The New Zealand Number Framework

The New Zealand number framework describes the different ways students think when solving number problems as strategy stages.

Stage Zero: Emergent

Students at this stage are unable to consistently count a given number of objects because they lack the knowledge of counting sequences and/or the ability to match things one-to-one.

8 + 5 = I could only get out 3 counters

Stage One: One-to-One Counting

Students can count and form a set of objects up to ten but cannot solve simple problems that involve joining and separating sets like 8+5

8 + 5 = I got out 8 counters and then I got out 5 counters. Then I got stuck

Stage Two: Counting from One on Materials

Given a joining or separating of sets problem, students rely on counting physical materials, like their fingers. They count all the objects in both sets to find an answer.

8 + 5 = I used my fingers and counted like this: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13

Stage Three: Counting from One by Imaging

Students count all of the objects in simple joining and separating problems. Students are able to image visual patterns of the objects in their minds and count them.

8+ 5 = I counted in my head like this: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13

Stage 4: Advanced Counting (Counting On)

Students at this stage understand that the end number in a counting sequence measures the whole set and can relate addition and subtraction of objects to the forward and backward number sequences by ones, tens, etc.

9 + 4 = I thought of 9 and then counted on like this: 10, 11, 12, 13: or *32 + 21 = 32, 42, 52, 53*

Students are also able to co-ordinate equivalent counts. This is the beginning of grouping to solve multiplication and division problems

8 x 3 = I counted in 3's like this: 3, 6, 9, 12, ……but then found it quite difficult so I went ***12****, 13, 14,* ***15****,16, 17,* ***18****, 19, 20,* ***21****, 22, 23,* ***24***

Stage Five: Early Additive

Students at this stage have begun to recognise that numbers are abstract units that can be treated simultaneously as wholes or can be partitioned and recombined. This is called *part whole thinking*. Students may use doubles or teen numbers.

9 + 4 = I took 1 off the 4 and added it to the 9 to make 10. Then I added the 3 to the 10 to make 13.

Stage Six: Advanced Additive Part Whole

Students use a range of part-whole strategies to solve and estimate the answers to addition and subtractions problems. They see numbers as whole units but also understand that they can be "nested" within these units.
Students are able to solve multiplication answers from known facts. These students are also able solve problems involving fractions using a combination of multiplication and addition-based strategies.

6 x 6 is (6 x 5) + 6 = 30 + 6 = 36

$\frac{3}{4}$ of 24 as $\frac{1}{4}$ of 20 =5 because 4 x 5 =20, so $\frac{3}{4}$ of 20 is 15, and $\frac{1}{4}$ of 4 is 1 because 4 x 1 = 4 so $\frac{3}{4}$ of 4 is 3. Therefore $\frac{3}{4}$ of 24 is 5 + 3 = 18

Stage Seven: Advanced Multiplicative Part-Whole

Students working at this stage are learning to choose appropriately from a range of part-whole strategies to solve and estimate the answers to problems multiplication and division.

27 x 6 is (20 + 7) x 6 = (20 x 6) = 120, (7 x 6) = 42, 120 + 42 = 162
or
2 x 27 = 54. 3 x 54 = 162

A key feature of this stage is using reversibility, solving division problems using multiplication. They are also able to solve and estimate the answers to problems involving fractions using multiplication and division.

2/3 of □ =18, $\frac{1}{2}$ of 18 = 9, □= 3 x 9 = 27

Stage Eight: Advanced Proportional Part-Whole

Students at this stage are able to select from a wide range of strategies to solve and estimate the answers to problems involving fractions, proportions and ratios. They use common factors and include strategies for the multiplication of decimals and the calculation of percentages.

You can make 21 glasses of lemonade from 28 lemons. How many glasses can you make using 12 lemons?
21:28 is equivalent to □:8? 21 is $\frac{3}{4}$ of 28, 9 is $\frac{3}{4}$ of 12.