure F-1: Year 6 addition and subtraction – final status by gender

Ethnicity	No change	Became early additive	Became advanced additive
New Zealand European	34.57%	56.69%	8.72%
Māori	43.49%	50.57%	5.93%
Pasifika	53.65%	42.75%	3.86%
Asian	37.72%	50.29%	11.97%
Other	36.75%	57.29%	5.94%

Table F-2: Year 6 addition and subtraction – final status by ethnicity

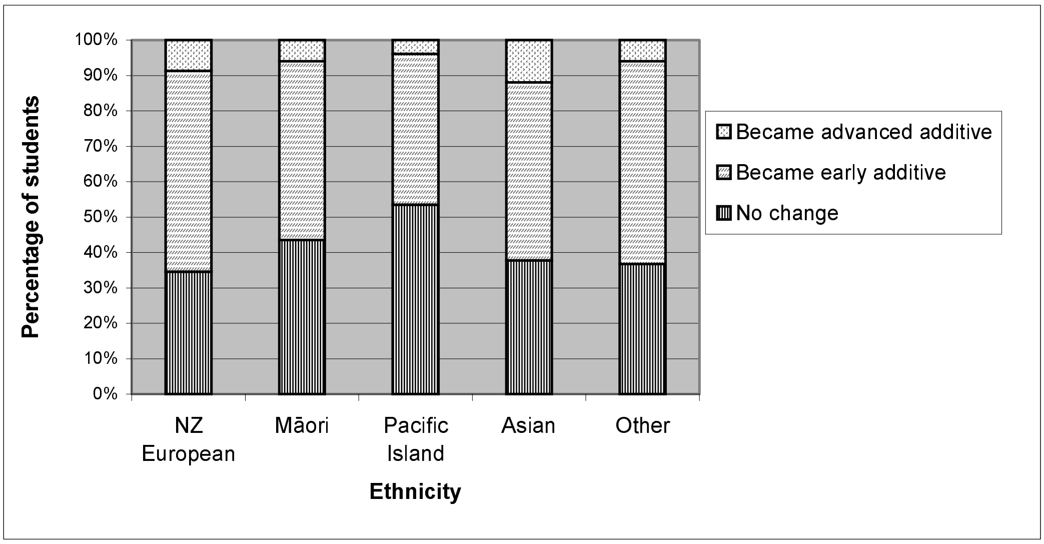


Figure F-2: Year 6 addition and subtraction – final status by ethnicity

Decile	No change	Early additive	Advanced additive
1	52.21%	43.57%	4.20%
2	42.85%	50.79%	6.34%
3	42.53%	51.22%	6.23%
4	36.63%	56.75%	6.60%
5	34.70%	56.47%	8.82%
6	38.72%	55.88%	5.39%
7	43.77%	49.78%	6.43%
8	32.24%	60.65%	7.10%
9	29.67%	54.21%	16.11%
10	24.31%	64.70%	10.98%

Table F-3: Year 6 addition and subtraction – final status by decile

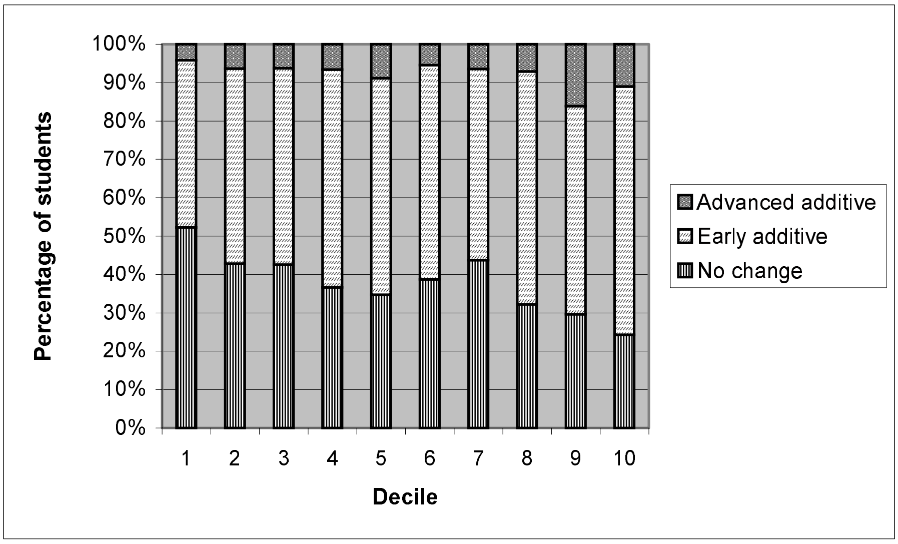


Figure F-3: Year 6 addition and subtraction – final status by decile

Region	No change	Early additive	Advanced additive
Auckland	45.55%	46.75%	7.69%
Christchurch	37.74%	57.59%	4.65%
Massey	39.12%	54.77%	6.10%
Northland	40.35%	50.87%	8.77%
Otago	22.45%	68.44%	9.09%
Southland	34.97%	51.12%	13.90%
Waikato	36.09%	57.51%	6.39%
Wellington	40.93%	54.14%	4.92%

Table F-4: Year 6 addition and subtraction – final status by region

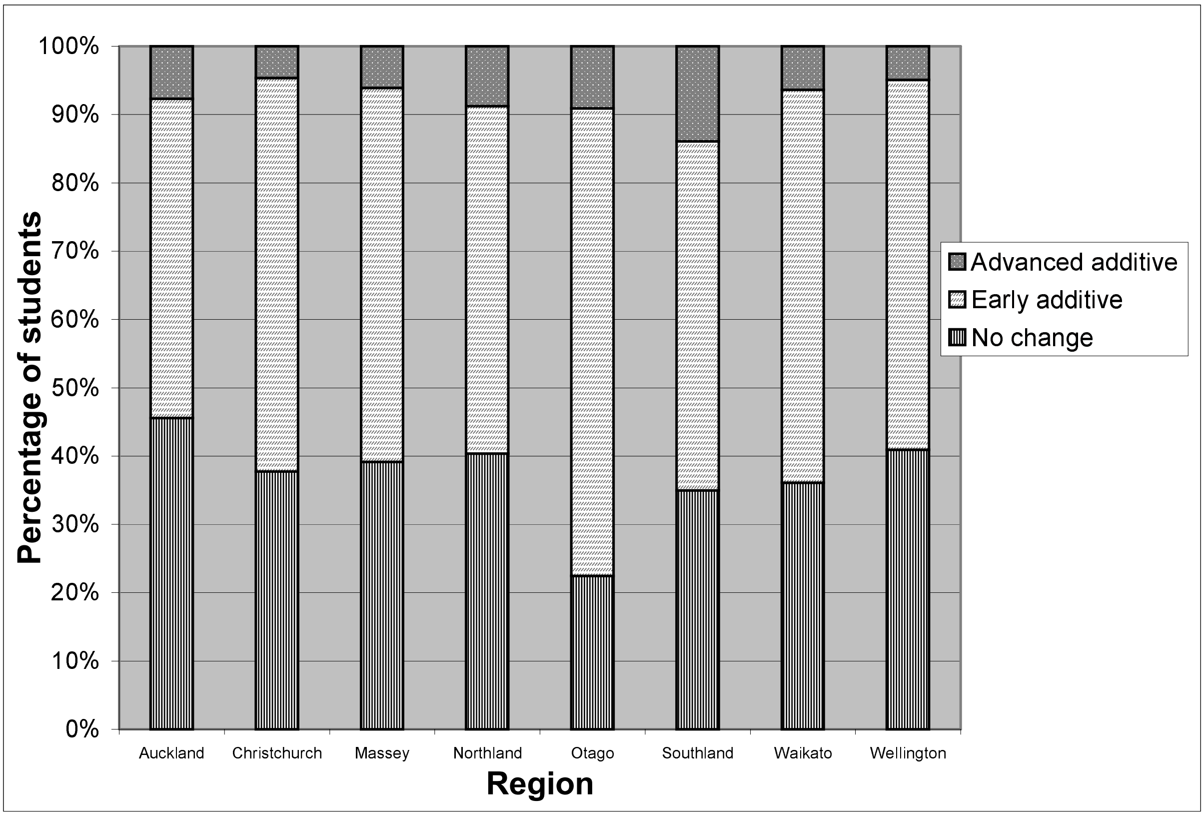


Figure F-4: Year 6 addition and subtraction – final status by region

Appendix G: Year 6 – Knowledge Base of Students Who Were Advanced Counters

	Emergent	Initial up to 10	up to 10	up to 20	up to 100	up to 1,000	up to 1,000,000
Counting-based	2%	0%	0%	3%	27%	54%	13%
Part-whole	0%	0%	0%	0%	5%	57%	38%
Counting-based	2%	0%	1%	4%	32%	50%	12%
Part-whole	0%	0%	0%	0%	8%	56%	35%

Table G-1: Year 6 FNWS and BNWS – knowledge for counting-based and part-whole

	Not assessed	Non-grouping with 5s and within 10	With 5s and within 10	With 10s	10s in 100	10s and 100s in whole numbers	10s, 100s, and 1,000s in whole numbers	10ths, 100ths, and 1,000ths in decimals
Counting-based	2%	6%	20%	46%	22%	3%	0%	0%
Part-whole	0%	0%	5%	36%	44%	12%	2%	1%

Table G-2: Year 6 grouping – knowledge for counting-based and part-whole

	Ordered unit fractions	Co-ordinated numerators and denominators	Equivalent fractions	Ordered fractions	Co-ordinated numerators and denominators	Equivalent fractions	Ordered fractions
Counting-based	15%	21%	32%	27%	5%	0%	0%
Part-whole	1%	9%	24%	45%	20%	1%	0%

Table G-3: Year 6 fractions – knowledge for counting-based and part-whole

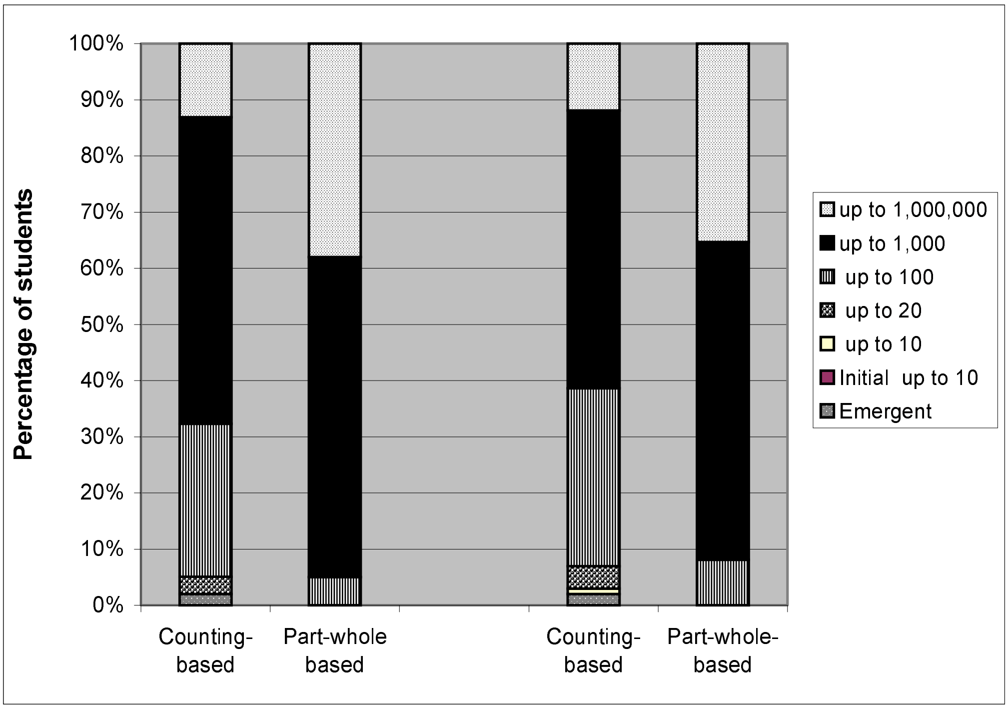


Figure G-1: Year 6 FNWS and BNWS – knowledge for counting-based and part-whole

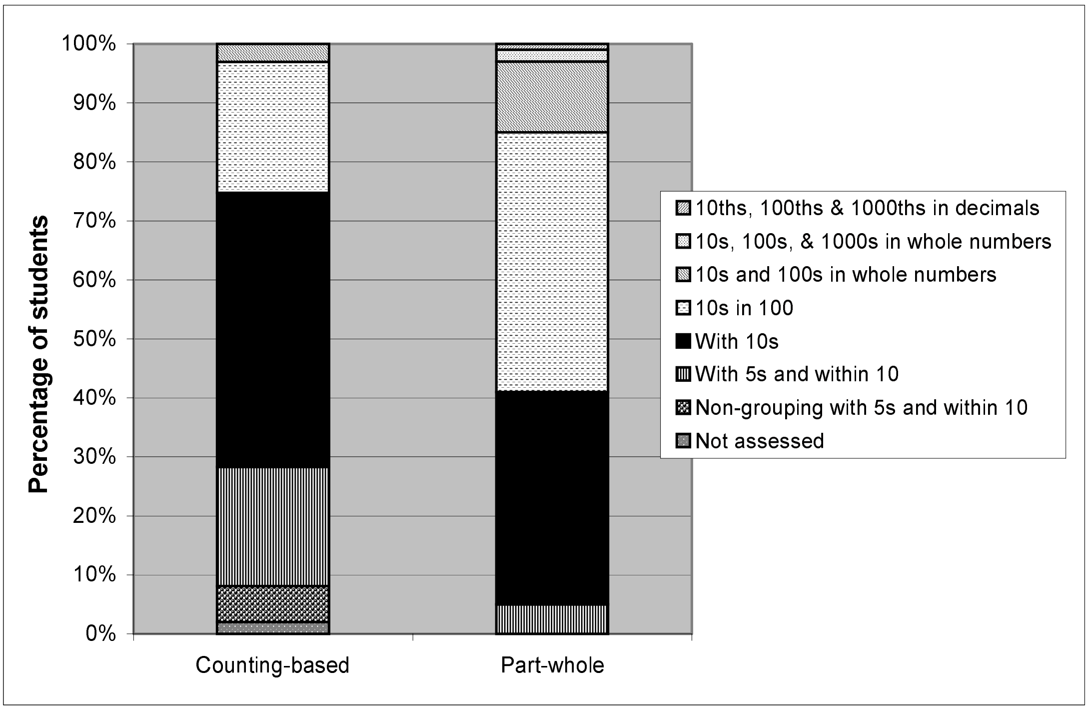


Figure G-2: Year 6 grouping – knowledge for counting-based and part-whole

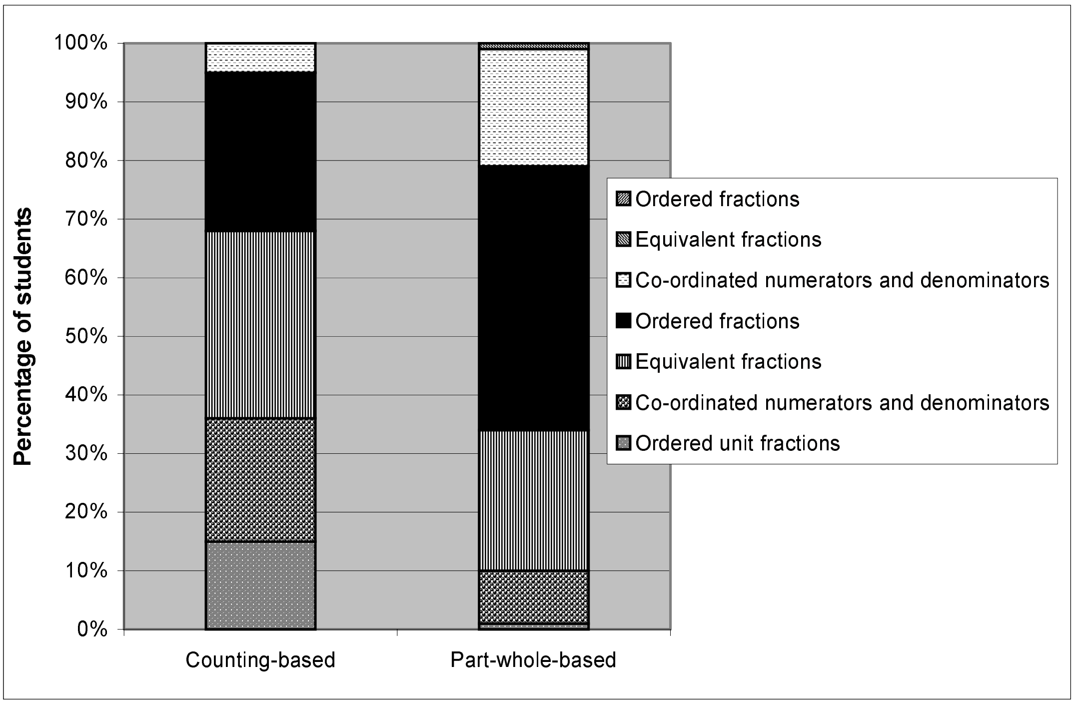


Figure G-3: Year 6 fractions – knowledge for counting-based and part-whole