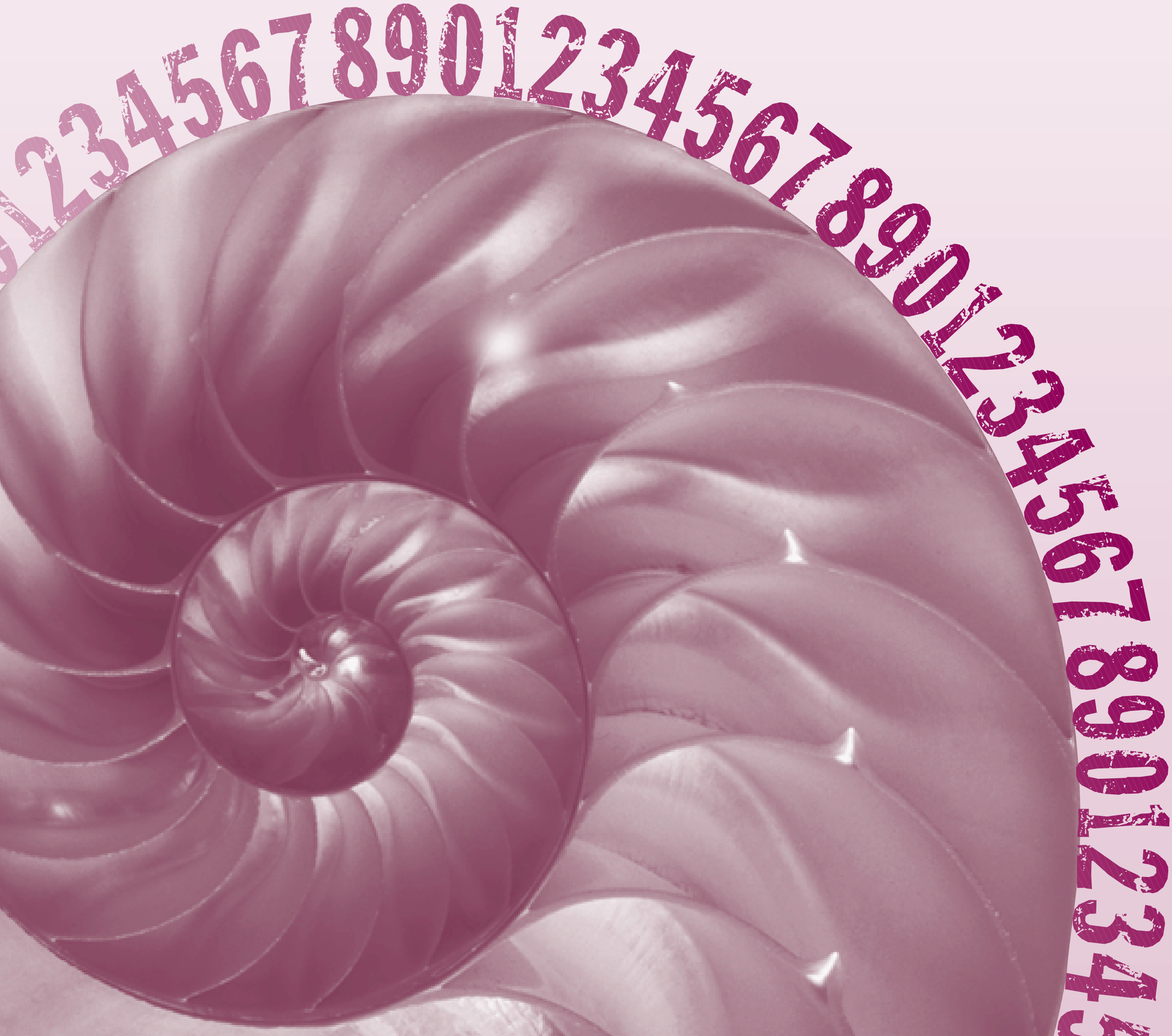
MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Book 5

Teaching Addition, Subtraction, and Place Value



Numeracy Professional Development Projects

This book revised 2007

EFFECTIVE MATHEMATICS TEACHING

The Numeracy Professional Development Projects assist teachers to develop the characteristics of effective teachers of numeracy.

Effective teachers of numeracy demonstrate some distinctive characteristics.[1] They:

- have high expectations of students' success in numeracy;
- emphasise the connections between different mathematical ideas;
- promote the selection and use of strategies that are efficient and effective and emphasise the development of mental skills;
- challenge students to think by explaining, listening, and problem solving;
- encourage purposeful discussion, in whole classes, in small groups, and with individual students;
- use systematic assessment and recording methods to monitor student progress and to record their strategies for calculation to inform planning and teaching.

The focus of the Numeracy Professional Development Projects is number and algebra. A key component of the projects is the Number Framework. This Framework provides teachers with:

- the means to effectively assess students' current levels of thinking in number;
- guidance for instruction;
- the opportunity to broaden their knowledge of how children acquire number concepts and to increase their understanding of how they can help children to progress.[2]

The components of the professional development programme allow us to gather and analyse information about children's learning in mathematics more rigorously and respond to children's learning needs more effectively. While, in the early stages, our efforts may focus on becoming familiar with the individual components of the programme, such as the progressions of the Framework or carrying out the diagnostic interview, we should not lose sight of the fact that they are merely tools in improving our professional practice. Ultimately, the success of the programme lies in the extent to which we are able to synthesise and integrate its various components into the art of effective mathematics teaching as we respond to the individual learning needs of the children in our classrooms.

[1] Askew et al (2000) *Effective Teachers of Numeracy*. London: King's College.

[2] See also the research evidence associated with formative assessment in mathematics: Wiliam, Dylan (1999) "Formative Assessment in Mathematics" in *Equals*, 5(2); 5(3); 6(1).

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2007 revision: Pages 47–49 are replacement material.

This book is also available on the New Zealand Maths website, at www.nzmaths.co.nz/Numeracy/2008numPDFs/pdfs.aspx

Support material for Book 5 is also available on the nzmaths website.

Teaching Addition, Subtraction, and Place Value

Teaching for Number Strategies

The activities in this book are specifically designed to develop students' mental strategies. They are targeted to meet the learning needs of students at particular strategy stages. All of the activities describe examples of using the teaching model from *Book 3: Getting Started.* The model develops students' strategies between and through phases of *Using Materials, Using Imaging,* and *Using Number properties.*

Each activity is based on a specific learning outcome. The outcome is described in the "I am learning to…" statement in the box at the beginning of each activity. These learning outcomes link to the Strategy Learning Outcomes provided in *Book 3: Getting Started*.

The following key is used in each of the teaching numeracy books. Shading indicates which stage or stages the given activity is most appropriate for. Note that CA, "Counting All," refers to all three *Counting from One* stages.

Strategy Stage

E →	Emergent
CA →	Counting All (One-to-one Counting, Counting from One on Materials or by Imaging)
AC →	Advanced Counting
EA →	Early Additive Part-Whole
AA →	Advanced Additive–Early Multiplicative Part-Whole
AM →	Advanced Multiplicative–Early Proportional Part-Whole
AP →	Advanced Proportional Part-Whole

The table of contents below details the main sections in this book. These sections reflect the strategy stages as described on pages 15–17 of *Book One: The Number Framework.*

Teaching Addition, Subtraction, and Place Value

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Learning Experiences for Emergent and One-to-one Counting

Knowledge Being Developed Simultaneously

Simultaneously with students moving from emergent and to one-to-one counting ideas, the following knowledge needs development:

- Counting forwards from a given number to another number in the range of 1 to 20.
- Counting backwards from 10 to 1.
- Recognising and writing numerals from 1 to 20.
- Recognising instantly numbers from 1 to 10 on fingers, using a five strategy. For example, eight is shown as five and three rather than four and four.

The key concept for one-to-one counting is that counting determines the number of objects in a set. Because activities that are designed to teach the counting words in order and to help students realise that counting produces the number of a set are usually closely linked, the knowledge activities and strategy activities are mixed together.

Lucky Dip

Equipment: Numeral cards 1 to 10 (Material Master 4–1). A container.

Using Materials

Activity: Show the students a card and ask the students what number it is. "Draw" the number in the air with your hand. "Draw" the number on the mat in large writing. Encourage your students also to "draw" the number in the air.

Example: Repeat with further cards.

Match It Up

Equipment: Numeral cards 1 to 10 and dot cards 1 to 10 (Material Master 5–11).

Using Materials

Activity: The students place the dot cards face down in one row and the numeral cards face down in a parallel row. Then they take turns to turn over a card from each row and see if the numeral card and the dot card match. If there is a match, the student keeps the pair. The game continues until all the pairs are matched.

Example: Repeat the game.

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Counting as We Go

Equipment: Objects to pass around.

Using Materials

Activity: The students get into groups and arrange themselves in circles. Nominate a student in each group to start at one. They then pass an object around and count as it passes each student. The students count as far as they can.

Example: Repeat counting from one.

Challenging activity: The group selects a single-digit number. Repeat the above activity but count backwards from the selected number. Before counting backwards, the students predict who will be number one. They check their prediction by passing an object and counting down out loud.

Challenging example: Give all the groups the same starting number. All the groups count forwards. Play some music. When you stop the music, each student draws the group's current number in the air. Record the numbers of all the groups on the board or modelling book and discuss whose number is biggest.

How Many Now?

Equipment: Marbles or heavy counters, Container.

Using Materials

Activity: The students close their eyes and listen and count as you drop objects into the container. At the end, ask how many objects are in the container. Check by emptying the container and counting them.

Example: Repeat with students in pairs. One student does the dropping, and the other does the counting. Then they swap roles.

How Many ...?

Equipment: None.

Using Materials

Activity: Divide the class into three groups. In each group, one student taps the boys on the shoulder, and the group counts how many boys.

Example: Repeat for the girls.

Challenging example: For the whole class, boys and girls line up in pairs to determine how many more/fewer boys there are than girls.

Loud and Soft

Equipment: Two puppets.

Using Materials

Activity: One puppet speaks loudly, and the other speaks softly. Counting from one, the puppets say the numbers alternately, and the students count with the puppets loudly and then softly.

If you have a puppet that can squeak, get the students to close their eyes and count the squeaks silently. Then ask how many squeaks the puppet made.

Example: Repeat by counting up to different numbers.

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Tick Tock

Equipment: A weighted object on the end of a piece of string.

Using Materials

Activity: An object swings like a pendulum. The students count at both ends of the swing. The students predict how many swings it will take for John to do 20 star jumps and then count as John does the jumps.

Example: Repeat with other students and other exercises.

Before and After

Equipment: Counters, numeral cards 1 to 10 (Material Master 4–1) and game boards (Material Master 5–4).

Using Materials

Activity: Each student takes turns to pull a card from the stack of cards. The student chooses the number before or after the number they have pulled out and then covers this number on their game board. If the student cannot cover a square, the next student has their turn. The pulled out card is replaced in the stack so that it can be pulled out again. The game ends when a student covers all their numbers.

Example: Play the game again.

Clapping

Equipment: None.

Using Materials

Activity: The students count from one, clapping their hands in time.

Example: Repeat but start from numbers other than one. Also count backwards from various numbers.

Activity: The students count from one, clapping their hands and slapping their knees alternately in time.

Examples: Repeat but start from numbers other than one. Also count backwards from various numbers.

Activity: The students count from one, clapping hands and slapping knees alternately in time. However, they do not say the numbers out loud when they slap their knees.

Examples: Repeat but start from numbers other than one. Also count backwards from various numbers.

Activity: Count backwards and forwards from some number by slapping knees, then chest, then clapping hands.

Example: Repeat by varying the parts of the body used.

Walk the Bridge

Equipment: Large numeral cards 1 to 10 (Material Master 4–3).

Using Materials

Activity: Place numeral cards on the ground in order from 1 to 10 to form a bridge.

The students count aloud as one student steps on the number. The student who is "walking the bridge" may decide to walk forwards or backwards. The other students watch closely to produce the forwards or backwards counting sequence.

Example: Have the student stand on a number and discuss what is before and after this number.

Ordering Numerals

Equipment: Numeral cards 1 to 10 (Material Master 4–1).

Using Materials

The students order the numeral cards from 1 to 10 forwards or backwards. They practise saying the sequence as they point at the cards.

Example: Ask them to point to a number. Then ask, "What number comes before this one?" "What number comes after this one?"

Up or Down

Equipment: Pegs. Vertical number lines (Material Master 5–3).

Using Materials

Activity: Each student has a vertical number line from 1 down to 20 and places a peg on 1 and a peg on 20. Choose a student to think of a mystery number between 1 and 20. The rest of the students ask the chooser if a number they say is up or down from the mystery number. The students move their pegs to narrow the range of answers until the right number is found.

Dice Groups

Equipment: Dice. Counters. Playing boards (Material Master 5–5).

Using Materials

Each small group has a dice and a playing board. Each student places their counter at the start. The students take turns to throw the dice and move their counter that number of squares. The winner is the first to throw exactly the right number to finish on their last move.

How Many Cubes?

Equipment: Wooden cubes or unilink cubes.

Using Materials

In pairs, each student picks up a handful of cubes and says how many they think they have before counting them. Each student counts their partner's cubes. Repeat the process.

Then each student picks up a handful of cubes and briefly shows the cubes to their partner. The pair discuss who they think has got more. They check this by lining up the cubes side by side. Discuss how many more are in one line than the other.

Caterpillar Legs

Equipment: Pegs. Numeral cards 1 to 10 (Material Master 4–1). Caterpillar legs (Material Master 5–6).

Using Materials

Each student has some blank caterpillar legs to which numeral cards are added. If the students have little number recognition, the numeral cards should initially be 1, 2, or 3. Gradually increase the number size. The students add the correct number of legs (i.e., pegs) then order the caterpillars from the smallest number of legs to the largest.

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Petals and Flower Centres

Equipment: Numeral cards 1 to 10 (Material Master 4–1). Counters.

Using Materials

The students surround the centre of a flower (a numeral card) with the correct number of petals (counters).

Feed the Elephant

Equipment: Paper cups. Counters or beans or "Checkout Rapua" material in the SEA kit. Laminated elephants (Material Master 5–7).

Using Materials

Add single-digit numbers to the "speech bubbles" on the elephants and clip them to the paper cups. In pairs, the students feed the elephants the correct amount of food (counters etc.) and check their partner's answer.

In pairs, they order the cups either forwards or backwards. Using numbers from six to nine, the students feed five yellow "bananas" (yellow counters), and the rest are green-coloured "vegetables" (green counters).

Birthday Cake

Equipment: Iceblock sticks. Numeral cards 1 to 10 (Material Master 4–1). Laminated birthday cake (Material Master 5–8).

Using Materials

In pairs, the students select cards for the age of the student having a party and match this with "candles" (iceblock sticks) on the birthday cake.

Repeat this but ask questions like "How many more candles will be needed in two years' time?" and "How many more candles will be needed when the student goes from seven to nine years of age?"

Murtles 5 and ...

Equipment: Counters. Wooden cubes labelled 0, 1, 2, 3, 4, 5. Turtle game board (Material Master 5–9).

Using Materials

Each student has a turtle game board. In pairs, the students alternately toss a wooden cube labelled 0 to 5, add five onto this number, and put a counter on their board. The winner is the first person to cover all the numbers on the turtle.

Facts to 10

Equipment: Tens frames (Material Master 4–6). Playing boards (Material Master 5–10). Counters.

Using Materials

Have a pile of tens frames with five or more dots on each. One person shows a tens frame briefly. That player says how many empty spaces there are on the frame. If correct, the player moves their counter forwards that number of spaces on the playing board. Turns rotate. The first person to reach the end is the winner.

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Counting

Refer to the Knowledge Activities, *Book 4: Teaching Number Knowledge,* page 11.

Number Fans

Refer to the Knowledge Activities, *Book 4: Teaching Number Knowledge,* page 4.

Number Mat and Lily Pads

Refer to the Knowledge Activities, *Book 4: Teaching Number Knowledge,* page 2.

Pipe Cleaner Numbers

Refer to the Knowledge Activities, *Book 4: Teaching Number Knowledge,* page 4.

Beep

Refer to the Knowledge Activities, *Book 4: Teaching Number Knowledge,* page 12.

Learning Experiences to Move Students from One-to-one Counting to Counting from One on Materials

Required Knowledge

Before attempting to develop students' counting from one on materials ideas, check that they can:

- count forwards from a given number in the range of 1 to 10;
- count backwards from 10 to 1;
- recognise and write numerals from 1 to 10;
- recognise instantly numbers from 1 to 5 on fingers.

Adding and Subtracting with Counters

I am learning to add and subtract small numbers on materials.

Equipment: Counters.

Using Materials

Problem: "Gary has four bottles of drinks in the cupboard, and he buys three more bottles for his birthday party. How many bottles of drink does Gary have now?"

Record 4 + 3 on the board or modelling book . The students solve the problem using counters. Record 4 + 3 = 7 on the board or modelling book.

Examples: Word stories and recording for: 3 + 5   2 + 5   6 + 2   2 + 8   4 + 4   7 + 2 ...

Problem: "Jules puts eight boiled eggs on the breakfast table. Mary and Robyn eat one egg each. How many eggs are left?"

Record 8 – 2 on the board or modelling book . The students use counters to solve the problem. Record 8 – 2 = 6 on the board or modelling book.

Examples: Word stories and recording for: 6 – 3   10 – 3   7 – 6   9 – 1   6 – 5 ...

Adding and Subtracting with One Hand

I am learning to add and subtract with numbers up to five using one hand.

Equipment: None.

Using Materials

Problem: "Jill has three apples, and she buys two more apples. How many apples does she have altogether?"

Record $3 + 2$ on the board or modelling book. The students model three fingers and then two more fingers on the same hand. The students need to recognise that three fingers and two fingers equals five fingers without counting to solve the problem. Record $3 + 2 = 5$ on the board or modelling book.

Examples: Word stories and recording for: $1 + 4$ $3 + 1$ $5 - 2$ $4 - 3$ $2 + 3$ ...

Problem: "When Pippa goes to sleep, she has three Easter eggs in her bedroom. While she is asleep, her mother gives her some more Easter eggs. She wakes up in the morning to find that she has five Easter eggs. How many eggs did her mother give her?"

Discuss and record $3 + \square = 5$ on the board or modelling book. Encourage the students to act out the problem for themselves. Record $3 + 2 = 5$ on the board or modelling book.

Examples: Word stories and recording for: $1 + \square = 4$ $4 + \square = 5$ $5 - \square = 2$ $4 - \square = 3$ ...

Problem: "Mr Brown is taking five children to have a birthday party at a hamburger shop. Peter and Roberta are already in the car. How many children are on the pavement waiting to get into the car?"

Record $5 - 2$ on the board or modelling book. The students use one hand to solve the problem. Record $5 - 2 = 3$ on the board or modelling book.

Examples: Word stories and recording for: $5 - 4$ $4 - 3$ $5 - 2$ $3 - 2$ $5 - 3$ ...

Problem: "When Mrs Hemi goes out to the shops, there are five marshmallows in the jar. When she comes home, she finds that her son Bryce has eaten some of them, and there are two marshmallows left. How many marshmallows has Bryce eaten?"

Record $5 - \square = 2$ on the board or modelling book. The students use one hand to solve the problem.
Record $5 - 3 = 2$ on the board or modelling book.

Examples: Word stories and recording for: $5 - \square = 4$ $4 - \square = 3$ $5 - \square = 1$ $3 - \square = 1$ $4 - \square = 3$ ...

Learning Experiences for Counting from One on Materials

Fly Flip

I am learning to solve problems like 5 + □ = 9. The missing number is between 1 and 5.

Equipment: Fly flips (Material Master 4–5).

Using Materials

Problem: "There are eight flies on this fly flip. (The students can see the eight on the bottom and five flies on their side of the fly flip.) How many flies are on the other side?"

Record 5 + □ = 8 on the board or modelling book. Encourage the students to model eight as five fingers and three fingers and then work out that there must be three flies on the reverse side.
Record 5 + 3 = 8 on the board or modelling book.

Examples: Repeat for the nine, five, six, 10, and seven fly flips.

Problem: Show the students the eight fly flip with three flies visible to them. Ask how many flies they can see and then what number they can see.

Discuss how many flies are hidden from the students. The students use their fingers if needed. Discuss the answer and record 3 + 5 = 8 on the board or modelling book.

Examples: Repeat for the nine, five, six, 10, and seven fly flips.

Using Fives

I am learning to use materials to solve problems in addition and subtraction that involve five as one of the numbers.

Equipment: None.

Using Materials

Problem: "Charlotte has $7, and she buys an ice cream for $2. How much money does she have left?"

Record 7 – 2 on the board or modelling book. The students model seven fingers and try to solve the problem by removing the hand showing two fingers. Discuss the answer.
Record 7 – 2 = 5 on the board or modelling book.

Examples: Word stories and recording for: 3 + 5   8 – 3   4 + 5   10 – 5   7 – 5   10 – 5   5 + 5 ...

Problem: "Grant has nine marbles. On the way to school, he drops some. When he gets to school, he finds that he has only five marbles left. How many marbles has he dropped?"

Record 9 – □ = 5 on the board or modelling book. The students use their fingers to work out that four marbles were dropped.

Examples: Word stories and recording for: 10 – □ = 5   5 + □ = 10   9 – □ = 4   3 + □ = 8   5 + □ = 8 ...

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Challenging Hands Problems

"Crossing over" from one hand to the other requires the beginning of part-whole thinking, which is challenging for students who use counting from one on materials. For example, to work out 4 + 3, the students put four on one hand and then may struggle to "see" three fingers split into one on one hand plus two more on the other hand. When the students realise this, they can see that 4 + 3 is the same as 5 + 2. Then they can "see" that the answer to 4 + 3 is 7.

> I am learning to solve addition and subtraction problems with two hands that involve crossing over from one hand to the other to work out the answer.

Equipment: None.

Using Materials

Problem: "Mitch has seven teddy bears, and he gives three of them to his friend. How many teddy bears does Mitch now have?"

Record 7 – 3 on the board or modelling book. The students model seven on their fingers as five and two and discuss removing the two then one more from the five to solve the problem. Record 7 – 3 = 4 on the board or modelling book.

Examples: Word stories and recording for: 8 – 4 7 – 4 4 + 3 10 – 6 9 – 6 10 – 8 2 + 6 3 + 6 7 – 6 ...

Challenging problem: "Fiona has $9. She buys a cake and a drink at the cafe. When she gets home, she has $3 in her purse. How much did she spend at the cafe?"

Record 9 – □ = 3 on the board or modelling book. The students use their fingers to work out that six is the answer. Record the answer on the board or modelling book.

Challenging examples: Word stories and recording for: 10 – □ = 2 3 + □ = 9 9 – □ = 2 2 + □ = 8 ...

Teens and Fingers

Key Ideas

Check that the students know that "teen" means 10 so they can decode a "teen" word as ones plus 10. For example, sixteen means six and 10. Check that in the two teen words that do not quite fit the pattern, namely, fifteen and thirteen, the students know that "fif" means five and "thir" means three. The students also need to know that the two unsystematic "teen" words, twelve and eleven, actually mean 10 and two and 10 and one respectively. (See Knowledge Activities, *Book 4: Teaching Number Knowledge,* page 3, for an activity.)

> I am learning to solve addition and subtraction problems that involve the "teen" numbers and using groups of five fingers.

Equipment: None.

Using Materials

Problem: "Vincent has 14 snack packs, and he eats one every day of the week for his lunch at school. How many packs does he have left at the end of one week?"

Record 14 – 5 on the board or modelling book. Put the students in pairs and get them to negotiate how to show 14 fingers between them. Normally, one student shows 10 as five and five and the other shows four. Removing five from 10 leaves five. So the answer is four from one student and five from the other, which together is nine. Record 14 – 5 = 9 on the board or modelling book.

Note that none of the problems below require going across a five. For example, 12 – 4 is not asked because it would require the students to use part-whole thinking by removing two and then two more out of the hands that show 10. This type of problem is delayed until students reach the part-whole stages.

Examples: Word stories and recording for: 14 – 4 8 + 5 7 + 5 20 – 5 5 + 10 17 – 7 12 – 5 ...

Challenging examples: Word stories and recording for: 14 – □ = 9 8 + □ = 13 4 + □ = 14 20 – □ = 10 □ + 4 = 14 ...

Ones and Tens

I am learning to count objects by creating groups of 10 from materials.

Equipment: Materials suitable for bundling into tens, for example, sticks with rubber bands around bundles of 10 or beans in lots of 10 in film canisters or plastic bags. If available, arrow cards (Material Master 4–14).

Using Materials

Problem: "Your job at the factory is to bundle up sweets and send them to the shops. Each bundle has exactly 10 sweets in it. At the end of the day, you have to write down how many sweets you have packed."

Give pairs of students about 50 items of loose material and get them to create bundles/ containers of 10. Record answers for all the pairs in a table on the board or modelling book or show the answer on the arrow cards.

Number of Bundles of 10	Ones Left Over

Many students will be able to read two-digit numbers but not realise that they represent ones and tens. This activity is designed to help them to learn this. In particular, many may not realise that "ty" at the end of words means "tens". It's important that students learn to decode words like "sixty" as six tens. You can assist this by rubbing out the headings in the table to leave the numbers only.

Example: With loose materials, the students repeat the grouping and recording objects in ones and tens.

Problem: "Jerry has 43 lollies, and Mark has 34 lollies. Who has more lollies?"

Record 43 and 34 on the board or modelling book. The students model both 43 and 34 with bundled materials and discuss the fact that 43 is more. Record "43 is more than 34" on the board or modelling book.

Examples: Word stories and recording for these pairs: Which number is larger: 56 or 65? 14 or 41? 25 or 52? 32 or 23? ...

To study this problem in greater depth, see Vince Wright, "The Bubblegum Machine", *Connected 2 1999.*

More Ones and Tens

I am learning to model two- and three-digit numbers on play money.

Equipment: $1, $10, and $100 notes (Material Master 4–9). Arrow cards (Material Master 4–14).

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Using Materials

Problem: In pairs, one student shows two-digit numbers on the arrow cards, and the other student makes up this amount of money with $1 and $10 notes. Then students swap roles.

Challenging example: Repeat with three-digit numbers.

Learning Experiences to Move Students from Counting from One on Materials to Counting from One by Imaging

Required Knowledge

Before attempting to develop students' counting from one by imaging ideas, check that they can:

- count forwards from a given number to another number in the range of 1 to 20;
- count backwards from 20 to 1;
- recognise and write numerals from 1 to 20;
- recognise instantly numbers from 1 to 10 on their fingers using a five strategy. For example, eight is shown as five and three rather than four and four.

Using One Hand

I am learning to image numbers up to five to solve addition and subtraction problems.

Equipment: None.

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Using Materials

Problem: "Last month, Moira read two *Junior Journals*. This month, she read three more. how many *Junior Journals* has she read altogether? "

Record 2 + 3 on the board or modelling book. The students solve the problem on one hand. Record 2 + 3 = 5 on the board or modelling book.

Example: Word stories and recording for: 1 + 4 5 – 1 4 – 2 2 + 2 5 – 4 3 + 2 ...

Using Imaging

Shielding: Problem: "Mr Randall has five Greedy Cat books, and he gives out some of them to some of his students to read. He has three books left. How many books did Mr Randall give out?"

Record 5 – ☐ = 3 on the board or modelling book. The students each put a hand behind their back and think about the missing number. If necessary, the students fold back to ***Using Materials*** by bringing their hands in front of them and solving the problem. Record 5 – 2 = 3 on the board or modelling book.

Examples: Word stories and recording for: 1 + ☐ = 4 5 – ☐ = 0 4 – ☐ = 2 3 + ☐ = 5 ...

Imaging Only: When the students can easily solve problems by imaging with shielding, move on to imaging only.

Problem: "What is the missing number: 5 – ☐ = 1?"

Record 5 – □ = 1 on the board or modelling book. The students think about the solution without using their fingers. Record 5 – 4 = 1 on the board or modelling book.

Examples: Word stories and recording for: 1 + □ = 5   3 + □ = 5   4 – □ = 1
1 + □ = 3 ...

Using Tens Frames

I am learning to image numbers up to five to solve addition and subtraction problems.

Equipment: Pre-printed tens frames (Material Master 4–6). Counters.

Repeat the activity **Using One Hand**, above, but replace one hand by using one column of an empty tens frame.

Both Hands

I am learning to image numbers up to 10 to solve addition and subtraction problems.

Equipment: None.

Using Materials

The students achieving success proceed to ***Using Imaging****. Otherwise, they proceed to the next activity at some later time.*

Problem: "Tana has seven plastic bears, and he hides some. Now his sister can see two bears. How many bears has Tana hidden?"

Record 7 – □ = 2 on the board or modelling book. The students model seven fingers as five and two and solve the problem. Record the answer on the board or modelling book.

Examples: Word stories and recording for: 8 – □ = 5   10 – □ = 8   5 + □ = 7
3 + □ = 8   7 + □ = 10 ...

Using Imaging

Shielding: Problem: "Margaret has five stickers on her schoolbag, and she adds three more. How many stickers does she have now?"

Record 5 + 3 on the board or modelling book. The students put both hands behind their backs and think about the answer. If necessary, fold back to ***Using Materials***. (The students bring their hands out in front of them and solve the problem.) Record the answer on the board or modelling book.

Examples: Word stories and recording for: 10 – □ = 9   6 – □ = 1   5 + □ = 9
3 + □ = 8   5 + □ = 10 ...

Imaging Only: Problem: "What is the missing number: 9 – □ = 4?"

Record 9 – □ = 4 on the board or modelling book. The students think about the solution without using their fingers at all. Record the answer on the board or modelling book.

Examples: Word stories and recording for: 6 – □ = 5   9 – □ = 4
1 + □ = 6   5 + □ = 7   7 + □ = 10 ...

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Imaging with Tens Frames

I am learning to image numbers up to 10 to solve addition and subtraction problems.

Equipment: Pre-printed tens frames (Material Master 4–6), and counters, or a magnetic whiteboard with tens frames drawn on it and magnetic counters.

Using Materials

Problem: "Brita has 10 rabbits, and she gives two rabbits to her friend. How many rabbits does Brita have now?"

Record 10 – 2 on the board or modelling book. The students put 10 counters on blank tens frames and remove two. Without counting, encourage the students to recognise instantly that there are eight counters left. Record the answer on the board or modelling book.

Examples: Word stories and recording for: 10 – 3   4 + 5   10 – 6   3 + 5
2 + □ = 7 ...

Using Imaging

Shielding: Problem: "Jonah has five jelly beans, and he gets three more jelly beans from a friend. How many jelly beans does Jonah now have?"

Record 5 + 3 on the board or modelling book. Out of the students' sight, build five on an empty tens frame. Show the students the tens frame and ask them to describe how you have built it. Drawing pictures in the air may help. Add three, out of the students' sight, and ask them to explain how you built it. Ask how many counters there are altogether. If necessary, fold back to ***Using Materials*** by showing the students the hidden counters. Record 5 + 3 = 8 on the board or modelling book.

Examples: Word stories and recording for: 4 + 5   10 – 4   6 – 5   5 + 2   10 – 8
3 + □ = 8 ...

Imaging Only: Problem: "What is the missing number in 8 – □ = 3?"

Record 8 – □ = 3 on the board or modelling book. The students think about the solution without seeing the tens frames. Record the answer on the board or modelling book.

Examples: Word stories and recording for: 9 – □ = 4   8 – □ = 5   2 + □ = 10
2 + □ = 5   1 + □ = 10 ...

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Learning Experiences for Counting from One by Imaging

What's Hidden?

I am learning to image numbers up to five to solve addition and subtraction problems.

Equipment: Plastic bears or counters. Opaque containers.

Using Imaging

Problem: "Here are three bears, and there are some more hidden under the container. Altogether, there are five bears. How many bears are hidden?"

Record 3 + ☐ = 5 on the board or modelling book. The students solve the problem by imaging the numbers. Fold back, if necessary, to ***Using Materials*** by showing what is hidden.

Examples: Put the students into pairs. One student hides some bears, and the other solves the problem. Then reverse the roles. Encourage the students to record the problems and answers.

Imaging Many Hands

I am learning to image numbers up to 20 to solve addition and subtraction problems.

Equipment: None.

Using Imaging

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Shielding: Problem: "Leanne has eight sweets, and she buys five more. How many does she have now?"

Record 8 + 5 on the board or modelling book. In pairs, the students discuss who would show eight fingers and who would show five fingers *without* showing the fingers. They discuss how they would put five and five together to give 10 and three fingers. Fold back to ***Using Materials*** by showing the fingers if necessary. Have the group discuss their methods.

Record 8 + 5 = 13 on the board or modelling book.

Examples: Word stories and recording for: 20 – 5   5 + 9   15 – 10   20 – 15
17 – 7   12 + 5   7 + ☐ = 12 ...

Imaging Only: Problem: "What does 5 + 7 equal?"

Record 5 + 7 on the board or modelling book. Each student solves the problem independently by imaging five and seven.

Examples: Word stories and recording for: 11 – 5   5 + 6   11 – 10   12 – 2
15 – 5   11 + 5   6 + ☐ = 11 ...

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Making Tens

I am learning to use imaging to solve problems in addition and subtraction that involve 10 as one of the numbers.

Equipment: Pre-printed tens frames (Material Master 4–6). Counters. A blank card.

Using Materials

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Problem: "Scott has seven black sweets, and he needs 10. How many more sweets does Scott have to get?"

Record 7 + □ = 10 on the board or modelling book. Get out a tens frame that shows seven dots. Discuss why the answer is three. Put three counters on the tens frame. Record 3 on the board or modelling book.

Examples: Word stories and recording for: 9 + □ = 10   6 + □ = 10   5 + □ = 10   4 + □ = 10   8 + □ = 10 ...

Problem: "What is 10 – 4?"

Record 10 – 4 on the board or modelling book. Take a tens frame with 10 dots on it and discuss which four can be removed. Cover these four with a blank card to show that six are left.

Examples: Word stories and recording for: 10 – 1   10 – 7   10 – 3   10 – 5   10 –9   10 – 8 ...

Using Imaging

Shielding: Problem: "Solve 4 + □ = 10." Have four on a tens frame hidden from the students under the blank card. Ask the students to solve 4 + □ = 10 by imaging the spaces needed to fill the card up. Discuss what the students are imaging. Get them to draw what they see in the air. Fold back if necessary. Record the answer on the board or modelling book.

Examples: Word stories and recording for: 3 + □ = 10   6 + □ = 10   6 + 4   2 + 8   3 + 7 ...

Imaging Only: Word stories and recording for: 10 – 5   1 + □ = 10   6 + 4   □ + 8 = 10   10 – 8   □ + 2 = 10 ...

Crossing the Five Barrier

I am learning to solve addition and subtraction by imaging tens frames that involves crossing over from one column of five to the other column to work out the answer.

Equipment: Pre-printed tens frames (Material Master 4–6). Counters. Blank card.

Using Materials

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Problem: "Work out 3 + 4."

Record 3 + 4 on the board or modelling book. Show a tens frame pre-printed with three dots. Ask the students to point where four more counters would go. Ensure they first image two to complete a column of five, then add two more in the other column. Check this by adding four counters. Record the answer on the board or modelling book.

Examples: Word stories and recording for: 3 + 3   6 + 3   2 + □ = 6   6 – □ = 2   6 – 4   4 + 4   8 – □ = 4 ...

Using Imaging

Shielding: Problem. "Find 7 – □ = 4."

Record 7 – □ = 4 on the board or modelling book. Hide a pre-printed tens frame with seven dots and ask the students to image the tens frame. Then have them image removing three dots. Let them draw in the air if this helps. Fold back, if necessary, by showing seven dots and shielding three of the dots. Record the answer on the board or modelling book.

Examples: Word stories and recording for: 3 + □ = 7 □ + 4 = 8 4 + □ = 9
□ + 2 = 8 3 + 4 8 – 2 ...

Imaging Only: Examples: Word stories and recording for: 2 + □ = 6 8 – □ = 4 ...

Challenging example: Problem: "Angus has some money. He spends $4. Now he has $3. How much did he start with?"

Examples: Word stories and recording for: □ – 3 = 3 □ – 4 = 3 □ – 3 = 6
□ – 2 = 8 ...

Fingers Again

Repeat ***Crossing the Five Barrier*** above but use hands in place of tens frames.

Ten Sweets per Packet

Key Ideas

Check that the students can:

- identify that "ty" words are "tens" (For example, 80 means eight tens);
- decode the ones and tens notation, for example, 36 means three tens and six ones.

I am learning to use addition facts like 4 + 5 = 9 to work out 40 + 50.

Equipment: A Slavonic abacus, or place value materials in bundles or containers of 10 objects (for example, beans).

Using Materials

Problem: "There are 10 sweets in each packet. Miranda has three packets, and her grandfather buys her two more packets. How many sweets does Miranda have altogether?"

Discuss why you write, "3 packets (of 10) + 2 packets (of 10)". Below this, record 30 + 20 on the board or modelling book. Model 30 by three rows of 10 on the abacus (or on the bundled or container materials). Ask the students how to model 20. Discuss why the answer is 50 and record 30 + 20 = 50 on the board or modelling book.

Examples: Word stories and recording for: 3 packets + 1 packet 5 packets – 1 packet
1 packet + 5 packets 60 – 20 80 + 10 50 – 40 ...

Using Imaging

Shielding: Problem: "A packet contains 10 sweets. If Edward has seven packets and Andrew has one packet, how many sweets do they have altogether?"

Record, "7 packets of 10 + 1 packet of 10", and, below this, 70 + 10 on the board or modelling book. Model 70 on the abacus out of sight of the students. Ask them what you have done. Model 10 more.

Record 70 + 10 = 80 on the board or modelling book.

Examples: 5 packets + 3 packets 5 packets – 1 packet 3 packets – 2 packets
10 + 50 30 + 20 ...

Imaging Only: Examples: Word stories and recording for: 5 packets + 4 packets
50 – 40 90 – 40 50 + 10 80 + 10 ...

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Learning Experiences to Move Students from Counting from One by Imaging to Advanced Counting

Required Knowledge

Before attempting to develop students' advanced counting ideas, check that they can:

- count on and back from numbers between one and 100;
- count from one by imaging to solve addition and subtraction problems;
- count forwards or backwards to add or subtract one from the previous number of objects in a set.

  Being able to do this is a critical predictor that students will be able to count on or count back when solving addition or subtraction problems. Refer to the Algebra unit, "Counting on Counting" found on www.nzmaths.co.nz. This unit contains key ideas to help students to generalise their counting.

Assessment example:

Show the students seven objects and then hide them under a container. Place one more object on top of the container. Ask how many objects there are altogether. Also put nine objects in the container and remove one. Ask how many are now in the container.

Number Tiles

Key Ideas

Check that the students understand that the last number in a line of numbered objects gives the number in the set only if the numbers are in order and no number is missing. For example, there are not nine tiles below because the 7 is missing:

1	2	3	4	5	6	8	9

I am learning to add by counting on when the larger number is given first.

Equipment: A set of small cards or tiles labelled 1 to 20.

The students achieving success proceed to ***Using Imaging****. Otherwise, they proceed to the next activity at some later time.*

Using Materials

Problem: "Michael has nine sweets in one bag and two sweets in another bag. How many sweets does he have altogether?"

Record 9 + 2 on the board or modelling book. Get the students to arrange tiles (in place of sweets) one to nine in order. Shield numbers 10 and 11 in your hand. Ask what the numbers on the sweets (tiles) in your hand are and then what 9 + 2 equals. Then lay down tiles 10 and 11 to check that the answer is 11. Record 9 + 2 = 11 on the board or modelling book.

Examples: (Second number is five or less.) Word stories and recording for: 4 + 2 7 + 2 12 + 2 7 + 3 12 + 3 16 + 3 ...

The students achieving success proceed to ***Using Number Properties****. Otherwise, they proceed to the next activity at some later time.*

Using Imaging

Shielding: Solve eight oranges plus four oranges.

Record 8 + 4 on the board or modelling book. Turn the tiles numbered one to eight face down to hide the numbers. Hold the next four tiles (9 to 12) in your hand and ask the students to image what numbers you have.

Ask what 8 + 4 equals. If necessary, fold back to turning the eight tiles over and add tiles 9 to 12 to the end. Some students may need to use their fingers to count on. Record 8 + 4 = 12 on the board or modelling book.

Examples: Word stories and recording for: 12 + 2   9 + 2   11 + 3   12 + 3
13 + 3   8 + 4   7 + 4   11 + 5 ...

Imaging Only: Examples: Word stories and recording for: 11 + 4   7 + 2   13 + 5
11 + 5   18 + 2   6 + 5   13 + 5 ...

Using Number Properties

By increasing the size of the first number in addition problems, the students are encouraged to let go of using materials or imaging and concentrate on the properties of the numbers instead.

Examples: Word stories and recording for: 29 + 4   46 + 5   63 + 5   78 + 3
34 + 4   89 + 3 ...

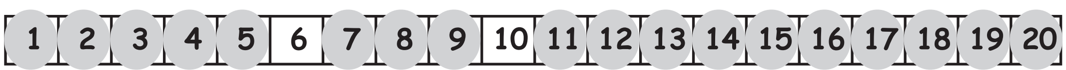
Challenging examples: 158 + 3   198 + 5   212 + 4   238 + 4   394 + 5
117 + 2   392 + 4 ...

Note that these problems require students to have knowledge of number sequences beyond 100.

The Number Strip

Key Ideas

As with **Number Tiles** above, the students must realise that the last number in a set of tiles gives the number of tiles only if each number has a counter on it. For example, there are not 20 counters shown below because counters 6 and 10 are missing.



I am learning to add by counting on when the larger number is given first.

Equipment: A set of number strips labelled 1 to 20 (Material Master 5–1). Two sets of different-coloured transparent counters.

Using Materials

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Problem: "Josie has nine toy cars, and her mother gives her four more as a present. How many toy cars does Josie have?"

Record 9 + 4 on the board or modelling book. The students place nine counters of one colour on the number strip from one to nine. The students have four counters of another colour. Before putting the counters on the strip, ask what numbers they go on and then what 9 + 4 is. The students confirm the answer by placing four counters on squares 10, 11, 12, and 13. Record the answer on the board or modelling book.

Examples: (Second number is five or less.) Word stories and recording for: 6 + 2
9 + 2   7 + 3   9 + 3   11 + 4 ...

Using Imaging

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Shielding: Problem: "Work out seven cats plus four cats."

Turn the number strip over and encourage the students to image seven counters. Then ask what squares the four counters will go on and so what 7 + 4 is. Fold back, if necessary, by building one to seven in one colour and 8 to 11 in another colour. Some students may need to count on with their fingers.

Examples: Word stories and recording for: 13 + 2   11 + 2   6 + 3   8 + 4
11 + 5   12 + 2 ...

Imaging Only: Examples: Word stories and recording for: 15 + 3   8 + 4   13 + 5
9 + 3   7 + 5   13 + 3   15 + 4 ...

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Using Number Properties

Provide addition problems in which one first number has two digits and the second number is a single digit below five. This encourages the use of number properties.

Examples: Word stories and recording for: 75 + 2 46 + 2 49 + 2 53 + 3 87 + 3 91 + 4 67 + 4 ...

The Bears' Picnic

I am learning to add by counting on when the larger number is given first.

Equipment: Set of plastic bears (or counters) for counting. An opaque container.

Using Materials

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Problem: "Eight bears go into their cave. Bobby and Bernice Bear arrive late to join the family. How many bears are there now in the cave?"

Record 8 + 2 on the board or modelling book. Hide eight bears in the container. Add Bobby Bear and then ask how many bears are now in the cave. Add Bernice Bear and then ask how many bears are now in the cave. Ask what 8 + 2 equals. Record 8 + 2 = 10 on the board or modelling book.

Examples: (Encourage the students to use their fingers to track the counting on.) Word stories and recording for: 12 + 2 7 + 2 13 + 2 7 + 3 11 + 3 14 + 3 7 + 4 11 + 4 ...

Using Imaging

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Shielding and *Imaging Only:* Examples: Word stories and recording for: 11 + 2 9 + 3 12 + 4 14 + 5 7 + 4 12 + 5 ...

Using Number Properties

Examples: Word stories and recording for: 39 + 4 66 + 4 33 + 5 28 + 3 87 + 5 ...

Frog Jumps

I am learning to add by counting on with the larger number first.

Equipment: Number strips (Material Master 5–1).

Using Materials

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Problem: "Freddo the Frog can jump one space at time. He makes four jumps from the number 14. Where does Freddo end up?

What is 14 + 4?"

Record 14 + 4 on the board or modelling book. The students track four jumps directly on number strips to give 18. Record 14 + 4 = 18 on the board or modelling book.

Examples: Word stories and recording for: 8 + 3 12 + 4 19 + 5 7 + 6 11 + 5 12 + 2 6 + 5 ...

Using Imaging

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Shielding and *Imaging Only:* Examples: Word stories and recording for: 12 + 3 19 + 4 17 + 5 8 + 5 21 + 2 ...

Using Number Properties

Examples: Word stories and recording for: 89 + 4 93 + 5 56 + 3 78 + 4 51 + 3 ...

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Learning Experiences for Advanced Counting

The Bigger Number First

I am learning to count on from the larger number even when the smaller number is given first.

Equipment: Counters.

Using Materials

Problem: "Jo has three cakes, and Marie has eight cakes. How many cakes are there together?"

Record 3 + 8 on the board or modelling book. The students model three counters and eight counters and work out the answer. Discuss the methods used. The students need to see why counting on from the eight, not from the three, is the more efficient method.

Examples: Word stories and recording for: 2 + 11   3 + 9   4 + 7 ...

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Using Imaging

The students who know how to count on from the first number and have connected this to counting on from the second number in the ***Using Materials*** phase may proceed straight to the ***Using Number Properties*** stage.

Using Number Properties

Pose a mixture of problems in which the larger number varies between being first and second.

Examples: Word stories and recording for: 2 + 99   4 + 67   78 + 5   5 + 47   2 + 65   31 + 3 ...

Challenging examples: Word stories and recording for: 67 + 2 + 3   55 + 4 + 3   2 + 88 + 4   1 + 3 + 78 ...

Change Unknown

I am learning to count on to solve problems like 4 + □ = 7.

Equipment: Sets of counters.

Using Materials

Problem: "A class is growing beans from bean seeds. When they leave school on Monday, six seeds have sprouted. When they come to school on Tuesday morning, they find that eight seeds have sprouted altogether. How many seeds have sprouted overnight?"

Discuss why the problem amounts to solving 6 + □ = 8. The students solve the problem with counters and discuss their methods. Encourage the students to count on by pointing at their fingers and saying "seven, eight, so the answer is two".
Record 6 + 2 = 8 on the board or modelling book.

Examples: Word stories and recording for: 4 + □ = 6   5 + □ = 6   7 + □ = 8   7 + □ = 9   6 + □ = 8   6 + □ = 9

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Using Imaging

Shielding and *Imaging Only:* Examples: Word stories and recording for: 12 + □ = 14   8 + □ = 10   12 + □ = 15   7 + □ = 11   8 + □ = 11   9 + □ = 12 ...

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

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Using Number Properties

Examples: Word stories and recording for: $87 + \square = 89$ $43 + \square = 45$
$79 + \square = 83$ $51 + \square = 54$ $\square + 56 = 59$ $\square + 43 = 45$ $58 + \square = 63$ ...

Counting Back

I am learning to count down to solve subtraction problems.

Equipment: A set of small cards or tiles labelled 1 to 20.

Using Materials

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Problem: "Michelle has 11 sweets and eats four. How many are left?"

Record 11 – 4 on the board or modelling book. Model the 11 sweets with tiles labelled in order 1 to 11. Ask which four sweets Michelle would eat and how many would be left. The students need to understand that removing sweets one to four does not solve the problem but removing sweets 11, 10, nine, and then eight does because the set one to seven remains. Record 11 – 4 = 7 on the board or modelling book.

Examples: Encourage the students to track on their fingers which sweets get eaten using word stories and recording for: 13 – 4 9 – 2 19 – 3 12 – 4 11 – 5
9 – 3 11 – 4 ...

Using Imaging

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Shielding and *Imaging Only:* Examples: Word stories and recording for: 12 – 3
19 – 2 17 – 3 8 – 2 21 – 4 15 – 5 ...

Using Number Properties

Examples: Word stories and recording for: 81 – 2 90 – 2 78 – 3 62 – 4
92 – 4 ...

Adding Tens

Key Ideas

Before attempting this activity, check that the students can:

- recall instantly the addition facts with answers up to 10;
- interchange instantly the "ty" words with "tens", e.g., 60 means six tens, 50 means five tens.

The activities that follow are important because they simultaneously help to develop addition, subtraction, *and* place value concepts.

I am learning to add tens to a number by counting on in tens or adding the tens together.

Equipment: Ice-cream containers. Ones and tens material – bundled and loose iceblock sticks, sets of 10 beans in film canisters and loose beans, unilink cubes, the Slavonic abacus, play money, place value blocks or tens frames.

This activity can be repeated using different kinds of material.

Using Materials

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Problem: "Hemi has 39 sweets, and he buys another packet of 20 sweets. How many sweets does he have altogether?"

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Record 39 + 20 on the board or modelling book. Get the students to model 39 and then 20 on ones and tens materials. The students work out the answer using materials and then discuss the answer. Record 39 + 20 = 59 on the board or modelling book.

Examples: Word stories and recording for: 35 + 20 42 + 10 20 + 34 30 + 34 21 + 50 40 + 27 ...

Using Imaging

Shielding: Problem: "Find 34 + 20."

Record 34 + 20 on the board or modelling book and ask the students how they would build 34 and 20 separately. Then hide 34 and 20 under an ice-cream container. Ask the students how many tens there are under the container. Discuss the idea that because 3 + 2 = 5, then three tens and two tens equals five tens, so 34 + 20 must be 54. (It's here that the instant recall of basic facts is needed.)
Record 34 + 20 = 54 on the board or modelling book.

Shielding and *Imaging Only:* Examples: Word stories and recording for: 18 + 20 30 + 24 23 + 40 13 + 50 10 + 46 ...

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Using Number Properties

Examples: Word stories and recording for: 87 + 10 78 + 20 20 + 62 46 + 50 80 + 17 ...

Challenging examples: The students will need to understand the meaning of three-digit numbers to do these: 340 + 20 640 + 30 423 + 20 50 + 204 ...

Subtracting Tens

I am learning to subtract tens from a number by counting back in tens or subtracting the tens first.

Equipment: Ones and tens material – bundled and loose iceblock sticks, film canisters with 10 beans in each canister and loose beans, unilink cubes, the Slavonic abacus, play money, place value blocks or tens frames.

Required Knowledge

Before attempting this activity, check that the students can instantly recall the single-digit subtraction facts.

Using Materials

Examples: Word stories and recording for: 45 – 20 52 – 10 42 – 20 35 – 30 63 – 50 48 – 30 ...

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Using Imaging

Shielding and *Imaging Only:* Examples: (Instant recall of single-digit subtraction facts is needed here.) Word stories and recording for: 48 – 40 51 – 20 53 – 50 27 – 20 64 – 10 43 – 40 57 – 50 71 – 40 ...

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Using Number Properties

Examples: Word stories and recording for: 97 – 10 78 – 30 88 – 20 90 – 30 71 – 60 ...

Challenging examples: The students will need to understand the meaning of three-digit numbers to do these: 240 – 20 340 – 40 443 – 20 570 – 20 ...

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Adding Ones and Tens

The answers should not exceed 99, and there should be no problem created by the ones adding up to 10 or more. For example, 56 + 32 is fine, but 56 + 87 and 45 + 28 are not. Problems like the latter two, where the ones add up to 10 or more, are delayed until the students have part-whole thinking strategies.

I am learning to add two-digit numbers where the sum of the ones digits is less than 10.

Equipment: Ones and tens material – bundled and loose iceblock sticks, film canisters with 10 beans in each and loose beans, unilink cubes, the Slavonic abacus, play money, place value blocks, or tens frames.

Using Materials

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Problem: "Ray has $34, and he gets $25 for a birthday present. How much money does Ray have now?"

Record 34 + 25 on the board or modelling book. The students model 34 and 25 using the chosen materials and group the ones and tens.
Discuss the answer and record 34 + 25 = 59 on the board or modelling book.

Examples: Word stories and recording for: 45 + 22 52 + 13 42 + 25 35 + 43 53 + 25 43 + 22 ...

Using Imaging

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Shielding and *Imaging Only:* Examples: Word stories and recording for: 14 + 43 31 + 25 23 + 41 24 + 25 32 + 26 38 + 21 13 + 41 25 + 23 44 + 24 ...

Using Number Properties

Examples: Word stories and recording for: 87 + 12 73 + 26 24 + 52 16 + 62 81 + 17 ...

Challenging examples: The students will need to understand the meaning of three-digit numbers to do these: 241 + 21 342 + 44 643 + 21 27 + 210 303 + 44 25 + 510 ...

Subtracting Ones and Tens

The ones digit in the beginning number should be greater than or equal to the ones digit in the subtracted number to avoid a renaming problem. For example, 56 – 34 is fine, but 57 – 29 is not. Problems like the latter are delayed until the students have part-whole thinking strategies.

I am learning to subtract two-digit numbers that do not involve renaming.

Equipment: Ones and tens materials.

Using Materials

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Examples: Word stories and recording for: 45 – 21 52 – 21 42 – 32 35 – 14 63 – 61 38 – 28 ...

Using Imaging

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Shielding and *Imaging Only:* Examples: Instant recall of single-digit subtraction facts is needed here. Word stories and recording for: 46 – 24 55 – 25 43 – 12 37 – 21 34 – 13 46 – 41 ...

Using Number Properties

Examples: Word stories and recording for: 97 – 11 78 – 38 99 – 62 56 – 46 88 – 17 ...

Challenging examples: The students will need to understand the meaning of three-digit numbers to do these: 240 – 20 347 – 47 443 – 21 577 – 26 ...

The Missing Ones and Tens

I am learning to solve problems like 45 + □ = 67, which do not involve renaming.

Equipment: Ones and tens materials, for example, bundled and loose iceblock sticks, 10 beans in film canisters and loose beans, unilink cubes, the Slavonic abacus, play money, place value blocks or tens frames.

Using Materials

Problem: "Marlene is planning a big party for 34 friends. Everyone needs a can of drink at the party, but she's got only 21 cans in her cupboard. How many cans of drink will she have to buy at the shop?"

Record this on the board or modelling book and discuss why this amounts to solving 21 + □ = 34.

The students use ones and tens materials to solve the problem, and their solutions are discussed.

Examples: Word stories and recording for: 15 + □ = 36 21 + □ = 36 20 + □ = 46 31 + □ = 37 42 + □ = 52 34 + □ = 36 ...

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Using Imaging

Shielding and *Imaging Only:* Examples: Word stories and recording for: 25 + □ = 46 31 + □ = 41 30 + □ = 41 11 + □ = 18 42 + □ = 52 4 + □ = 36 ...

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Using Number Properties

Examples: Word stories and recording for: 25 + □ = 86 31 + □ = 91 20 + □ = 85 ...

Challenging examples: The students will need to understand the meaning of three-digit numbers to do these: 241 + □ = 248 320 + □ = 345 643 + □ = 647 99 + □ = 109 ...

The Thousands Book

I am learning to add and subtract ones and tens by moving left, right, up, and down on charts with numbers up to 1 000.

Equipment: Thousands book (Material Master 4–7).

Using Materials

Problem: "Add 20 to 55 using the first sheet in the thousands book.

Locate 55, then drop down two squares to 75."
Record 55 + 20 = 75 on the board or modelling book.

Examples to add/subtract ones by going right/left and add/subtract tens by going down/up: 35 + 30 65 – 4 65 + 3 75 – 40 45 + 20 95 – 5 33 + 6 75 – 30 ...

Examples: Use the thousands book with numbers from 100 to 1 000.

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Learning Experiences to Move Students from Advanced Counting to Early Additive Part-Whole Thinking

Required Knowledge

Before attempting to develop students' early part-whole ideas, check that they can:

- identify ones and tens in any two-digit number;
- instantly recall addition of single-digit numbers up to a total of 10 and know the related subtraction facts;
- instantly recall doubles from 1 + 1 to 9 + 9;
- count on or back by imaging to solve addition and subtraction problems;
- identify any number from 1 to 10 on a tens frame instantly.

Make Ten

I am learning to add three or more numbers by first making up pairs that add up to 10.

Equipment: Counters.

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Using Materials

Problem: "Tina catches six fish, Miriam catches seven fish, and Liam catches four fish. How many do they catch altogether?"

Discuss and record 6 + 7 + 4 on the board or modelling book. The students model this with three piles of counters. Ask how these numbers can be added by combining the six and four piles. Encourage them to add the six and four first to give 10, which makes the answer (17) obvious. Record the answer on the board or modelling book.

Examples: Word stories and recording for a "make tens" strategy: 5 + 2 + 5   9 + 5 + 1
8 + 3 + 7 + 2   3 + 5 + 5 + 7   4 + 6 + 4 + 9 + 6   3 + 6 + 4 + 9 + 7...

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Using Imaging

Examples: Word stories and recording for a "make tens" strategy: 6 + 2 + 4   8 + 5 + 2
8 + 1 + 9 + 2   1 + 5 + 5 + 9   4 + 6 + 4 + 9 + 6   1 + 5 + 5 + 9 + 7...

Using Properties of Numbers

Examples: Record these numbers on paper and cross out pairs that make a 10 to get the answers: 8 + 6 + 4 + 7 + 2   7 + 8 + 4 + 3 + 2   5 + 2 + 3 + 7 + 8 + 5
1 + 2 + 5 + 7 + 9 + 5 + 3 ...

Compatible Numbers

I am learning to use compatible numbers to solve problems like 5 + 3 + 6 – 8, by first adding five and three to get eight then removing the eight.

Equipment: Counters.

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Using Materials

Problem: "Tina has six tomatoes, Miriam has two tomatoes, and Liam has three tomatoes. They use nine tomatoes for a salad. How many tomatoes are left?"

Discuss how to record the problem, then record 6 + 2 + 3 – 9 on the board or modelling book. The students model piles of six, two, and three counters. Discuss which two piles add to nine and remove them to leave the pile with two counters. Record = 2 on the board or modelling book.

Examples: Word stories and recording for: 5 + 2 + 5 – 10   9 + 5 + 1 – 6
8 + 2 + 7 – 9   4 + 5 – 9   3 + 5 + 5 – 8   4 + 6 + 4 + 3 – 7 ...

Using Imaging

Examples: Word stories and recording for: 4 + 2 + 5 – 9   8 + 5 + 2 – 7
10 + 2 + 7 – 12   10 + 5 – 15   3 + 2 + 6 – 5   2 + 6 + 4 + 3 – 7 ...

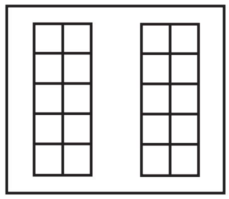
Using Number Properties

Examples: Record these numbers on paper and cross out the pairs and the numbers subtracted to get the answers: 8 + 6 + 4 – 10   7 + 8 + 2 – 9   7 + 3 + 3 + 7 – 10 – 10
1 + 2 + 5 + 2 – 7 – 3   9 + 3 + 4 – 7 – 2 ...

Subtraction in Parts

I am learning to subtract by splitting numbers into parts instead of counting down.

Equipment: A magnetic whiteboard with two tens frames drawn on it and magnetised counters (or use two blank tens frames [Material Master 4–6] and counters), bundled sticks (ones and tens) or beans in film canisters (ones and tens).



Using Materials

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Problem: "Brian has 14 oranges and eats six of them. How many are left?"

Record 14 – 6 on the board or modelling book. Model 10 and four counters on the tens frames. Ask the students, without them touching the tens frames, to say what remains when six are removed. Invite a student who says that eight is the answer to come and demonstrate how he/she got the answer. Typically, a student removes four from the four frame and then two from the frame with 10 on it to leave eight. Sometimes he/she will remove six from the 10 to leave four and add four and four to give eight. Either method is fine. Record 14 – 6 = 8 on the board or modelling book.

Examples: Word stories and recording for: 15 – 6   20 – 5   16 – 9   12 – 4
20 – 8   11 – 4   13 – 6 ...

Problem: "Tara has $34 and spends $5. How much money does she have left?"

Record 34 – 5 on the board or modelling book. Let the students model 34 with bundled sticks or beans in film canisters. The students break a 10 to get the answer. Discuss how the students get 29.

Examples: Word stories and recording for: 45 – 6   30 – 5   46 – 7   32 – 7
40 – 8   41 – 4   33 – 6 ...

Using Imaging

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Shielding: Problem: "Find 43 – 5."

Record 43 – 5 on the board or modelling book. Model 43 on sticks or beans out of sight of the students. Ask the students to image removing five and describe what is happening. Fold back, if necessary, to ***Using Materials***. Record 43 – 5 = 38 on the board or modelling book.

Examples: Word stories and recording for: 35 – 7   40 – 9   26 – 8   31 – 5
30 – 8   31 – 3   40 – 7 ...

Using Number Properties

Examples: Word stories and recording for: 75 – 8   90 – 8   96 – 9   61 – 5
70 – 7   51 – 6   81 – 9 ...

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*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Up over the Tens

I am learning to add by splitting numbers into parts to make tens.

Equipment: Animal strips (Material Master 5–2).

Using Materials

Problem: "The farmer has 28 cows. She buys some more cows at the stock sale. Now she has 33 cows. How many cows did she buy?"

Record 28 + □ = 33 on the board or modelling book. Tell the students to model 28 cows with two 10-animal strips and an eight-animal strip. Tell them you want them to make a complete 10 by modelling 30 animals. Before they put a strip on, the students should predict that they need a two. Let them check by doing this. Now ask them to predict what strip they need to reach 33 and check it. Ask how many animals were added on altogether. Record 28 + 5 = 33 on the board or modelling book.

Examples: Word stories and recording for: 7 + □ = 13 26 + □ = 31 8 + □ = 13 18 + □ = 23 25 + □ = 32 17 + □ = 24 ...

Using Imaging

Shielding and *Imaging Only:* Examples: Word stories and recording for: 17 + □ = 23 16 + □ = 25 28 + □ = 35 14 + □ = 23 35 + □ = 41 23 + □ = 31 ...

Using Number Properties

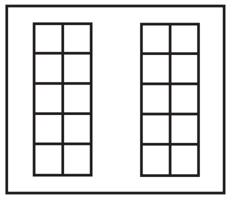
Examples: Word stories and recording for: 77 + □ = 83 56 + □ = 65 68 + □ = 75 54 + □ = 63 65 + □ = 71 22 + □ = 31 ...

Learning Experiences for Early Additive Part-Whole

Adding in Parts

I am learning to add by splitting numbers into parts.

Equipment: Preferably, a magnetic whiteboard with two tens frames drawn on it and magnetised counters or two blank tens frames (Material Master 4–6) and counters. Bundled sticks (ones and tens) or beans in film canisters (ones and tens).



Using Materials

Problem: "Peter has eight oranges and six apples. How much fruit has he got altogether?"

Record 8 + 6 on the board or modelling book. Model eight on a tens frame. Ask the students, without touching the tens frames, to say where six more go. Then say what 8 + 6 equals. Invite a student who says that 14 is the answer to come and demonstrate how he/she got the answer. Record 8 + 6 = 14 on the board or modelling book.

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Examples: Word stories and recording for: 5 + 6   9 + 7   8 + 5   7 + 6   8 + 7   4 + 8   3 + 9 ...

Problem: "Tere has $38 and saves $5 more. How much money does he have now?"

Record 38 + 5 on the board or modelling book. Let the students model 38 with bundled sticks or beans in film canisters. The students make a bundled 10 to get the answer. Discuss how the students get 43 as the answer.

Examples: Word stories and recording for: 45 + 6   29 + 7   8 + 45   7 + 46   28 + 7   4 + 28   33 + 9 ...

Using Imaging

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Shielding: Problem: "38 + 7 equals what?"

For a problem like 38 + 7, the students need to make the connection that removing two from the seven and adding it to 38 creates a 40. There is five left, so the answer is 45. For those students who do not make the connections, fold back to showing the materials and work through the solution.

Shielding and *Imaging Only:* Examples: Word stories and recording for: 35 + 7   42 + 9   6 + 38   6 + 35   36 + 8   5 + 36 ...

Using Number Properties

Examples: Word stories and recording for: 75 + 8   9 + 48   6 + 67   75 + 7   74 + 7   94 + 7   9 + 89 ...

Comparisons

I am learning to add and subtract using comparisons of sets without counting down or counting up.

Equipment: Counters or unilink cubes.

Using Materials

Problem: "Martin has $7, and David has $3 more than Martin. How much money does David have?"

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

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In pairs, the students build a line of seven counters for Martin and experiment with building David's line alongside Martin's. Discuss why the answer is found by 7 + 3. Record 7 + 3 = 10 on the board or modelling book.

Problem: "Tusi has $11 and Mere has $8 less than Tusi. How much money does Mere have?"

In groups, the students build a line of 11 counters for Tusi and experiment with building Mere's line. Discuss why the answer is found by 11 – 8. Record 11 – 8 = 3 on the board or modelling book.

Examples: Comparison word stories and recording for: 8 + 3 11 – 2 7 – 3 11 – 5 7 + 3 4 + 5 10 – 8 ...

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Using Imaging

Shielding and *Imaging Only:* Examples: Comparison word stories and recording for: 2 + 6 12 – 4 7 – 6 11 – 2 4 + 5 2 + 11 10 – 4 ...

Using Number Properties

Examples: Comparison word stories and recording for: 28 + 6 42 – 4 71 – 6 31 – 2 44 + 5 32 + 11 60 – 4 ...

More Comparisons

> I am learning to add and subtract using comparisons of sets without counting down or counting up.

Equipment: Counters or unilink cubes.

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Using Materials

Problem: "Sarah has $14, and Denise has $8. How much more money does Sarah have than Denise?"

In pairs, one student is Sarah and the other Denise; they build 14 and 8, typically in lines with counters or unilink cubes. They need to make the connection that the problem is solved by working out either 14 – 8 or 8 + □ = 14. Record whichever of these arises in the discussion on the board or modelling book. Discuss why the answer is six and record this on the board or modelling book.

Examples: Comparison word stories and recording for: 3 + □ = 7 5 + □ = 10 9 + □ = 13 6 + □ = 11 □ + 8 = 12 ...

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Using Imaging

Problem: "Karen has $8, and Fiona has $3. How much more money does Karen have than Fiona?"

Shield parallel lines of eight counters and three counters. Ask the students to image the extra ones on Karen's line and how many there are. Record 3 + 5 = 8 or 8 – 3 = 5 on the board or modelling book, depending on which way the students solved the problem.

Shielding: Examples: Comparison word stories and recording for: 2 + □ = 7 8 + □ = 15 8 + □ = 13 9 + □ = 15 ...

Imaging Only: Examples: Comparison word stories and recording for: 10 + □ = 17 11 + □ = 19 12 + □ = 16 16 + □ = 20 ...

Using Number Properties

Examples: Comparison word stories and recording for: 77 + □ = 81 38 + □ = 42 56 + □ = 64 68 + □ = 77 □ + 87 = 95 ...

How Many Ten-dollar Notes?

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I am learning how many tens there are in numbers less than 1 000.

Equipment: Play money $1, $10, $100 (Material Master 4–9).

Using Materials

Problem: “Mrs Jones takes her class to the circus. She has $237 to pay for the students to get in. Admission is $10 per person. She has 25 students in her class. Does she have enough money?”

The students solve the problem in groups with play money. Record 237 on the board or modelling book and discuss the meaning of the digit 2. “How many tens is this worth?” Then ask how many tens are needed altogether. Then answer the question “Is there enough money?” “No.”

Examples: Word stories and recording for: $167 for 13 students

$203 for 41 students $203 for 21 students

$199 for 18 students $167 for 17 students ...

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Problem: “Mrs Wineta collects $10 from each student in her class to take them to the circus. She collects from 17 students. How much money has she got?”

Examples: Word stories and recording for: 15 ten-dollar notes

26 ten-dollar notes   13 ten-dollar notes   21 ten-dollar notes ...

Using Imaging

Shielding and *Imaging Only:* Examples: Word stories and recording for:

12 ten-dollar notes   29 ten-dollar notes   19 ten-dollar notes

31 ten-dollar notes   34 ten-dollar notes   45 ten-dollar notes ...

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Using Number Properties

Problem: “Boxes of chocolates cost $10 each. How many boxes can Charlotte buy if she has $589 to spend?” Discuss the solution.

Examples: Word stories and recording for: $867 $701 $327 $991 $563 ...

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Learning Experiences for Moving Students from Early Additive Part-Whole to Advanced Additive–Early Multiplicative Part-Whole

Required Knowledge

Before attempting to develop students' advanced part-whole ideas, check that they can:

- instantly recall the addition of single-digit numbers up to a total of 20 and know the related subtraction facts;
- find how many tens and hundreds there are in any number up to 10 000.

Saving Hundreds

I am learning how knowing 10 ones make one 10 and 10 tens make 100 can help me solve problems like 567 + □ = 800.

Equipment: Play money $1, $10, $100 (Material Master 4–9).

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Using Materials

Problem: "Julia has to save $400 for a bike. She has $287 saved. How much more money does Julia have to save?"

Record 287 + □ = 400 on the board or modelling book. Select three students and say which students will be in charge of the hundreds, tens, and ones respectively. Let the "hundreds" student count out two hundreds, the "tens" student count out eight tens, and the "ones" student count out seven ones. The "ones" student is given three more ones, which he/she swaps for one 10, gives to the "tens" student, and sits down. Record "3" on the board or modelling book. Now the "tens" student has nine tens and needs to get one more 10 to make 100. When he/she has 10 tens, he/she swaps them for 100, gives this to the "hundreds" student, and sits down. Record 10 on the board or modelling book. The "hundreds" student receives another hundred. Record 100 on the board or modelling book. Discuss why the answer to 287 + □ = 400 is 113. Notice that the order in which the extra money is given out can be varied. For example, the "tens" student may be given $20 initially. The students will need to understand why, eventually, this student will have to exchange one of these ten-dollar notes for 10 ones at the bank.

Examples: 484 + □ = 500 345 + □ = 400 290 + □ = 400 488 + □ = 600
67 + □ = 200 ...

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Using Imaging

Shielding and *Imaging Only:* 98 + □ = 500 345 + □ = 400 290 + □ = 400
□ + 488 = 700 □ + 381 = 500 ...

Using Number Properties

Examples: Word stories and recording for: 106 + □ = 800 279 + □ = 700
378 + □ = 1 000 136 + □ = 800 809 + □ = 900 □ + 378 = 1 000 ...

Challenging examples: 2 990 + □ = 5 000 456 + □ = 4 000 6 + □ = 8 000
□ + 450 = 5 000 □ + 1 950 = 9 000 ...

Jumping the Number Line

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"Tidy" numbers end in a zero. They are frequently useful numbers to reach during an addition or subtraction calculation.

> I am learning to jump through a tidy number on a number line to solve problems like 17 + □ = 91.

Equipment: A large class number line (Material Master 4–8). Student sheets of number lines by tens with open number lines printed on the back (Material Master 5–12).

Using Materials

Problem: "Freddo the Frog lives at number 28 on the number line. He wants to visit his friend at number 81. How far does he have to jump to get there?"

Record 28 + □ = 81 on the board or modelling book.

Suggest that Freddo will first jump to 30 because it's a "tidy" number. Show this jump with an arrow and ring the jump of 2. Discuss how far Freddo has to go. Some students will jump by tens to 80 and then go one more. Some will jump 50 then one more, and a few will jump 51 directly to 81. Show these jumps with arrows and ring the numbers. In all cases, focus attention on the ringed numbers always giving the answer 53. Discuss which way is best. The students now do individual work with you observing their methods.

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Examples: Give the students the first sheet from Material Master 5–12 and get them to write the following seven problems down against each number line: 39 + □ = 61
48 + □ = 81 57 + □ = 85 29 + □ = 78 18 + □ = 60 27 + □ = 93
36 + □ = 90

The students do the problems and then discuss the answers as a whole group.

Using Imaging

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Problem: "Freddo jumps from 18 to 73. How far is this?"

Discuss recording this as 18 + □ = 73. Draw a large empty number line on the board or modelling book and discuss where to place the 18 and 73. Without adding 30, 40, 50, 60, and 70 to the empty number line, discuss how to jump from 18 to 20 and then to 73 in two steps. So, the only numbers on the number line are 18, 20, and 73. Record the answer 55.

Examples: Get the students to turn over their sheet to use the empty number lines. It has seven empty number lines. Get them to write the following seven problems down against each number line: 29 + □ = 62 58 + □ = 93 27 + □ = 86
29 + □ = 78 48 + □ = 70 29 + □ = 83 46 + □ = 83

Regroup to discuss the answers.

Using Number Properties

Examples: Word stories and recording for: 19 + □ = 62 36 + □ = 94
8 + □ = 84 39 + □ = 75 37 + □ = 75 25 + □ = 81 ...

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*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Don't Subtract – Add!

I am learning that problems like 34 + ☐ = 51 and 51 – 34 = ☐ have the same answer.

Equipment: Play money $1 and $10 (Material Master 4–9).

Using Materials

Problem: "Moana has $63 and gives Miles $59. How much money does Moana have left?"

The students pair off. One student acts as Moana. This student gets $63 as six tens and three ones. The other student is to be paid $59. They discuss how this can be done. Usually, Moana would give Miles five tens. Then Moana has to realise that she must swap the remaining ten-dollar note for 10 ones. Then have a group discussion. Discuss the solutions to 63 – 59 and 59 + ☐ = 63. "Why are both answers the same?"

Examples: Word stories and recording for: 80 – 78 83 – 77 63 – 57 33 – 26 50 – 49 23 – 18 ...

Using Imaging

Shielding: Problem: "Murray has $92, and he spends $87."

Record 92 – 87 on the board. Shield $87 and ask the group of students how many are required to make $92. Discuss why you have added $3 to $87 to make $90 and then added $2 more to make $92. So the answer is five. Discuss the link to 92 – 87. Fold back to **Using Materials** if necessary. Record the answer on the board or modelling book.

Using Number Properties

Problem: "Jill has $73, and she spends $28. How much money does she have left?"

Discuss why this is the same as 28 + ☐ = 73 and use the method in **Jumping the Number Line** above to solve it.

Examples: Word stories and recording for: 81 – 18 75 – 39 93 – 57 103 – 88 110 – 89 64 – 38 ...

Learning Experiences for Advanced Additive–Early Multiplicative Part-Whole

Problems like 23 + ☐ = 71

This problem can be solved by jumping up to 30 and then jumping to 71 as in **Jumping the Number Line** above. However, because 23 is some distance from 30, another method, namely, adding 50 to the 23 first, is worth learning.

I am learning to solve problems like 13 + ☐ = 91 by jumping up by a tidy number on a number line, then jumping back a small number.

Equipment: A large class number line (Material Master 4–8). A student sheet of number lines with number lines printed on the back (Material Master 5–12).

Using Materials

Problem: "23 plus what is 71?"

Record 23 + ☐ = 71 on the board or modelling book. On the class number line, jump 50 from 23 to reach 73. Discuss why we need to jump back two and so why the answer is 48. Record 23 + 48 = 71 on the board or modelling book.

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Examples: Record and do these problems on the first side of the sheet.

24 + ☐ = 82   52 + ☐ = 90   25 + ☐ = 94   13 + ☐ = 71   12 + ☐ = 91
54 + ☐ = 72   45 + ☐ = 83   14 + ☐ = 83 ...

Using Imaging

Imaging examples are not likely to be needed, especially for students who were successful in **Jumping the Number Line**. Use imaging for those students who are struggling.

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Using Number Properties

Examples: Word stories and recording for: 24 + ☐ = 82   55 + ☐ = 93
24 + ☐ = 92   12 + ☐ = 61   82 + ☐ = 101

How Many Tens and Hundreds?

Key Ideas

The students need to know that 10 hundreds make 1 000 and vice versa and 10 thousands make one 10 000 and vice versa.

I am learning how many hundreds there are in numbers over 1 000.

Equipment: Play money from $1 to $10,000 (Material Master 4–9).

Using Materials

Problem: "The Bank of Mathematics has run out of $1,000 notes. Alison wants to withdraw $2,315 in $1, $10, and $100 notes. How many one-hundred-dollar notes does she get?"

Discuss the answer and record it on the board or modelling book.

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Examples: Word stories and recording for: $2,601 $3,190 $1,555 $1,209
$2,001 $1,222 $2,081 ...

Using Imaging

Problem: "Tickets to a concert cost $100 each. How many tickets can you buy if you have $3,215?"

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

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Record $3,215 on the board or modelling book. Shield three one thousands, two one hundreds, one 10, and five ones. Ask the students what they can see. Discuss how many one-hundred-dollar notes they could get by exchanging the thousands. Discuss which notes are irrelevant (the 10 and the ones). Record the answer on the board or modelling book.

Shielding and *Imaging Only:* Examples: Find and record the number of hundreds in: $1,608 $2,897 $2,782 $3,519 $3,091 $4,000 ...

Using Number Properties

Examples: Find and record the number of hundreds in: 3 459 8 012 9 090 6 088 3 280 5 823 7 721 2 083 ...

Challenging examples: Find and record the number of hundreds in: 13 409 28 002 78 370 12 088 45 290 82 356 21 344 ...

Find the number of tens in: 3 709 8 002 8 579 5 208 4 829 82 333 12 897 30 897 89 000 50 890

Problems like 37 + □ = 79

In an example like this, there is no need to jump up to 40 and then jump up to 79. Indeed, this makes a simple problem harder. This is because the seven in 37 is less than the nine in 79.

I am learning to add mentally the ones and tens separately when appropriate.

Equipment: $1 and $10 play money (Material Master 4–9) or a large class number line (Material Master 4–8).

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Using Materials

Problem: "Toni has $37 and is saving to buy a skateboard costing $79. How much does she need to save?"

Discuss the problem and record 37 + □ = 79 on the board or modelling book. In pairs, the students solve the problem with play money (or use the large class number line). As a group, discuss why the answer is 42. Two methods of solution are likely: With money, three tens and four tens make seven tens (remember the four tens). And 7 + 2 = 9. So the answer is four tens and two makes 42.

With a number line, 37 + 4 tens = 77, 77 + 2 = 79. So the answer is 40 + 2 = 42. Record 42 on the board or modelling book.

Examples: 27 + □ = 69 12 + □ = 67 75 + □ = 95 61 + □ = 83 □ + 26 = 96 □ + 51 = 82 □ + 36 = 59 ...

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Using Imaging

Shielding: Problem: "Shelley has $62 and needs a total of $89. How much does she need to save?"

Record 62 + □ = 89 on the board or modelling book. Model $62 (or use a number line) out of sight and ask the students to image what to do next. Record the answer on the board or modelling book. Fold back if necessary.

Examples: Word stories and recording for: 37 + □ = 79 12 + □ = 69 75 + □ = 95 61 + □ = 83 □ + 36 = 77 □ + 23 = 49 □ + 68 = 98 ...

Using Number Properties

Examples: Word stories and recording for: 17 + □ = 99 5 + □ = 67 5 + □ = 88 4 + □ = 89 □ + 56 = 99 □ + 58 = 88 □ + 48 = 59 ...

Challenging examples: 387 + □ = 399 212 + □ = 777 45 + □ = 795 681 + □ = 999 □ + 256 = 759 ...

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Problems like ☐ + 29 = 81

This is related to **Problems like 23 + ☐ = 71** above.

> I am learning to reverse problems like ☐ + 29 = 81 to 29 + ☐ = 81 and then use an appropriate mental method to solve the problem.

Equipment: None.

Using Number Properties

Problem: "Taine has some cows. Taine buys 29 more cows. Now he has 81 cows. How many cows did Taine have before?"

Here the unknown number comes first. Discuss why this may be written as ☐ + 29 = 81. Discuss reversing the problem to 29 + ☐ = 81 and solve it.

By a wise selection from previous methods, students solve these problems.

Examples: Word stories and recording for: ☐ + 38 = 72 ☐ + 33 = 83 ☐ + 18 = 43 ☐ + 9 = 62 ☐ + 23 = 72 ☐ + 21 = 88 ☐ + 25 = 70

When One Number Is Near a Hundred

There are at least two useful methods for solving such a problem. For example, in the case of 78 + 99: either 78 + 100 = 178, but this is one too many, so the answer is 177, or transfer one from the 78 to the 99.

So 78 + 99 is the same as 77 + 100 = 177.

> I am learning to solve some addition and subtraction problems by adjusting one number to the nearest hundred.

Equipment: $1, $10, $100 play money (Material Master 4–9).

Using Materials

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Problem: "Work out $99 + $78."

Record 99 + 78 on the board or modelling book. Draw an empty number line on the board. Show on the number line why 100 + 78 is one too many. Record the answer on the board or modelling book.

Examples: Draw empty number lines and work out: 56 + 99 67 + 98 97 + 63 38 + 298 123 – 99 456 – 98 ...

Problem: "Work out $98 + $38."

Record 98 + 38 on the board or modelling book. The students model $98 and $38 with play money. Discuss what transferring $2 from the $38 to the $98 does and how this shows that the answer is $136. Record the answer on the board or modelling book.

Examples: Using play money, work out: 151 – 99 62 + 98 97 + 73 338 – 98 103 + 197 295 + 456 ...

Using Imaging

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Problem: "Work out $175 – $99."

Encourage the students to image this in two ways, namely, by number line and using play money, and to choose the method they like better.

Shielding: Examples: Word stories and recording for: 141 – 98 55 + 98 97 + 86 18 + 298 603 – 198 299 + 456 ...

Using Number Properties

Examples: Word stories and recording for: 834 – 99 456 – 98 297 + 198 818 + 698 1 000 – 797 1 200 – 998 ...

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Problems like 73 – 19 = □

Previously this problem was reversed to solve 19 + □ = 73 by adding. Another useful and powerful method here is to subtract 20 from 73 to give 53 and add one to give 54. A problem students often have with this second method is whether to add or subtract the one. The number line makes it obvious why one is added. Eventually, the students need to choose between these two methods for themselves.

> I am learning to solve problems like 73 – 19 by first subtracting a tidy number then adding on a small number to get the answer.

Equipment: A large class number line (Material Master 4–8).

Using Materials

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Problem: "Frances the Frog lives at 73 on the number line. She is capable of leaps of any size. She wants to make a backwards jump of 19. Where does Frances end up?"

Discuss this and record 73 – 19 on the board or modelling book. On the class number line, show 73 and say that Frances decides to make a big jump of 20. Discuss whether Frances has jumped too far and how Frances will need to add a jump of one rather than subtract one.

Examples: Draw blank number lines and get the students to show the jumps needed to solve these problems: 72 – 59 98 – 79 45 – 18 66 – 48 72 – 17 103 – 88 ...

Using Imaging

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Problem: "Frances jumps from 72 back 39."

Record 72 – 39 on the board or modelling book. Encourage the students to image the number line. Drawing pictures in the air may be helpful. If this proves too hard, fold back to **Using Materials** by drawing the number line on the board or modelling book and discuss the solution.

Examples: Word stories and recording for: 82 – 29 68 – 19 65 – 28 96 – 28 82 – 47 ...

Using Number Properties

Examples: Word stories and recording for: 123 – 18 228 – 19 91 – 28 596 – 89 312 – 9 991 – 87 ...

Equal Additions

For a problem like 445 – 398, the fact that 398 is very close to a tidy number, namely, 400, suggests that a useful way of solving it is by equal additions, in this case, of 2. The problem then becomes 447 – 400, whose answer is obviously 47.

> I am learning to solve subtraction problems by equal additions that turn one of the numbers into a tidy number.

Equipment: Play money (Material Master 4–9).

Using Materials

Problem: "Debbie has $445 in her bank account, and her younger sister Christine has $398. How much more money does Debbie have?"

Make piles of $445 and $398. "Now suppose that Grandma gives Christine $2 to give her a 'tidy' amount of money. To be fair, Grandma gives Debbie $2 also." Discuss why

445 – 398 has the same answer as 447 – 400 and then record 445 – 398 = 47 on the board or modelling book.

Examples: Word stories and recording for: 367 – 299 546 – 497 662 – 596 761 – 596 334 – 95 567 – 296 ...

Using Number Properties

Examples: Word stories and recording for: 900 – 298 701 – 399 760 – 96 905 – 96 507 – 296 865 – 590 1 000 – 396 ...

People's Ages

I am learning to apply mental subtraction methods to an application.

Equipment: None.

Using Materials and Using Number Properties

Problem: "John is interested in the history of his family. He sees from family records that his great-grandmother was born on 25 October 1879, and she died on 13 December 1945. How old was she when she died?"

Write and discuss why the problem can be written as 1879 + □ = 1945.

Model 1879 and 1945 on an open number line and insert 1900 on the number line. Discuss the solution. "From 1879 to 1900 is 21, and from 1900 to 1945 is 45. So great-grandmother was 21 + 45 = 66 years old when she died." Record this on the board or modelling book.

Problem: "Jonah Rogers was born on 12/03/1823 and died on 14/09/1891. How old was he when he died?"

Discuss why 1891 – 1823 is a suitable calculation to make, which reduces to finding 91 – 23. Record the answer on the board or modelling book.

Examples: "How long did these people live?" Record the answers on the board or modelling book.

Born	***Died***
21/09/1853	17/11/1901
13/11/1937	01/12/1980
01/03/1899	12/12/1987
28/04/1848	01/05/1924
30/08/1888	21/09/1923
23/05/1902	21/07/1933

Problem: "On a tour of England, Malcolm Smith finds the gravestone of one of his ancestors, which reads "Joseph Smith. Born 22nd July 1638. Died 12th April 1665." How old was Joseph when he died?"

Discuss why 1665 – 1638 = 27 produces an answer that is one too large (because 12th April precedes 22nd July). So Joseph was 26 when he died.

Challenging examples: "How long did these people live?" Record the answers on the board or modelling book.

Born	***Died***
21/12/1943	12/01/2003
13/12/1956	01/12/1991
01/03/1856	12/12/1911
28/09/1924	12/01/2002
30/08/1878	21/03/1922
23/12/1919	21/07/1991

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*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

A Balancing Act

Many students regard the "=" sign as meaning only "get the answer". That is what "equals" means on a calculator. But the notion of equals is more general than this. In the problem below, the equals sign means that the total on the left of the sign is the same as the total on the right.

> I am learning that the answer on the left of the equals sign is the same as the answer on the right of the equals sign.

Equipment: Counters. (Balances and weights are alternative materials.)

Using Materials

Problem: "Maggie and John are given a bag of sweets each. Both bags have the same number of sweets in them. Maggie shares her bag of sweets with her friend Jan. Maggie gets four sweets, and Jan gets five. John shares his bag with Anne. John gives Anne six sweets. How many sweets does John have left?"

Discuss why this problem is 4 + 5 = □ + 6 and record it on the board or modelling book. In pairs, nominate one person to be Maggie and the other to be John. Maggie creates piles of four and five while John has to work out that a pile of six leaves three for the other pile. Common wrong answers are nine and 15. Discuss why they occur and why they are wrong. (Nine comes from adding the left side of the equation and 15 from adding all the numbers on both sides. In both cases, the student views "equals" as meaning "get the answer" rather than "balance each side".)

Examples: Word stories and recording for: 2 + 4 = □ + 3   5 + 5 = □ + 6
5 + □ = 4 + 3   □ + 6 = 6 + 3   □ + 6 = 8 + 2   1 + 7 = 4 + □ ...

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Using Imaging

Problem: "Solve: 9 + 3 = □ + 7."

Record the problem on the board or modelling book and discuss how to build piles of counters. Show 9 + 3 in two piles of counters. In separate piles, place five under a container and seven in sight beside it. Encourage the students to think about the equality of the two sides of the equation.

9 + 3 is 12, so switch attention to the other two piles. Record the answer on the board or modelling book.

Examples: Word stories and recording for: 5 + 4 = □ + 3   6 + 5 = □ + 3
5 + □ = 4 + 6   □ + 2 = 5 + 3   □ + 6 = 8 + 3   3 + 8 = 4 + □ ...

Using Number Properties

Problem: "Solve 67 + □ = 66 + 44"

Discuss why the answer is one less than 44 (because 67 is one more than 66 and "balance" has to be maintained). Record 43 in the square.

Examples: Word stories and recording for: 42 + 38 = □ + 39   55 + 35 = □ + 34
105 + □ = 103 + 43   □ + 65 = 67 + 33   □ + 180 = 190 + 38
1 020 + □ = 1 010 + 67

Challenging examples: 442 – 38 = □ – 39   585 – 35 = □ – 34
105 – 56 = 103 – □   □ – 65 = 667 – 62   □ – 280 = 790 – 29 ...

Near Doubles

I am learning to solve addition problems where the two numbers are easily related to doubles.

Equipment: Play money (Material Master 4–9).

Using Materials

Problem: "Work out 153 + 147."

Record 153 + 147 on the board or modelling book. Show $153 and $147 with play money. Discuss the fact that transferring $3 gives $150 and $150, so the answer is obviously $300. Record the answer on the board or modelling book.

Problem: "To work out 79 + 79, Harry works out $80 + $80 with play money first and then says 79 + 79 must be 158." Discuss Harry's method. Record the answer on the board or modelling book.

Examples: Work out: 149 + 149 352 + 348 350 + 352 104 + 96 ...

Using Imaging

Examples: Word stories and recording for: 440 + 439 248 + 252 349 + 350 202 + 198 ...

Using Number Properties

Examples: Word stories and recording for: 150 + 149 448 + 452 147 + 153 502 + 498 1 009 + 991 9 996 + 10 004 ...

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*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

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Three or More at a Time

I am learning to look at the addition and subtraction of three or more numbers to calculate easy combinations first.

Equipment: Play money (Material Master 4–9).

Using Materials

Problem: "Find $89 + $527 + $11."

Record the problem on the board or modelling book. This looks difficult. However, modelling 89 and 527 and 11 with play money and combining 89 + 11 as 100 makes it obvious that the answer is 100 + 527 = 627. Record the answer on the board or modelling book.

Using Number Properties

Examples: Use materials or imaging only if necessary. Most students should be able to transfer straight to ***Using Number Properties***: Word stories and recording for:
970 + 3 213 + 30   99 + 89 + 11 + 1   678 + 234 – 78   234 – 89 –11
992 + 3 456 + 8   2 345 + 5 + 995 – 345 ...

Problems like 67 – ☐ = 34

I am learning to solve problems like 67 – ☐ = 34 by solving 34 + ☐ = 67 or by finding 67 – 34.

Equipment: Ones and tens materials.

Using Materials

Problem: "At her party, Glynnis provides 67 sweets. At the end of the party, she has 34 left. How many sweets did her guests eat?"

The students model 67 as six tens and seven ones and experiment to see why 34 will be left. The connection the students need to make is that the problem can be solved by working out 67 – 34 or by solving 34 + ☐ = 67.

Examples: Word stories and recording for: 86 – ☐ = 61   67 – ☐ = 21
63 – ☐ = 17   500 – ☐ = 46 ...

Using Number Properties

Examples: Word stories and recording for: 84 – ☐ = 61   77 – ☐ = 25
42 – ☐ = 26   200 – ☐ = 156 ...

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Large Numbers Roll Over

I am learning how a number rolls over when 10 of any unit occur in an addition, and how the number rolls back when 10 of any unit occur in a subtraction

Key Ideas

Check that the students understand that 10 one thousands equals one ten thousand *not,* as is commonly thought, one million.

Equipment: Play money $1, $10, $100, $1000, $10 000 (Material Master 4–9).

Using Materials

Problem: "Work out $9,993 + $9."

Record $9,993 + $9 on the board or modelling book. With play money, model $9,993 and $9. Discuss why the 12 single dollars must be swapped for a ten-dollar note and two single dollar notes. Discuss why nine tens plus the extra ten-dollar note makes 10 tens, which must be swapped for a one-hundred-dollar note. Continue these swaps until there is a single ten-thousand-dollar note and two single dollars. Record the answer of $10,002 on the board or modelling book.

Problem: "Work out $10,003 – $4."

Record $10,003 – $4 on the board or modelling book. Using play money, break the $10,000 down to 10 one-thousand-dollar notes, break the one-thousand-dollar note down to 10 one-hundred-dollar notes, and so on until there are 13 single dollars. Record the answer of $9,999 on the board or modelling book.

Examples: Word stories and recording for: $9,988 + $19 $6 + $52,994
$116 + $9,884 $40,003 – $7 $20,000 – $100 $999 + $1,004
$1,001– $45 $50,003 – $5 ...

Using Number Properties

Examples: Word stories and recording for: 8 992 + 9 6 + 12 996 16 + 6 684
44 503 – 7 18 900 + 102 99 + 12 099 50 + 6 150 102 003 – 5 ...

A Standard Written Form for Addition

Key Ideas

12 tens = 120, there are 34 tens in 345, etc.

10 hundreds = 1 000 and 10 one thousands = one 10 000.

Because the students are already part-whole thinkers, they already have all the understandings needed for a standard written form. Standard written forms bear no relation to efficient mental ways of computation. (They are designed to be good for pencil and paper.) Written forms are needed when the numbers get too large to handle mentally.

I am learning a standard method of doing an addition problem on paper when the numbers are too difficult to find the answer mentally.

Equipment: Pencil and paper.

Using Materials

Problem: "Work out 235 + 386 in a standard written method."

Record the following on the board or modelling book and discuss the reason for every step.

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

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*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

											10		
235	→	200	+	30	+	5	→	200	+		30	+	5
+ 386	→	300	+	80	+	6	→	300	+		80	+	6
													1

	100		10					
→	200	+	30	+	5			
→	300	+	80	+	6			
	600	+	20	+	1	→	621	

Examples: 234 + 478   459 + 328   308 + 536   889 + 67 ...

Using Imaging

Problem: "Find 235 + 487 by a standard written method. Use a compact form."

Discuss how five and seven produce two ones and one 10 and 12 tens produce two tens and one hundred.

```
 1 1
 235
+487
----
 722
```

Examples: 484 + 468   279 + 326   508 + 536
89 + 557   367 + 902   78 + 970 ...

Using Number Properties

Examples: Find the answers by using a standard written form: 3 404 + 478
4 079 + 2 327   588 + 4 536   59 + 4 556   3 268 + 8 902   78 + 970 ...

Decomposition – A Written Form for Subtraction

For students who are good at multi-digit addition and subtraction, learning a standard written subtraction is straightforward, provided they understand the core idea that the particular decomposition needed in a given subtraction depends on what is subtracted.

> I am learning a standard method of doing a subtraction problem on paper when the numbers are too difficult to find the answer mentally.

Equipment: Play money if needed (Material Master 4–9).

Using Materials

Problems:

1. "To work out 856 – 138, Jane rearranges 856 as 800 + 40 + 16. Why does she do this?" Explain, using play money, if necessary. (In the decomposition method of subtraction, there are sufficient hundreds and tens to solve the problem, but there are insufficient ones.) "So find 856 – 138."
2. "To work out 856 – 162, Jane rearranges 856 as 700 + 150 + 6. Why does she do this?" Explain, using play money, if necessary. (In the decomposition method of subtraction, there are sufficient hundreds and ones to solve the problem, but there are insufficient tens.) "So find 856 – 162."
3. "To work out 856 – 168, Jane rearranges 856 as 700 + 140 + 16. Why does she do this?" Explain, using play money, if necessary. "So find 856 – 168."
4. "To work out 856 – 123, Jane does not have to rearrange 856 at all. Why not?" Explain, using play money, if necessary. "So find 856 – 123."

Examples: In each of these subtractions, explain how to split up 953 to solve the problem, then find the answers: 953 – 234   953 – 184   953 – 594   953 – 284   953 – 388 ...

Now establish a standard written form for subtraction using a similar method to **A Standard Written Form for Addition** above.

A good way to do this is to explain why 546 – 278 requires 546 to be renamed 4 hundreds + 13 tens and 16 ones and link this to the setting out on the right.

```
   13
 4 14 16
  5 4 6
 -2 7 8
 ------
  2 6 8
```

Examples: 456 – 259   1 034 – 429   781 – 678 ...

Ask the students to explain why 953 – 631 and 953 – 630 do not need the written form.

Mixing the Methods – Mental Exercises for the Day

It is strongly recommended that teachers offer advanced additive–early multiplicative part-whole students a regular daily dose of mental calculation. It is suggested that you record only one problem on the board or modelling book at a time and do not allow the students to use pencil and paper. Make sure that they all have time to solve each problem. Do not allow early finishers to call out the answer. Then you can discuss each problem for the variety of solutions as well as deciding which was easiest.

I am learning to select wisely from my range of mental strategies to solve addition and subtraction problems and discuss my methods with other students.

Some Problem Sets

Record the problems on the board or modelling book in the horizontal form.

Set 1

45 + 58	67 + □ = 121	8 001 – 7 998	26 + □ = 52	81 – 67	456 + 144
789 – 85	□ + 58 = 189	33 + 809 + 67 + 91			

Set 2

28 + 72	191 + □ = 210	7 001 – 21	39 + □ = 77	234 – 99	6 091 + 109
2 782 – 15	□ + 123 = 149	616 + 407 – 16 + 93			

Set 3

999 + 702	287 + □ = 400	2 067 – 999	45 + □ = 91	771 – 37	316 + 684
709 – 70	□ + 88 = 200	7 898 – 6 000 – 98 – 100			

Set 4

38 + 128	14 + □ = 101	9 000 – 8 985	102 – □ = 34	800 – 33	78 + 124
4 444 – 145	□ + 8 = 1 003	4 700 – 498 + 200 –2			

Set 5

405 + 58	880 + □ = 921	8 789 – 7 678	80 – □ = 41	701 – 96	8 888 + 122
781 – 45	□ + 48 = 789	6 000 – 979 – 11 – 10			

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Mental or Written?

I am learning to select wisely between using a mental method and a written method for addition and subtraction problems.

Equipment: None.

Using Number Properties

Problem: "Which is the better way to solve these problems – mentally or using the standard written forms? 997 + 1 234   4 546 – 2 788."

Discuss why 997 + 1 234 is easy to solve mentally, using strategies such as,
1 000 + 1 234 = 2 234,   2 234 – 3 = 2 231.

Doing 4 546 – 2 788 mentally will be beyond most students, so the standard written form is needed for this one.

Examples: A mixture of addition and subtraction problems (Material Master 5–13).

Estimation as a Check

When students use the standard written forms or use a calculator, it's essential that they demonstrate good number sense in rejecting answers that are obviously wrong.

I am learning to check any addition and subtraction problem I cannot solve mentally by estimating the answer.

Equipment: None.

Using Number Properties

Problem: "To work out 4 567 + 4 890, Maureen uses her calculator or pencil and paper. Her answer is 8 457. Without working out the exact answer, why must Maureen's answer be wrong?"

Discuss why a quick estimate shows that the answer must exceed 9 000.

Examples: Discuss whether these answers are definitely wrong or not:

2 365 + 7 694 = 10 059	1 788 – 891 = 497	8 502 – 6 934 = 1 568
789 + 34 + 309 = 1 532	34 567 + 8 790 = 43 357	
9 897 + 34 567 = 4 464	123 089 – 45 678 = 57 411 ...	

Learning Experiences for Moving Students From Advanced Additive–Early Multiplicative Part-Whole to Advanced Multiplicative–Early Proportional Part-Whole

Introducing Decimal Fraction Place Value

Students' first encounter with a decimal point in a number is likely to be with money. Unfortunately, this is not truly a decimal point – it is a device that separates two whole numbers, the dollars and the cents. So it is unwise to introduce decimal fractions through money, even though this is quite a common practice.

Fractions need to be thoroughly understood before introducing decimal fractions.

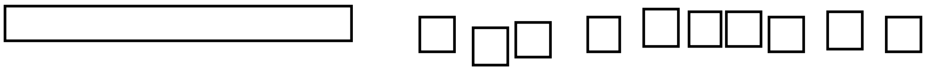
I am learning how tenths arise out of division.

Equipment: Unilink cubes. Sets of ten connected unilink cubes wrapped in paper.

Using Materials

Problem: Henry has 6 bars of chocolate to share among 5 friends. How much does each get?

A wrapped bar and a loose bar are shown:



Henry has 6 wrapped bars. After sharing out 1 whole bar to each friend, the last bar needs to be unwrapped. Then each friend gets another two-tenths of a bar.

The teacher writes 6 ÷ 5 = 1 whole + 2 tenths on the whiteboard or modelling book.

Using Materials, Using Imaging, and Using Number Properties

The teacher can select problems from the set below for any stage. When teachers make up their own examples, they should make sure the answers are to 1 d.p.
So 7 ÷ 5 = 1.4 is suitable, but 7 ÷ 4 = 1.75 is not.

Problems

8 ÷ 5 = □ wholes + □ tenths	7 ÷ 5 = □ wholes + □ tenths
5 ÷ 2 = □ wholes + □ tenths	2 ÷ 5 = □ wholes + □ tenths
1 ÷ 2 = □ wholes + □ tenths	6 ÷ 4 = □ wholes + □ tenths
4 ÷ 5 = □ wholes + □ tenths	7 ÷ 2 = □ wholes + □ tenths
10 ÷ 4 = □ wholes + □ tenths	4 ÷ 10 = □ wholes + □ tenths
3 ÷ 10 = □ wholes + □ tenths	9 ÷ 2 = □ wholes + □ tenths

The Decimal Fraction Point

The presence of a decimal point always signals that the number of tenths is the digit immediately to the right of the point. This can be the cause of some confusion for students who read, say, 0.39 as meaning 39 hundredths. It does not *mean* this, although it does *equal* it. 0.39 actually means 3 tenths + 9 hundredths.

I am learning to use the shorthand for tenths.

Equipment: Calculators.

Using Materials

None.

Using Imaging

None.

Using Number Properties

Problem: Complete 13 ÷ 2 = □ wholes + □ tenths. Predict the decimal point version of the answer, then check on a calculator. (Answer: 6.5)

When teachers make up their own examples, they should make sure the answers are to 1 d.p. So 13 ÷ 5 = 2.6 is suitable, but 11 ÷ 4 = 2.75 is not.

Examples:

Find each answer and write it in two forms, that is, the words in wholes and tenths and the decimal point shorthand:

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21 ÷ 5	22 ÷ 4	10 ÷ 4	13 ÷ 2	15 ÷ 2
13 ÷ 5	14 ÷ 4	18 ÷ 4	17 ÷ 2	25 ÷ 2
26 ÷ 5	30 ÷ 4	2 ÷ 4	1 ÷ 2	15 ÷ 2

Adding with Decimal Fractions

The advantage of doing operations with decimal fractions is that it embeds the notion that the number after a decimal point always represents tenths. It also emphasises the canon of place value, which states that the maximum digit in any place is 9. For addition and multiplication, ten subunits must be replaced by one unit, and for subtraction and division, one unit must be broken into ten subunits. (The canonical form is defined as the standard, conventional, and logical way of writing numbers.)

I am learning to add with 1 decimal place fractions.

Equipment: Unilink cubes. Sets of ten connected unilink cubes wrapped in paper.

Using Materials

Problem: Work out 3.9 + 2.4

Students model 3.9 + 2.4 as 3 wrapped bars (wholes) and 9 small pieces (tenths) and as 2 wrapped bars and 4 small pieces. Collecting them gives 5 wholes and 13 tenths. By the canon of place value, 10 tenths are rewrapped to give 1 whole.
So 3.9 + 2.3 = 6.1

Using Materials, Using Imaging, and Using Number Properties

When teachers make up their own examples, they should generally make sure that there are 10 or more tenths in the addition so that 10 tenths has to be swapped for 1 whole. So 3.3 + 1.9 is suitable, but
3.4 + 4.4 is not.

The teacher can select problems from the set below for any stage.

3.9 + 2.4	2.6 + 1.6	1.4 + 0.8	3.5 + 2.5
0.6 + 1.7	5.8 + 0.8	2.2+ 1.9	4.2 + 2.3
2.9 + 1.9	0.6 + 1.6	3.2 + 0.8	2.4 + 1.5
10.4 + 10.9	18.5 + 1.7	0.4 + 20.8	15.2 + 2.3

Subtraction with Tenths

I am learning to subtract with 1 decimal place fractions.

Equipment: Unilink cubes. Sets of ten connected unilink cubes wrapped in paper.

Using Materials

Students work out 3.3 – 1.7. It is likely that students will do 3.3 – 1 = 2.3 first. Then they may unwrap a bar and remove 7 tenths from the 13 tenths to leave 6 tenths.
So 3.3 – 1.7 = 1.6

Using Materials, Using Imaging, and Using Number Properties

The teacher can select problems from the set below for any stage.

When teachers make up their own examples, the digit in the tenths column of the number to be subtracted needs to be greater than the digit in the tenths column of the first number. So 3.3 – 1.7 is suitable, but 5.6 – 3.4 is not.

Problems:

4.6 – 1.7	3.3 – 1.6	5 – 3.6	4.2 – 0.4
3.6 – 2.9	2.3 – 1.8	3 – 2.9	3.3 – 0.6
7.2 – 3.5	9.2 – 7.3	11 – 1.6	12.8 – 0.9
15.8 – 8.9	15.1 – 1.8	16 – 3.9	10.3 – 0.8

Multiplication with Tenths

I am learning to multiply a whole number by a decimal fraction.

Equipment: Unilink cubes, and sets of ten connected unilink cubes wrapped in paper.

Using Materials

Students work out 3 x 1.4. It is likely that students will group the 3 wholes. Then they group the 12 tenths into 1 whole + 2 tenths.
So 3 x 1.4 = 4.2

Using Materials, Using Imaging, and Using Number Properties

The teacher can select problems from the set below for any stage.

When teachers make up their own examples, after grouping the tenths there needs normally to be at least 10 tenths. So 2 x 4.7 is suitable, but 4 x 3.2 is not.

Problems:

3 x 1.5	2 x 1.7	5 x 1.2	4 x 0.4
2 x 2.8	3 x 2.5	6 x 0.5	5 x 2.4
5 x 4.2	2 x 6.6	3 x 4.5	8 x 0.4
6 x 0.5	2 x 10.8	3 x 9.5	5 x 6.4
2 x 6.6	3 x 4.5	5 x 10.2	10 x 0.4

Division with Tenths

I am learning to do simple division by a whole number with tenths in the answer.

Equipment: Unilink cubes. Sets of ten connected unilink cubes wrapped in paper.

Using Materials

Students work out 5.2 ÷ 4. It is likely that students will give out 1 whole to each share, then convert the 1 whole + 2 tenths that are left over into 12 tenths and share out 3 tenths to each. So 5.2 ÷ 4 = 1.3

Using Materials, Using Imaging, and Using Number Properties

The teacher can select problems from the set below for any stage.

When teachers make up their own examples, wholes need to be broken down into tenths, and the answer is to 1 d.p. So 3.2 ÷ 4 = 0.8 is suitable, but 3.4 ÷ 4 = 0.85 is not.

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Problems:

$5.2 \div 2$	$1.6 \div 4$	$6 \div 4$	$1.2 \div 3$
$1.2 \div 6$	$7.5 \div 5$	$2.1 \div 3$	$2.1 \div 7$
$14 \div 4$	$3.6 \div 6$	$12 \div 8$	$4.5 \div 3$
$41.2 \div 4$	$21.6 \div 4$	$22 \div 4$	$19.2 \div 3$

Dollars and Bills

I am learning to add integers (positive and negative numbers).

Equipment: Play Money, including $1 coins [Material Master 4–9]. Bills [Material Master 4–38]. Dice labelled $1, $2, $3, $5, $10.

Using Materials

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Tell the students that you want them to imagine getting pocket money and spending some of it. Rolling the dice will tell them how much pocket money they get this week, and taking a bill card from the pile will tell them how much they are going to spend.

Problem: Pocket Money: $3; Bill $2.

"Will you have any spare money this week? How do you know?"

Record the calculation as ${}^{+}3 + {}^{-}2 = {}^{+}1$.

Look for the students to identify that some of the dollars are used up to pay the bill. Highlight that $2 pocket money and a $2 bill result in no money over nor any money owing.

Ask what other combinations of dollars and bills will result in no money over and no money owing. Record these as: ${}^{+}1 + {}^{-}1 = 0$, ${}^{+}3 + {}^{-}3 = 0$, ${}^{+}4 + {}^{-}4 = 0$, etc.

Examples: Pose pocket money and bills stories for:
${}^{-}2 + {}^{+}5 = {}^{+}3$, ${}^{+}10 + {}^{-}5 = {}^{+}5$, ${}^{-}10 + {}^{+}2 = {}^{-}8$, ${}^{+}1 + {}^{-}3 = {}^{-}2$, ${}^{+}10 + {}^{-}1 = {}^{+}9$, ${}^{-}5 + {}^{+}2 = {}^{-}3$.

Discuss whether the order of the addends makes a difference to the answer. This is the commutative property that also applies to whole numbers. Be sure that the students understand that it is possible to owe money in a given week, and in financial terms this is called a negative balance.

Using Imaging

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Have one student who is hidden from the group roll the dice and take a bill card to establish their pocket money and their spending for one week. Get that student to call out their pocket money and spending amounts for their classmates. Get the rest of the students to describe what the situation looks like and what the hidden student's balance for the week will be.

Using Number Properties

Pose problems about a person's income and expenditure over a whole year. Examples might be: ${}^{+}150 + {}^{-}225 = {}^{-}75$, ${}^{-}1000 + {}^{+}1500 = {}^{+}500$, ${}^{+}99 + {}^{-}48 = {}^{+}51$, ${}^{-}998 + {}^{+}1000 = {}^{+}2$

Hills and Dales

I am learning to add and subtract integers (positive and negative numbers).

Equipment: Hills and Dales made from Material Master 4–39. This can be made as a demonstration set or the students can be given their own sets.

Using Materials

Put one hill and one dale on the base. Explain how, if there is one hill and one dale, a bulldozer could be used to push the hill into the dale and leave a flat (zero) surface. Ask the students what other combinations of hills and dales would result in flat earth that

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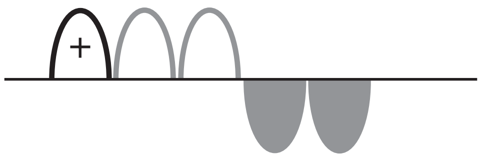
would give a zero surface: $^{+}2$ and $^{-}2$, $^{+}3$ and $^{-}3$, etc. Record these combinations as equations, for example: $^{+}3 + ^{-}3 = 0$.

Show how calculations like these can be done on a calculator. For example, press 3 + 3 ± =. Note that the ± (+/–) key changes the sign of the number shown in the calculator window from positive to negative (or vice versa).

Now place three hills and two dales on the base. Ask the students how they could describe this "landscape" using numbers ($^{+}3 + ^{-}2$). Ask them how they could use a bulldozer to fill in the two dales. What would remain? (One hill). Show them how to complete the equation $^{+}3 + ^{-}2 = ^{+}1$.



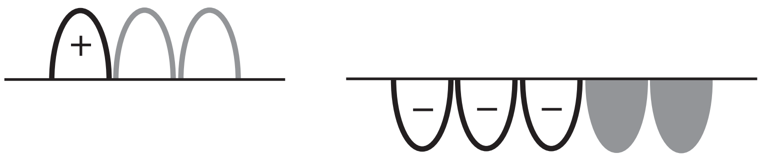
Get the students to perform the operation on a calculator as a check: 3 + 2 ± =. Give them a range of addition problems to solve, using hills and dales. The students can use calculators to confirm the answers.

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Good problems might be:

$^{-}1 + ^{+}4 = ^{+}3$	$^{+}3 + ^{+}2 = ^{+}5$	$^{-}2 + ^{-}3 = ^{-}5$	$^{+}5 + ^{-}3 = ^{+}2$	$^{+}1 + ^{-}4 = ^{-}3$
$^{-}3 + ^{+}7 = ^{+}4$	$^{-}3 + 0 = ^{-}3$	$^{-}2 + ^{+}2 = 0$	$0 + ^{-}3 = ^{-}3$	$^{-}1 + ^{-}4 = ^{-}5$

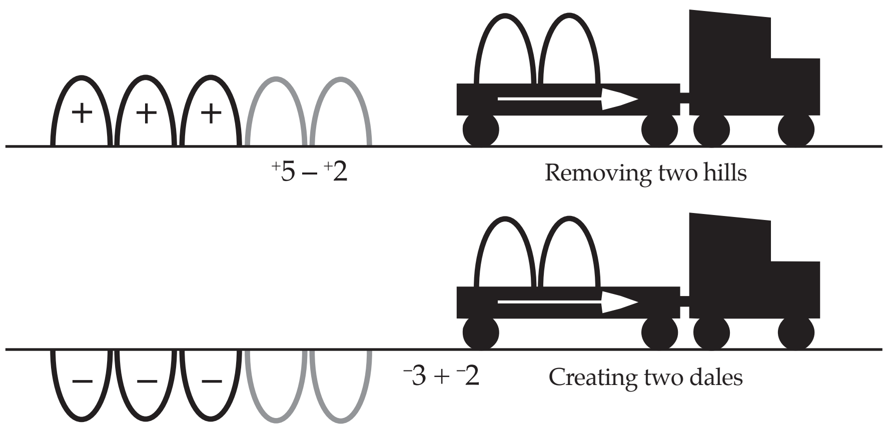
Now give your students some subtraction problems that involve *only* hills, or *only* dales, and get them to describe these problems using equations.
For example: $^{+}3 - ^{+}2 = ^{+}1$ and $^{-}5 - ^{-}2 = ^{-}3$.



Care needs to be taken when explaining how to do subtractions that involve *both* hills and dales. It can be useful to think of a subtraction as a removal. Suggest to your students that they imagine they are trucking contractors who earn their living from earthmoving. Ask them what similarity there is (in the amount of earth to be trucked in or away) in these pairs of situations:

- Removing a hill; creating a dale (by excavating it)
- Removing a dale (by filling it in); creating a hill.

Consider these two scenarios:



From the trucker's point of view, carting away the earth from the demolition of two hills or from the excavation of two dales is the same thing. (The same amount of earth has to be shifted.) Likewise, from a mathematics point of view, $^{+}3 - ^{+}2$ has the same answer as $^{+}3 + ^{-}2$. Both equal $^{+}1$. *Subtracting $^{+}2$ has the same effect as adding $^{-}2$.*

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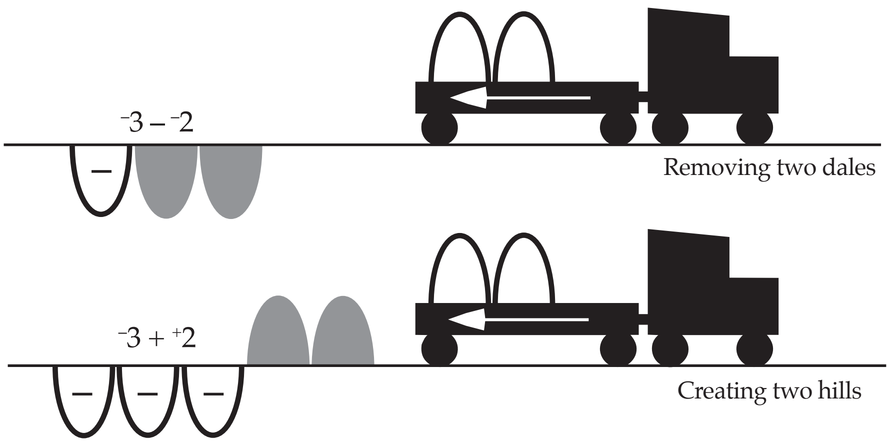
Similarly, from the trucker's point of view, removing two dales (by filling them in) requires exactly the same amount of earth as creating two hills. (The same amount of earth has to be shifted.) So $^{-}3 - {}^{-}2$ has the same answer as $^{-}3 + {}^{+}2$. Both equal $^{-}1$. *Subtracting* $^{-}2$ *has the same effect as adding* $^{+}2$.



Give your students a range of problems involving removing hills and dales, and ask them to rewrite each one (using the trucker's principle) so that the problem is entirely about hills, or entirely about dales, and then solve. Here are some to start with:

$^{+}4 - {}^{-}1$ [$= {}^{+}4 + {}^{+}1 = {}^{+}5$]	$^{-}6 - {}^{+}4$ [$= {}^{-}6 + {}^{-}4 = {}^{-}10$]	$^{+}5 - {}^{-}2$ [$= {}^{+}5 + {}^{+}2 = {}^{+}7$]
$^{-}6 - {}^{+}3$ [$= {}^{-}6 + {}^{-}3 = {}^{-}9$]	$^{+}4 - {}^{-}2$ [$= {}^{+}4 + {}^{+}2 = {}^{+}6$]	$^{-}7 - {}^{+}3$ [$= {}^{-}7 + {}^{-}3 = {}^{-}10$]
$^{+}8 - {}^{-}5$ [$= {}^{+}8 + {}^{+}5 = {}^{+}13$]	$^{-}1 - {}^{+}6$ [$= {}^{-}1 + {}^{-}6 = {}^{-}7$]	$^{+}12 - {}^{-}7$ [$= {}^{+}12 + {}^{+}7 = {}^{+}19$]
$^{-}4 - {}^{+}6$ [$= {}^{-}4 + {}^{-}6 = {}^{-}10$]	$^{+}2 - {}^{-}2$ [$= {}^{+}2 + {}^{+}2 = {}^{+}4$]	$^{-}9 - {}^{+}9$ [$= {}^{-}9 + {}^{-}9 = {}^{-}18$]

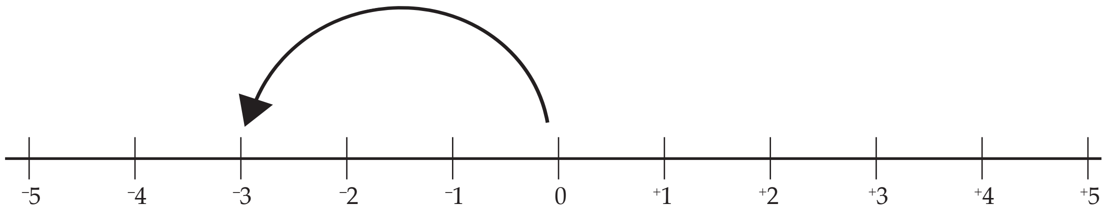
The students achieving success proceed to ***Using Number Properties****. Otherwise, they proceed to the next activity at some later time.*

Using Imaging

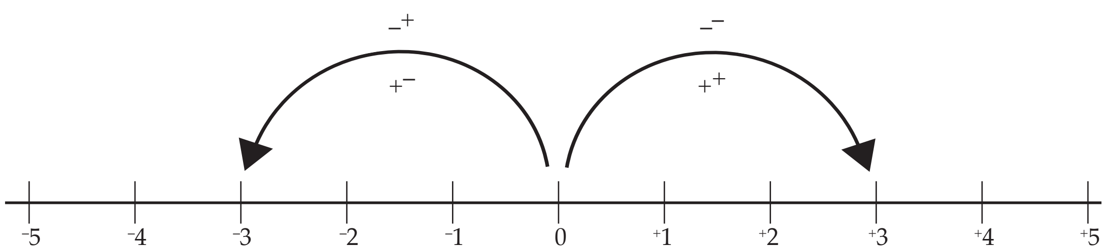
Provide equation sets so students can use both imaging of hills and dales and recursive relationships within the equations to establish results. For example:

$^{+}3 + {}^{+}2 = {}^{+}5$	$^{-}3 + {}^{+}2 = {}^{-}1$	$^{+}3 - {}^{+}2 = {}^{+}1$	$^{-}3 - {}^{+}2 = {}^{-}5$
$^{+}3 + {}^{+}1 = {}^{+}4$	$^{-}3 + {}^{+}1 = {}^{-}2$	$^{+}3 - {}^{+}1 = {}^{+}2$	$^{-}3 - {}^{+}1 = {}^{-}4$
$^{+}3 + 0 = {}^{+}3$	$^{-}3 + 0 = {}^{-}3$	$^{+}3 - 0 = {}^{+}3$	$^{-}3 - 0 = {}^{-}3$
$^{+}3 + {}^{-}1 = \square$	$^{-}3 + {}^{-}1 = \square$	$^{+}3 - {}^{-}1 = \square$	$^{-}3 - {}^{-}1 = \square$
$^{+}3 + {}^{-}2 = \square$	$^{-}3 + {}^{-}2 = \square$	$^{+}3 - {}^{-}2 = \square$	$^{-}3 - {}^{-}2 = \square$

Summarise the results of the calculations using an integer number line. Draw specific problems on number lines. For example, $0 + {}^{-}3 = {}^{-}3$



Develop a number line model that shows the directional effect of adding and subtracting positive and negative integers:



Give the students the opportunity to apply this model, folding back to materials and imaging of the hills and dales, if necessary, to confirm results.

Using Number Properties

The students can demonstrate their understanding of the number properties by solving integer addition and subtraction problems mentally. Suitable problems are:

${}^{+}3 + {}^{-}4 = {}^{-}1$	${}^{-}1 + {}^{+}5 = {}^{+}4$	${}^{+}3 - {}^{-}3 = {}^{+}6$	${}^{-}4 - {}^{+}2 = {}^{-}6$
${}^{+}2 + {}^{+}2 = {}^{+}4$	${}^{-}3 + {}^{-}1 = {}^{-}4$	${}^{+}3 - {}^{+}2 = {}^{+}1$	${}^{-}3 - {}^{+}2 = {}^{-}5$
${}^{+}6 - {}^{+}3 = {}^{+}3$	${}^{-}3 - {}^{-}2 = {}^{-}1$	${}^{-}1 - {}^{+}5 = {}^{-}6$	${}^{-}3 + {}^{+}2 = {}^{-}1$
${}^{+}39 + {}^{-}26 = {}^{+}13$	${}^{-}64 + {}^{+}58 = {}^{-}6$	${}^{+}72 - {}^{-}28 = {}^{+}100$	${}^{-}47 - {}^{+}45 = {}^{-}92$

Independent Activities

Material Master 4–40 is the game of Integer Invaders. Through this game, the students learn to add integers and plot co-ordinates on an integer number plane. Material Master 4–41 is the game of Integer Cover-up, which is a game to consolidate addition and subtraction of integers.

Average Ability

I am learning to use multiplication to add and subtract series of whole numbers, fractions, and decimals.

Equipment: Student hundreds boards. Different-coloured transparent counters.

Using Materials

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Give each student a hundreds board. Ask them to put counters of the same colour on any pair of numbers that add to 11. Tell them to continue the process, using a new colour to mark each pair. The students should notice that there are five pairs, 5 + 6, 4 + 7, 3 + 8, 2 + 9, and 1 + 10. Challenge the students to add all of the numbers they have covered, that is, 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10. Discuss strategies they may have used, like making tens. Note that the answer, 55, is the answer to 5 × 11. Ask the students why that is. They should be able to link this to the five pairs adding to 11, which they found through the counter exercise.

Challenge the students to find pairs of numbers that add to 21 and mark these pairs with coloured counters on their hundreds board.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

Tell them to add all the whole numbers, 1 + 2 + 3 + ... + 18 + 19 + 20. These form 10 pairs totalling 21, so the sum is 10 × 21 = 210. Challenge the students to then add all of the whole numbers 1 + 2 + 3 + ...+ 22 + 23 + 24. Some students may realise that they could simply add 21 + 22 + 23 + 24 = 90 onto the previous answer of 210. This is a good check on the multiplicative strategy of finding 12 pairs of 25 (1 + 24, 2 + 23, 3 + 22, ...).

Using Imaging

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Encourage imaging of adding arithmetic series (those with common differences) by posing the following problems. Allow the students access to the hundreds boards if necessary but encourage them to work out the problems without putting on counters.

Add all the whole numbers: 1 + 2 + 3 + ... + 98 + 99 + 100 (50 × 101 gives 5 050).

Add all the odd numbers: 1 + 3 + 5 + ... + 95 + 97 + 99 (25 × 100 gives 2 500).

Add all the even numbers: 2 + 4 + 6 + ... + 96 + 98 + 100 (25 × 102 gives 2 550).

Note that the answers to the odd and even series should total the answer for all the whole numbers: 2 500 + 2 550 = 5 050.

Add all these numbers: 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 (four pairs of 20 plus 10 gives 90).

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Using Number Properties

Promote generalisation of the sum of a series that has constant differences though examples like:

5 + 8 + 11 + 14 + 17 + 20 + 23 + 26 + 29 + 32 + 35 (five pairs of 40 plus 20 gives 220)

$\frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1 + 1\frac{1}{4} + 1\frac{1}{2} + 1\frac{3}{4} + 2 + 2\frac{1}{4} + 2\frac{1}{2} + 2\frac{3}{4}$ $(5 \times 3 + 1\frac{1}{2} = 16\frac{1}{2})$

1.4 + 1.3 + 1.2 + 1.1 +...+ 0.3 + 0.2 + 0.1 (7 × 1.5 = 10.5)

2 + 5 + 8 + 11 + 14 + 17 + 20 + 23 + 26 + 29 (5 × 31 = 155)

5.0 + 4.6 + 4.2 + 3.8 + 3.4 + 3.0 + 2.6 + 2.2 + 1.8 + 1.4 (5 × 6.4 = 32.0)

Multiple Ways to Add and Subtract

I am learning to use multiplication and division to solve addition and subtraction problems.

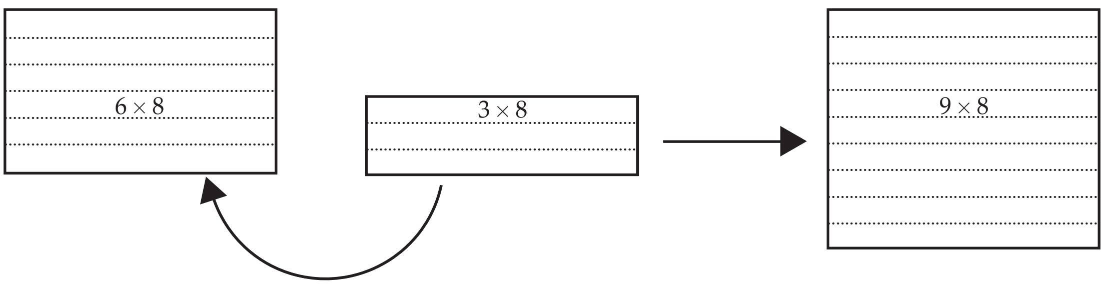
Equipment: Animal Cards.

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

Using Materials

Create two arrays using animal cards to represent 6 × 8 and 3 × 8. Ask the students how many animals are in each array. Pose the problem of adding these sets of animals together, that is 48 + 24. Allow the students to present their strategies.

Some students may note that (6 × 8) + (3 × 8) = 9 × 8 = 72 (combining sets of eight).



Provide two similar examples, getting the students to make the arrays then solve the resulting addition problem. Suitable problems include:

(5 × 7) + (4 × 7) = 9 × 7   (3 × 6) + (6 × 6) = 9 × 6
(4 × 9) + (6 × 9) = 10 × 9   (7 × 8) + (2 × 8) = 9 × 8

Ask the students to state what these problems have in common (both addends are multiples of a common number).

*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Using Imaging

Pose problems for the students to image, beginning with the addends instead of the factors.

If necessary, fold back to the materials. Possible examples include:

16 + 24 + 32 as (2 × 8) + (3 × 8) + (4 × 8) = 9 × 8
49 + 28 as (7 × 7) + (4 × 7) = 11 × 7
45 + 27 + 18 as (5 × 9) + (3 × 9) + (2 × 9) = 10 × 9

Extend the imaging to include subtraction problems:

81 – 27 as (9 × 9) – (3 × 9) = 6 × 9   64 – 48 as (8 × 8) – (6 × 8) = 2 × 8
63 – 35 as (9 × 7) – (5 × 7) = 4 × 7   90 – 36 as (10 × 9) – (4 × 9) = 6 × 9
54 – 36 as (9 × 6) – (6 × 6) = 3 × 6 or (6 × 9) – (4 × 9) = 2 × 9

Using Number Properties

Pose addition and subtraction problems where it is helpful to identify a common factor.

48 + 56 + 24 + 32 = 20 × 8   42 + 35 + 49 + 14 = 20 × 7
72 – 27 + 45 – 36 = 6 × 9   88 – 56 – 16 + 32 = 6 × 8
120 – 54 – 48 – 18 = 0 × 6   77 – 28 + 14 – 35 = 4 × 7

Learning Experiences to Move Students from Advanced Multiplicative–Early Proportional to Advanced Proportional

Combining Proportions

I am learning to combine proportions and work out what parts the total is made up of.

Equipment: unilink cubes. Plastic ice-cream containers or paper bags.

Using Materials

Tell the students to make up three stacks of 10 cubes made up as follows: 3 yellow with 7 blue, 5 yellow with 5 blue, and 4 yellow with 6 blue.

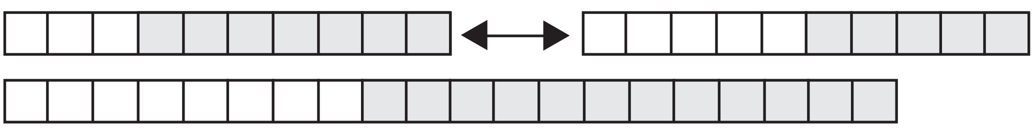
For each stack of cubes, ask the students to say what fraction and percentage of each stack is yellow or blue. Record their answers as:

*The students achieving success proceed to **Using Imaging**. Otherwise, they proceed to the next activity at some later time.*

	Yellow		**Blue**	
Stack A (3:7)	$\frac{3}{10}$	30%	$\frac{7}{10}$	70%
Stack B (5:5)	$\frac{5}{10}$	50%	$\frac{5}{10}$	50%
Stack C (4:6)	$\frac{4}{10}$	40%	$\frac{6}{10}$	60%

Problem: "If I combine stack A and stack B by putting them together in this container (or bag), what fraction and percentage of the total will each colour be?"

The students should observe that the combined number of yellows will be 3 + 5 = 8 and the combined total of blues will be 7 + 5 = 12. So the total fractions and percentages are $\frac{8}{20}$ = 40% (yellow) and $\frac{12}{20}$ = 60% (blue). Ask the students if they could have guessed the combined proportions from the fractions from each stack. Note that simply adding the fractions does not give the answer, that is, $\frac{3}{10} + \frac{5}{10} = \frac{8}{10}$ for yellows, as the initial fractions and the answer fractions have different denominators, 10 and 20. Recording the initial stacks as ratios may help, 3:7 combined with 5:5 gives a combined ratio of 8:12.



The students will need to recognise the subtle connection between ratios and fractions, that is, 8:12 means $\frac{8}{20}$ of the cubes are yellow, not $\frac{8}{12}$.

Combine other collections of stacks and ask the students to work out the fractions and percentages for each collection; Stack A with Stack C (7 yellow:13 blue), Stack B with Stack C (9 yellow:11 blue), Stack A, Stack B, and Stack C together (12 yellow:18 blue). With each problem, expect the students to anticipate the combined proportions.

Introduce a new stack (D) made from 40 cubes, 23 yellow and 17 blue. Tell the students that you are going to combine the 40 cube stack with each of the 10 cube stacks in turn to see what the combined proportions will be. Ask the students what they expect. They should note that the 40 cube stack proportions will have four times the "weight" in determining the combined proportions.

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*The students achieving success proceed to **Using Number Properties**. Otherwise, they proceed to the next activity at some later time.*

Stacks	Ratios		Fractions		Percentages	
	Separate	Combined	Yellow	Blue	Yellow	Blue
A and D	3:7 and 23:17	26:24	$\frac{26}{50}$	$\frac{24}{50}$	52%	48%
B and D	5:5 and 23:17	28:22	$\frac{28}{50}$	$\frac{22}{50}$	56%	44%
C and D	4:6 and 23:17	27:23	$\frac{27}{50}$	$\frac{23}{50}$	54%	46%

Using Imaging

Pose similar problems but mask the materials. Get the students to anticipate the combined proportions and express them as fractions and percentages. Expect the students to record their thinking on paper as an aid to communication.

Problems might be:
2 yellow:8 blue with 8 yellow:2 blue (10:10 and 50%:50%),
7 yellow:3 blue with 5 yellow:15 blue (12:18 and 40%:60%)
15 yellow:5 blue with 3 yellow:37 blue (18:42 and 30%:70%)

Using Number Properties

Increase the difficulty of the problems by making the bases of the proportions non-multiples of 10, using three colours (for example, white) and removing cubes rather than combining them. Record the problems in table form, for example:

Yellow	Blue	White
3	4	6
2	4	1

Other problems might be:
8 yellow:5 blue:4 white with 11 yellow:11 blue:11 white (19:16:15 and 38%:32%:30%)
4 yellow:10 blue:5 white with 8 yellow:2 blue:11 white (12:12:16 and 30%:30%:40%)
Begin with 14 yellow:11 blue:5 white and remove 7 yellow:2 blue:1 white (7:9:4 and 35%:45%:20%).

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