Answers and Teachers' Notes



Contents

Introduction	2
Answers	3
Teachers' Notes	8
Copymaster	30

MINISTRY OF EDUCATION

Te Tāhuhu o te Mātauranga

Introduction

The Figure It Out series is designed to support *Mathematics in the New Zealand Curriculum*. The booklets have been developed and trialled by classroom teachers and mathematics educators. The series builds on the strengths of a previous series of mathematics booklets published by the Ministry of Education, the School Mathematics supplementary booklets.

Figure It Out is intended to supplement existing school mathematics programmes and can be used in various ways. It provides activities and investigations that students can work on independently or co-operatively in pairs or groups. Teachers can select particular activities that provide extension to work done in the regular classroom programme. Alternatively, teachers may wish to use all or most of the activities in combination with other activities to create a classroom programme. The booklets can be used for homework activities, and the relevant section in the teachers' notes could be copied for parents. These notes may also provide useful information that could be given as hints to students.

There are eight booklets for level 3: one booklet for each content strand, one on problem solving, one on basic facts, and a theme booklet. Each booklet has its own *Answers and Teachers' Notes*. The notes include relevant achievement objectives, suggested teaching approaches, and suggested ways to extend the activities. The booklets in this set (level 3) are suitable for most students in year 5. However, teachers can decide whether to use the booklets with older or younger students who are also working at level 3.

The booklets have been written in such a way that students should be able to work on the material independently, either alone or in groups. Where applicable, each page starts with a list of equipment that the students will need to do the activities. Students should be encouraged to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Teachers could encourage students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Where possible, suggestions have been made to encourage discussion and oral presentation of answers, and teachers may wish to ask the students to do this even where the suggested instruction is to write down the answer.

The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.

(Mathematics in the New Zealand Curriculum, page 7)

Students will have various ways of solving problems or presenting the process they have used and the solution. Successful ways of solving problems should be acknowledged, and where more effective or efficient processes can be used, students can be encouraged to consider other ways of solving the problem.



Page 1: Three Chairs	Page 4: Monster Measure		
 Activity Answers will vary. a. Answers will vary, but the important parts to think about are the length of the person's back, the length of their legs, and the width of their bottom. bc. Answers will vary. Investigation Answers will vary.	Activity 1. a. 12.5 m b. 9.5 m c. 3 m d. 3 m e. 1.5 m 2. Answers will vary.		
Dago 2: Notwork Nightmaro	Page 5: Growth Industry		
Activity 1. 181 m 2.–3. Answers will vary.	Activity 1. $$2,000$ 2. Answers will vary. An example is: 4		
Page 3: Going the Distance	 Answers will vary. The example in question 2 would cost \$1,300. 		
Activity 1. There are many possible routes. Some routes will have equal distances while others will be different. Here is an example of a route with the distances measured. The scale is 1 cm : 1 km. Places Scale measure Distance Annie's town to Poto 4 cm 4 km	 The cheapest plan is a square 6 m x 6 m greenhouse. (This would cost \$1,200.) This gives a perimeter of 24 m, which is less than the perimeter of any other 36 m² floor plan. The cheapest plan is a square 8 m x 8 m greenhouse. This gives a perimeter of 32 m, which is less than the perimeter of any other 		

Annie's town to Poto	4 cm	4 km
Poto to Ohiti	7 cm	7 km
Ohiti to Dunchurch	5.5 cm	5.5 km
Dunchurch to Eastville	6 cm	6 km
Eastville to Totorere	6.5 cm	6.5 km
Totorere to Waimutu	3 cm	3 km
Waimutu to Manuhou	7.5 cm	7.5 km
Manuhou back to	3 cm	3 km
Annie's town		
Total	42.5 cm	42.5 km
	I	l

- . 18 km
- **3.** Answers will vary.

 64 m^2 floor plan.

Answers will vary. The cheapest in whole metres 6. would be a 6 m \times 4 m floor plan. Otherwise, a square with sides of $\sqrt{24}$ (4.899 m) is the most economical.

Page 6: Tivaevae Karakara

Activity One

- 1. 168 pieces
- 2. 88 pieces
- 3. 4 squares are used, but each flower covers approximately half of each square.

Activity Two

- 1. 72 squares
- 2. 48 squares

Page 7: An Absorbing Challenge

Activity

- 1. Answers will vary slightly because the part squares must be estimated as half, quarter, etc. Approximate areas are:
 - **a.** 40 square units
 - **b**. 57 square units
 - c. 80 square units
 - d. 67 square units
 - e. 57 square units
 - f. 75 square units
- 2. Answers will vary, depending on what qualities students think are important, for example, absorbing all liquid in a small area.

Pages 8-9: School Sculpture

Activity

Answers will vary.

Page 10: Cuboid Construction

Activity

- 1. a. Yes.
 - $9 \times 2 \times 2 = 36$ blocks Iosefa

 $12 \times 3 \times 1 = 36$ blocks Roma

1 **x** 6 **x** 6 = 36 blocks Liam

- Bee Har $3 \times 4 \times 3 = 36$ blocks
- **b.** 2 x 1 x 18; 1 x 1 x 36; 9 x 1 x 4; 3 x 2 x 6
- 2. Answers will vary. Some examples are:

















The volume of each cuboid will be 30 cubic units.



Page 11: Cut It Out Page 16: Wanted! Activity Activity One 1. 392 2. When a 3×3 square is cut from the paper, the Activity Two volume is $12 \times 12 \times 3 = 432 \text{ cm}^3$. This gives the maximum number of unit place value blocks Answers will vary. (432). Investigation Answers will vary. (The volume of **a** is $337^{1/2}$, cm³ and of **b** is $422^{1}/_{2}$ cm³.) Activity 1. Answers will vary. Page 12: Weighty Words 2. Practical activity Investigation Page 18: Fill Them Up 1.-3. Answers will vary. 4. 25 boxes Activity One Answers will vary. Activity Two Page 13: Solve It or Sink It Answers will vary. Activity Page 19: Hot Spots! 1.-2. Answers will vary. 3. a.-b. Answers will vary. You will use approximately half the number used in Activity One question 1. 1. According to the colouring in the map: c. Discussion points will vary. Page 14: Mini Mass Activity

Answers will vary.

Page 15: Would Wood Fit?

Activity

- 1.-2. Answers will vary depending on the dimensions of the blocks used.
- 3.-4. Answers will vary.

Answers will vary according to the duster's properties.

Page 17: Rainy Day Delights

Very warm	Warm	Mild
Kaitaia	Auckland	Waiouru
Tauranga	New Plymouth	Hokitika
Gisborne	Wellington	Milford Sound
Blenheim	Nelson	Dunedin
Alexandra	Christchurch	Invercaroill

- 2. Very warm places: those listed in the "Very warm" column above
- 3. Warm and mild places: those listed in the "Warm" and "Mild" columns
- 4. Answers will vary. The factors that should be discussed are distance from mountain ranges, on what side of a mountain range a town is, the prevailing wind directions, height above sea level, and latitude.

Activity Two

Answers will vary according to location.

Page 20: Burning Issues

Activity

- Answers will vary, but they should include 1. comment about what causes sunburn and the influence of the clouds.
- 2.

Place	UV index	Temperature	Weather
Kaitaia	12	26°C	sunny
Gisborne	12	33℃	partly cloudy
New Plymouth	10	28°C	sunny
Nelson	9	26°C	sunny
Christchurch	6	25°C	overcast
Invercargill	7	20°C	sunny







5.35 p.m.

6.25 p.m.

11.



6.10 p.m.

Page 21: On Time

Activity



6.25 a.m.



9.08 a.m.



12.30 p.m.







2.23 p.m.

Page 22: Breaking Records

Activity

2.

- **a.** Yes. Explanations will vary $\binom{60}{40} = 1.5$. 1.
 - b. Answers will vary.
 - Answers will vary.

Page 23: Cooking Time

Activity One

- 9 minutes 1.
- 2. 21 minutes

Activity Two

One solution:

Start both timers together. When the 2 minute timer has run out, start cooking the damper bread because there will be exactly 3 minutes to go on the 5 minute timer. When the 5 minute timer runs out, flip it over again and let it run for the full 5 minutes. 3 + 5 = 8

Activity Three

A possible meal:

- 4 potatoes at 3 minutes each = 4×3
 - = 12 minutes
- 4 corn cobs at 2.5 minutes each = 4×2.5
 - = 10 minutes
- Half a head of cauliflower at 10 minutes
 = 10 minutes
- 1.2 kg of chicken is the same as 500 g + 500 g + 200 g. This means the cooking time will be 10 minutes + 10 minutes + $\frac{2}{5}$ of 10 minutes = 24 minutes.

Total time = 56 minutes

Page 24: Holey Moley

Game

A game using measurement



Overview: Measurement			
Title	Content	Page in students' book	Page in teachers' notes
Three Chairs	Measuring lengths in metres, centimetres, and millimetres	1	9
Network Nightmare	Solving problems involving lengths in metres	2	10
Going the Distance	Using scale maps to measure distances	3	10
Monster Measure	Using scale to measure lengths	4	11
Growth Industry	Finding the areas and perimeters of rectangles	5	12
Tīvaevae Karakara	Exploring units of area	6	13
An Absorbing Challenge	Finding areas of irregular shapes	7	14
School Sculpture	Solving problems involving length and area	8–9	15
Cuboid Construction	Finding the volume of rectangular prisms	10	16
Cut It Out	Finding the volume of rectangular prisms	11	17
Weighty Words	Solving problems involving units of mass	12	18
Solve It or Sink It	Exploring weight and area	13	19
Mini Mass	Measuring the mass of small objects	14	20
Would Wood Fit?	Finding volumes with cubic metres	15	21
Wanted!	Performing measuring tasks using a range of units	16	22
Rainy Day Delights	Solving problems using units of length and capacity	17	23
Fill Them Up	Exploring capacity	18	24
Hot Spots!	Reading temperature in degrees Celsius	19	25
Burning Issues	Interpreting information on a weather map	20	26
On Time	Showing digital time as analogue time	21	27
Breaking Records	Measuring time in seconds and minutes	22	27
Cooking Time	Solving problems with time	23	28
Holey Moley	Solving problems with length	24	29

Page 1: Three Chairs

Achievement Objectives

- perform measuring tasks, using a range of units and scales (Measurement, level 3)
- effectively plan mathematical exploration (Mathematical Processes, problem solving, level 3)

Activity

This activity provides an opportunity for some purposeful measurement of length in a problem-solving context. It is particularly useful for measuring in millimetres, converting from millimetres to centimetres and metres, developing skills using a ruler or tape measure, and understanding the right angle.

You could read aloud or have available the story of Goldilocks and the three bears as an introduction to this activity.

There are four lengths worth considering: the height of the back of the chair (A), the length of the seat (B), the width of the seat (C), and the height of the seat above the floor (D). The most important of these is probably D, the height of the seat above the floor, as it affects the angles at which a person sits.



When students measure these lengths for the five people, be sensitive to anybody who shows signs of discomfort with the measurement process. It might help to measure the B and C lengths by placing a piece of large paper on a chair and sitting the student who is to be measured on the paper. The student could then mark the two lengths on the paper themselves and take their measurements from the marks on the paper.

Take the opportunity to discuss correct posture when sitting at a desk. This could be extended to include the correct arm and head positions for working on a computer. Discuss the ideal angle between lengths A and B and lengths B and D (in each case, a right angle).



Discuss the reasons why only a small number of chair sizes are available for classes. You could explain the impracticality of continually replacing chairs as students grow. A useful way of deciding an appropriate range of chair sizes would be to plot a stem-and-leaf graph (see *Mathematics in the New Zealand Curriculum*, page 216) for everyone in the class for the lengths B, C, and D.

Investigation

The school office probably has some brochures from suppliers of school furniture that show the sizes of class chairs available for purchase.

Page 2: Network Nightmare

Achievement Objectives

- demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (Measurement, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)

Activity

Students practise estimating and measuring length in metres in this activity.

Make sure that students fully understand questions 2 and 3. They could restate them in their own words or explain the key facts to a partner. Discuss some good strategies that students could use, such as trial and error, drawing a diagram, making a table, or trying a simpler example first. One way to simplify the task would be to network only one section of the school. This adjustment may be useful when students try to apply the problem to a large school.

Set up problem-solving groups (four students per group) and ask them to use estimation skills to come up with two or three options that they can then use to calculate accurately. The students could draw a scale plan of the school and use a ruler or drawing compass to solve the problem.

Tell students that the rule "only four computers to a cable" applies only to the problem given in this booklet. In reality, other networks can connect many more computers. However, the "only four computers to a cable" rule would be useful for question **2**. For question **3**, you may need to help students find information on cable prices.

Page 3: Going the Distance

Achievement Objectives

- demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (Measurement, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)

Activity

This activity involves estimating and measuring length in centimetres. The major focus is to calculate distances using a scale drawing.

This is an ideal activity to encourage estimation. Initially, you could ask students to estimate the distance in centimetres between two towns. Have them make a chart to help them convert this to the actual distance in kilometres, for example:

Places	Scale measure	Distance
Annie's town to Poto	4 cm	4 km
Poto to Ohiti	7 cm	7 km

Those who are confident with the idea of scale drawing could estimate the distance in kilometres straight away and explain their answer.

Encourage accuracy with the use of the ruler by insisting that students look directly down on the town they are measuring as they align their ruler and read the measure. Make sure that they start at the beginning of the scale on the ruler and not at the 1 centimetre mark.



Change the scale to provide some additional practice with calculating a scale distance, for example,

- 1 centimetre : 2 kilometres
- 1 centimetre : 4 kilometres
- 1 centimetre : 8 kilometres
- 1 centimetre : 16 kilometres

This could be extended to:

1 centimetre : 2.5 kilometres

1 centimetre : 7.25 kilometres

Make a point of focusing students' attention on how the scale is written using the colon and how to interpret the instruction "The scale is 1 centimetre to 4 kilometres". Help them to see that this is the same instruction as saying "1 centimetre on the drawing shows 4 kilometres of actual distance".

Ensure that students who make up some distance problems, as in question **3**, are responsible for checking the answers to their problems and are prepared to explain their answers if necessary. This will encourage them to make up sensible problems and help reinforce their understanding of scale measurement.

Page 4: Monster Measure

Achievement Objectives

- demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (Measurement, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)

Activity

In this activity, students will need to be able to measure accurately in centimetres and millimetres and to calculate using a scale.

If students are having difficulty drawing their own monster accurately, give them some centimetresquare grid paper.

Take the opportunity to build a strong understanding of the relationship between centimetres and millimetres by helping students convert 0.5 and 0.25 centimetres to millimetres. Compare the millimetre scale on their 30 centimetre ruler or on a metre ruler with the centimetre scale. A table of equivalences would make a good chart for the classroom wall:

1 cm = 1.0 cm	$\frac{1}{5}$ cm = 0.2 cm
= 10 mm	= 2 mm
${}^{1}/_{2} \text{ cm} = 0.5 \text{ cm}$ = 5 mm	${}^{1}/{}_{10} \text{ cm} = 0.1 \text{ cm}$ = 1 mm
$^{1}/_{4}$ cm = 0.25 cm	
= 2.5 mm	

You might like to relate the prefix centi- to the metre unit and highlight other words that refer to 100 by using "cent" as a prefix or syllable, for example, century, percent, dollars and cents. You could compare the prefix "mille" in millennium to the prefix "milli" in millimetre and point out the connection to 1 000.

It may be worth discussing with students why we have different units of measure like metres, centimetres, and millimetres. The choice of unit depends mainly on the degree of accuracy needed to complete the measuring task. The special relationship of 10 that connects the units of measure in our metric system can be used to reinforce our place value system. Introduce students to the decimetre (dm) to complete the unit of measure number pattern:

10 mm = 1 cm 10 cm = 1 dm 10 dm = 1 m

Page 5: Growth Industry

Achievement Objectives

- demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (Measurement, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical processes, communicating mathematical ideas, level 3)

Activity

This activity focuses on rectangles and the relationship between the perimeter and the area.

Use the 18 metre x 2 metre grid to identify the length, width, perimeter, and area. Students will need to see that question 1 is asking them to find the perimeter of the grid. If they have trouble connecting the grid plan (the array) with area and perimeter, it may be useful to have them use a simpler scenario. For example, they could make all the possible rectangles using 12 squares each time. Unit place value blocks would be useful for making the arrays.



3 cm x 4 cm

They could then complete this chart:

Rectangle Measures				
Area Length Width Perimeter				
12 cm ²	l cm	12 cm	26 cm	
12 cm ²	2 cm	6 cm	16 cm	
12 cm ²	3 cm	4 cm	14 cm	

Initially, have students use whole numbers only. Mixed fractions, such as a $1^{1/2}_{2}$ centimetre **x** 8 centimetre rectangle, would make an interesting extension to this activity.

This chart could then be used with the 36 square metre floor plan to describe each possible rectangular plan. Have the students extend their rectangular measure chart to include a cost column that will help with questions 3 to 6.

Students could make or draw a grid to a scale of 1 centimetre : 1 metre that will help them as they explore the various plans.

Rectangle Measures				
Area	Length	Width	Perimeter	Cost
36 m ² 36 m ² 36 m ² 36 m ²	2 m 3 m 4 m 6 m	18 m 12 m 9 m 6 m	40 m 30 m 26 m 24 m	\$2,000 \$1,500 \$1,300 \$1,200

Check that students have found all possible whole number combinations.

The most economical rectangular shape will always be a square. For this greenhouse problem, this means the 6 metre x 6 metre plan. Remember that all squares are rectangles, so this option belongs in the chart.

If some students go on to explore mixed fraction grids, they may find the more obvious grid options such as $1^{1}/_{2} \times 24$, $2^{1}/_{4} \times 16$, and $4^{1}/_{2} \times 8$ grids, but these will not be more economical than the square.

Encourage students to express the rules for calculating area and perimeter in their own words as well as in written form. Accept rules like "To find the perimeter, you can add all the sides" or "You add the length and width and double it". Eventually students may refine this to the usual p = 2(l + w), but it does not matter if they express the idea in a different form.

In question **6**, the most likely answer will be a plan that is 6 metres **x** 4 metres, so the scale grid plan will be 6 centimetres **x** 4 centimetres. Encourage students to make some grids to the same scale of 1 centimetre : 1 metre but measured in millimetres. The students may find that a scale grid of 48 millimetres **x** 50 millimetres (4.8 centimetres **x** 5 centimetres) would be more economical. If they do, help them to notice that this rectangle is closer to being a square than the 6 metre **x** 4 metre rectangle, so it is going to be more economical. As noted in the answers, the most economical floor plan is a square with sides of $\sqrt{24}$ (4.899 m).

Page 6: Tīvaevae Karakara

Achievement Objectives

- demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (Measurement, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)

Activity One

This activity provides some area problems in an interesting context.

Ask students how they solved question **1**. If necessary, explain that the quickest solution would be to calculate the length and the width and multiply the two.

Question 2 may present a difficulty for those who mistake this for a perimeter question. It is really asking for the area of the border. Other students may simply count the squares.



Discuss other ways of calculating the area of the border. Some useful hints may be: "Try to use the answer from question **1** to help you" or "Which part of the rectangle is not part of the border? How can you use this to help?" These questions may help students see that problems of this sort can be solved by subtracting the area of the smaller (10×8) rectangle from the larger (14×12) rectangle.

Question **3** asks the students to estimate the amount of area that the frangipani flower covers on each square. The answer gives the total squares used, but it also indicates that the actual area covered by each flower is half of each of the squares on which it appears.

Activity Two

In this activity, encourage your students to investigate different ways of calculating the answers for questions **1** and **2**. This may include subtracting the small (8 x 6) rectangle from the large (12 x 10) rectangle.

You may like to set some conditions for the patterns that students use in their designs, for example, each pattern should have an area of four squares but must be in more than four squares.

Page 7: An Absorbing Challenge

Achievement Objectives

- demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (Measurement, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)

Activity

This is a good activity for encouraging students to estimate and make sensible approximations of area.

When students have made an estimate, they are usually motivated to check the accuracy of their effort.

You may wish to have students work in problem-solving groups of four people to come up with a group list of the blots in order from largest blot to smallest blot, using estimation only. They could then split into pairs to answer question **1**. After they have been working for 10 minutes, bring the groups together to discuss their estimation strategies. Their ideas may include: "We coloured all the whole squares red and then the half squares green. We used different colours for the one-quarters, three-quarters, one-thirds, and two-thirds. We then added them up to make whole squares and found the total."

"After tallying the whole squares and crossing them off, we tried to pair up two squares that would join to make a whole. Then we tallied these and crossed them off as we went." "We copied the blot and then tried to cut and paste bits to make the blot into a rectangle. Then we could multiply the length by the width."

After this discussion, the groups who have not been able to work out approximate areas for the blots could be sent back to continue solving the problem.

To clarify question **2**, ask the groups: "How do we decide which towel soaked up the water best?" Students may decide that the best soaker would use the least area because it could hold more. Alternatively, they might say that the most regular shape must mean a better quality towel because it soaks up water more evenly. Or they could argue that the largest area blot may indicate the best towel because it might have been the fastest to soak up the water. Whatever their decision, the students' choice of best soaker should match their reasoning for the way to determine the best soaker.

Pages 8-9: School Sculpture

Achievement Objectives

- demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (Measurement, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)

Activity

In this activity, students use measurement to problem solve. The key challenge is to make the best use of space while positioning the buildings to make the most of the sun.

You may need to discuss the path that the sun takes in the southern hemisphere as it moves from east to west to show why the north side is the warm side and the south is cold. The students can choose which direction the compass points on their plan, but they should show this on the plan. They should also decide the position of the main entrance to the school.

If this activity is set up as a design competition, students may be more motivated.

Brainstorm some ideas about how to decide what "best use" means. You could encourage students to consider trees, gardens, paths, types of playgrounds, pool, sheds and an incinerator, sports fields, covered eating spaces outside, halls, staff parking, and pedestrian access. Useful hints might include: "Can people move around the school easily and without getting muddy?"

"Can parents and visitors find the office easily?"

"What buildings should be near each other?"

"Where are the best places for the library and the pool?"

"Does your plan have a special feature like a slope or a gully that can be landscaped?"

"Is the sports field big enough?"

"Will there be enough parking for staff and visitors?"

Make sure that the students estimate and then calculate the areas for each item on their plan, using the scale provided, otherwise they will not have achieved the measurement objectives.

Page 10: Cuboid Construction

Achievement Objectives

- demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (Measurement, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)

Activity

This activity focuses on measuring the volume of rectangular prisms.

At this point, the key to this activity is the "How do you know?" part of question **1**. This is when the technique for determining volume by multiplying the length, width, and depth should become clear. If students do not come up with this technique themselves, you could then demonstrate with the multilink cubes.



Use patterning to help students see the connection between length, width, depth, and volume. (Note that length and height refer to the same dimension.) Have the students make a chart to help them see the pattern. A spreadsheet will be a useful tool here if students know how to enter a formula in the Volume column.

Volume Chart				
Length (units)	Width (units)	Depth (units)	Volume (cubic units)	
36	1	1	36	
18	2	1	36	
12	3	1	36	
9	2	2	36	
9	4	1	36	
6	6	1	36	
6	3	2	36	
4	3	3	36	

Discuss the pattern on the chart and make explicit the $l \times w \times d = V$ rule for finding the volume of a rectangular prism. Highlight the connection between the area rule and the volume rule by comparing them: "The area rule is length multiplied by width, and this gives square units. The volume rule is length multiplied by width multiplied by depth, and this gives cubic units. So volume is the area multiplied by the depth." This can be written as the rules: $A = l \times w$

 $V = l \mathbf{x} w \mathbf{x} d$

so $V = A \times d$.

Students could then explore the cuboids made with 30 cubes and make their own volume chart.

Page 11: Cut It Out

Achievement Objectives

- demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (Measurement, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)

Activity

This activity investigates measuring the volume of rectangular prisms. It is an ideal opportunity to use and compare a variety of problem-solving strategies such as guess and check, draw a diagram, make a table, and look for a pattern.

Question 1 clarifies the conditions for the investigation and tests whether the student understands the problem. Let students use a variety of strategies to find the volume of unit place value blocks. After students share these strategies, discuss the efficiency of using the $l \times w \times d$ rule.

Question 2 encourages students to investigate the most efficient open box, namely the one that holds the most unit cubes. Point out that different size cut-out squares will result in different combinations of length, width, and depth measures and hence different volumes. The students could work individually or in pairs. It may be necessary to clarify the meaning of "set of small corner squares". It may help to ask "What size squares can be cut from each corner of the 18×18 grid?" This would show that it is possible to cut out squares ranging from 1×1 to 8×8 .

Ask the students to discuss and report on possible strategies for answering the question. You may wish to assign different strategies to different groups and use an observer to report and discuss the most efficient strategy. Make sure a group uses a systematic approach and makes records using a table such as:

Cut out	L (cm)	W (cm)	D (cm)	V (cm ³)
1 x 1	16	16	1	256
2 x 2	14	14	2	392
3 x 3	12	12	3	432
4 x 4	10	10	4	400
5x5	8	8	5	320
6 x 6	6	6	6	216
7 x 7	4	4	7	112
8 x 8	2	2	8	32

They may stop after the fourth box is recorded if the pattern has become obvious.

The investigation explores sides based on fractions. This could result in varying solutions. The practical solution using blocks will be different from the solution reached using the $l \times w \times d$ rule because the rule assumes that you can split the cubes into halves. Those who use blocks may claim that it is sensible to stack the top layer of cubes with a half cube standing above the box edge. This will give a very different result.



13 units

Page 12: Weighty Words

Achievement Objectives

- demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (Measurement, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)

Investigation

Here students will be measuring mass in grams and kilograms and will need to compare kilograms with tonnes.

Using a practical strategy is the key to this investigation. How to weigh the biggest book may be a problem students need to solve. Small kitchen scales may not handle the biggest book. Students may need to use bathroom scales and devise a way to be accurate with these scales.

Here is one possible solution: If the scales show that the book has a mass between the divisions on the scales, students could add some metric weights to the book until the scales read accurately. They would then be able to subtract the metric weights from the mass recorded on the scales to find the actual mass of the book.



Question **3** has some challenging variables to consider. Soft or hard covers and the different sizes of books will cause problems. Students may begin by weighing books individually, but this will take a long time. Discuss strategies for doing the task. Ideas may include:

- Work in groups. Sort books into size categories and then into type of covers.
 - Weigh 1 kilogram of books in a particular category and multiply that number of books by 20.
 - Take one book from each different category and weigh them together. Divide 20 kilograms by this mass and calculate the number of books from this.
- Students could weigh one book at a time while another student keeps a running total and stops at 20 kilograms. Then they could count the books.

In question 4, students will need to know that 1 tonne = 1 000 kilograms.

After students have been given some time to try to solve the problem, bring them together to discuss the strategies they are using. They may be tallying, adding, counting in twenties, or dividing 100 by 20 and then multiplying by five. Share two or three of these thinking strategies and have students record the solution they choose to use, showing each step they have taken.

Page 13: Solve It or Sink It

Achievement Objectives

- demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by using reasonable estimates (Measurement, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)

Activity

This activity integrates measurement of mass with technology in a problem-solving context. Students should measure the mass of the marbles in grams.

The boat will float well if it has a wide base. Students may need to push the bottom fold back up inside the boat a little to make this base.

Additional problem-solving instructions or questions could include:

"Explore ways of arranging the marbles to find the least mass and the greatest mass that the boat can hold."

"Does the size of the marbles change the mass that the boat can hold?"

"Discuss a plan to explore this question with a classmate. Write down each step of the plan and explore your findings."

In question **3**, have the students estimate the mass of marbles that the smaller boat size may hold. All the class estimates could be recorded on a stem-and-leaf graph (see *Mathematics in the New Zealand Curriculum*, page 216).

When the students compare the number of marbles that each boat holds, they could record this as a ratio or fraction, for example:

Number of	t marbl	les in	small	boat	=	15
Number of	f marbl	les in	large	boat	=	36

Using fractions reduces the effect of variations in design and construction styles. This may help students when they are comparing their results. It is also a good context for finding and recording fractions.

Have the students explore the class results to see whether there is a pattern, such as a number of students with equivalent (or nearly equivalent) fractions. Positioning each fraction on a number line between zero and one may help the students to see whether there is a cluster effect in the comparisons, for example:



If the results do not show a pattern, discuss some possible reasons with the class. Ideas may include:

- the way the marbles were placed in the boat
- boats used for the first time may float longer than boats used more than once
- the similarity or otherwise of the water conditions.

Page 14: Mini Mass

Achievement Objectives

- demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (Measurement, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)

Activity

This activity gets students to solve the problem of how to find the mass of small objects.

Use the illustrations and Suzanne's first statement to clarify the problem. Discuss solutions proposed by the students. Follow up with Suzanne's solution and ask: "Why did she choose to weigh 100 peas?"

Highlight the importance of choosing a number that your scales can measure but one that is easy to use to calculate the average mass. Discuss which other numbers would be useful. Mention that the calculator is a useful tool for finding the average mass. Ask the students "What keys on the calculator would Suzanne have had to press to find the average mass? What keys would she have had to press if the mass of 100 peas was 200 grams?"

Explain that the idea of average mass is a practical approximation of the mass of the small object. It does not have to be the mass of each thing.



It would be a good practice to have students look carefully at each group of objects to be weighed and remove any individual item that is very different from the rest, such as a half pea, a steel marble (rather than a glass one), or a broken nursery stick.

A class chart or spreadsheet that shows the different objects being weighed may be a good way to record the results of the investigation:

Object	Number chosen	Calculation used	Average mass of one object
Peas	100	100 g ÷ 100 g	1 g

Page 15: Would Wood Fit?

Achievement Objectives

- demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (Measurement, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)

Activity

This activity helps students develop a concept of a cubic metre through visualisation and then has them use cubic measures to find volumes. Junior classes may have suitable wooden blocks.

You might begin by having each student estimate and show with their hands an imaginary block that is 1 metre long, 1 metre wide, and 1 metre deep. Four students could each crouch at the corners of a metre square while four more students could stand beside them and hold out one hand at a height of 1 metre. Another student could then check all the edges of this human (metre-square) cube with a metre ruler to see how close they were to making a metre cube.



Another variation of this is to use a 4 metre piece of string and have four students make a metre square on the floor by holding a corner of the square with one hand. Again, have another student check with a metre rule. Use another 4 metre string for four more students who are standing to make another metre square and get a student to check this with a metre rule. The four students standing need to try to position the top square a metre above the square on the floor. The students should check this to see how close they are to making a cubic metre.

Students could use the 10 centimetre cube place value block as a wooden block. This is usually used to model the thousands place. One thousand of these would fit nicely into a metre cube. A bigger challenge would be to have students use a block that is 20 centimetres by 10 centimetres by 5 centimetres. You may choose to show students a diagram of this if you cannot find a block that is this size.



You may be able to borrow some suitable wooden blocks for this activity from a junior area's construction set. Using such blocks may be very challenging if their dimensions do not fit easily into a metre.

After estimating in questions **3** and **4**, discuss techniques for making good estimates. Students may think of the tree as square instead of round. They could measure the square and multiply that by the estimated height. The area of a circle inside a square is approximately three-quarters of the area of the square. This fact could help students make better estimates.



Have the students report on their estimation techniques and the reasons for their estimates.

If there are no large trees in the school playground, question **4** could be used as a homework exercise. The focus could then be on the method, because tree sizes will vary.

Page 16: Wanted!

Achievement Objectives

- demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (Measurement, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)

Activity One

This activity combines measuring length, area, and mass to estimate and check a familiar classroom object. Hide the duster before students try to describe it from memory. They can then adjust their description when they are shown the duster. After estimating its length, width, and depth in millimetres, pass the duster around so that students can feel it as they estimate its mass in grams.

Activity Two

This is a very open activity. Mention the need to package and mail the present and the cost of sending parcels and encourage students to keep the price of the actual present realistic. These factors could influence a discussion of possible presents. The students could produce a Christmas present list and then decide which one they will ask Tama's uncle to send.

Shops often have free catalogues of toys available as handouts. A collection of these flyers would be a helpful resource.

Page 17: Rainy Day Delights

Achievement Objectives

- demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (Measurement, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)

Activity

This activity provides a practical context for measuring capacity and constructing a scale for solving a technology problem.

Get students to read through the instructions. After reading the fifth and sixth points, ask students "What do you think is the reason for these two instructions?" The discussion should clarify the need for an accurate starting point and the difficulty that students would have if they tried to mark the starting point at the base of a bottle with a dimpled base. Make sure students fill carefully to the 0 mark. They may need to use a dropper, and they should have their eye level with the 0 mark on the scale.



When they punch the holes in the bottle cap, get students to use small nails, such as brads, and make sure they do not make too many holes. Two or three holes will be enough. (Large holes or too many holes could represent a flash flood!)



A challenging investigation would be to explore the rate of flow for different numbers of nail holes. Students could draw up a table that shows the amount poured after 1 minute. You could ask them to see if they can predict the increase in flow as they punch another hole in the cap.

Further information on weather and related topics can be found in the Ministry of Education's *Making Better Sense of Planet Earth and Beyond* (Wellington: Learning Media, 1999).

Page 18: Fill Them Up

Achievement Objectives

- demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (Measurement, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)

Activity One

This activity asks students to estimate and measure the capacity of different containers.

This activity can be done with the whole class or in groups. You will need to have a range of jars and enough of them for each student. Detergent bottle lids, soap powder scoops, pottles, yoghurt containers, plastic cups, coffee tins, pots, basins, and buckets are suitable.

You will also need measuring instruments. These should range from measuring spoons showing 5 millimetres to 1 or 2 litre measuring jugs.

Begin by having students estimate which are the smallest and which are the largest containers. Place these at either end of the front row of desks in the classroom. Students can discuss and vote for the position of each container as they arrange them from smallest to biggest. If any containers have the same capacity, they would be positioned in the same place. Use numbered cards to mark the position of each container. These will help you to keep track of how the positions of the containers have changed after students have measured each container.

Have some students each select one of the containers and mark it with the same number as the position card or write its name on the position card. Other students could each select a measuring instrument. Discuss the suitability of the instrument for measuring the capacity of the chosen container.

Students can then fill the containers with water and measure the amount of water in each container. They should do this several times to check the accuracy of their measurements.

When the class regroups, they can reposition the containers from smallest to largest and discuss any changes of position. If there are any surprises, students should discuss what the difficulty was in making the estimate.



Activity Two

Students will need to fill and pour with care so that the activity is done reasonably accurately. They could reduce spillage by using a funnel.

Have the students complete a chart that could feature a check column where they write the equation and use a calculator to see how accurate they were in measuring. For example:

Container size	Number of fills to make 3 L	Check equation		
200 mL	16	200 x 16 = 3 200 mL = 3.2 L		
250 mL	12	250 x 12 = 3 000 mL = 3 L		

Page 19: Hot Spots!

Achievement Objectives

- demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (Measurement, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)

Activity One

This activity provides an opportunity to read temperatures in degrees Celsius. Students will also order one-place decimal numerals.

Questions **2** and **3** may be used as an introduction to the activity to ensure that the students can read everyday temperature data and use the term "Celsius" as the unit of temperature.

You may prefer to begin by having students collect yesterday's temperature data from television, radio, the Internet, or from the daily newspaper. Use this to highlight the range of temperatures across the different regions of the country. Questions **2** and **3** could then be used to introduce the map. This may lead to other questions of interpretation based on logical reasoning. Help students explore questions such as: "At what time of the year do you think there are temperatures like the ones shown on the map?" "Why do you think some parts of the South Island are much warmer than other parts?"

This leads well to Question **4**. Include the proximity of the towns to the ocean, with its currents and sea breezes, as well as features such as latitude, altitude, and the position of mountain ranges to protect or channel the prevailing breezes. The cold southerly wind and the hot north-westerly wind that are common in parts of the South Island may need to be mentioned when interpreting the map.

Students may like to construct their own map showing their estimates of the average temperatures for the same places in New Zealand for each of the four seasons. They can check their estimates in the *New Zealand Official Yearbook*, the Metservice website (www.metservice.co.nz/knowledge), or a good New Zealand atlas.

You could also compare the more constant temperatures on small islands to the greater extremes in temperatures experienced in continental climates to show the effect of being close to the ocean. Areas such as Auckland, surrounded so closely by the Tasman Sea and the South Pacific Ocean, often have a microclimate like that of an island, while Central Otago, surrounded by mountains, has a climate similar to a continental climate.

Activity Two

In this activity, students could also explore the concept of microclimate and the importance of the site that the students have chosen to measure. If students choose different places, this effect will be obvious. Discuss the possible difference of positions, such as sunny or shady places, as well as sheltered or exposed places. Students will need to measure the temperature at the same time each day, regardless of the weather.



Page 20: Burning Issues

Achievement Objectives

- demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (Measurement, level 3)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 3)

Activity

This activity will encourage sound reasoning in a temperature, location, and weather conditions context. It also has important implications for health and physical education.

There is plenty of scope in this activity for class discussion of UV radiation and how it affects us. New Zealanders experience relatively intense sunburning ultraviolet (UV) rays, and students need to be aware of the dangers of this. The World Meteorological Organization and the World Health Organization now recommend using the UV index rather than burn time. The information below was provided by the National Institute of Water and Atmospheric Research Ltd (NIWA). It is included here as necessary background for the activity.

New Zealand was the first country (1989) to begin informing the public about the dangers of UV radiation. This information was provided to the media by the New Zealand Meteorological Service as a "time to burn" and was based on calculations assuming clear-sky conditions. Unfortunately, time to burn is a poorly defined quantity, which varies greatly from person to person. A new international standard, called the UV index, was introduced in New Zealand in the summer of 1994/95. It is based on UV intensity, so that when UV radiation is more intense, the index is higher (unlike burn time, which decreases with intensity). The UV scale is open-ended, but a UV index of greater than 10 is extreme, and a UV index less than 1 is low.

Students may still hear UV radiation being reported as burn time, and this activity is a good way of helping them understand why this is not as useful as the UV index.

Students need to understand the following points:

- If weather conditions are similar, UV levels are higher in the north of the country than in the south because in the north of the country, the sun is higher in the sky than in the south of the country at the same time of the day. When the sun is higher in the sky, the path through the atmosphere is shorter and therefore less of the sunlight is absorbed by the atmosphere.
- When it's sunny and there are no clouds in the sky, the UV level is high.
- When it's sunny but there are some clouds in the sky, UV levels can be higher or lower than if there are no clouds. This depends on whether the sun is partly obscured by cloud. On a partly cloudy day when there is no cloud in front of the sun, the UV level is higher than on days that have completely clear skies. (Clouds can reduce UV radiation by reflecting light rays back into space, but during partly cloudy conditions when the sun is not obscured, UV levels can increase by more than 20% because the clouds scatter the UV rays. When cloud is partly obscuring the sun, the UV level is lower at that point.)
- When it's totally overcast, the UV level is lower than it would be if there were no clouds in the sky.
- Temperature has nothing to do with UV levels. A hot, overcast day can have a lower UV level than a cool, cloudless day at the same time of the year.

Students will need to look carefully at the data supplied on this page. By looking at the UV indices for the cities with sun only, that is, Kaitaia, Tauranga, Auckland, Hamilton, New Plymouth, Nelson, Dunedin, and Invercargill, they can conclude that the UV level was higher in the north of the country than in the south on that day.

They can then see that the UV level was lower in Christchurch because it was totally overcast. This is also apparent by comparing Wellington (overcast) with Nelson and New Plymouth, the closest sunny cities on that day. Gisborne had a higher UV index than you might expect. (It had a UV index of 12, whereas Hamilton, to the north, had 11, and New Plymouth, to the south, had 10.) This is because it was partly cloudy, which, as explained above, can make the UV level higher than clear-sky levels. However, note that in Napier, the partly cloudy skies suppressed UV. Students can assume from this that at 1 p.m. on that day, the sun was covered by cloud.

Page 21: On Time

Achievement Objectives

- read and interpret everyday statements involving time (Measurement, level 3)
- show analogue time as digital time, and vice versa (Measurement, level 3)

Activity

In this activity, students convert digital time to analogue time using a photocopy of the clock faces copymaster found at the back of this booklet.

The context of Sonja's watch provides the logic for doing the task because she must keep her analogue watch for its built-in extras.

Page 22: Breaking Records

Achievement Objectives

- read and interpret everyday statements involving time (Measurement, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)

Activity

In this activity, students will measure the time taken to complete some simple, fun activities using minutes and seconds.

Question **1a** asks students to check Celina's claim. They may do this by choosing to multiply 1.5 seconds by 40, which gives 1 minute, and would conclude from this that Celina is correct. They may even add 1.5 to itself 40 times, using the constant function on their calculator.

Point out to students that if they did not know the rate of 1.5 seconds per join, they would not know what to add or multiply.

Ask the students to explore a method that Celina might use to work out the mean time for joining paper clips. Make sure that students know how many seconds are in a minute and how to work in the common unit of seconds. Students may need to use a calculator to solve this, using the traditional method of dividing 60 seconds by 40 to find the rate of 1.5 seconds per join. Discuss the way of saying and writing the unit of rate as seconds per join.

Once students understand or have worked out a method of finding the mean, there is still a little twist to the question because there are actually only 39 joins needed to link together 40 paper clips. If Celina's average rate was extremely accurate, she would be able to join 41 paper clips in 1 minute.

In question **1b**, make sure that students have a good routine for starting, stopping, and recording the results of their experiments.

The estimate for a world record in the activities for question **2** should be based on sound logic following trials in the classroom. You may wish to find the top three performers in the class for each activity and have them share their strategies for their speed.

Achievement Objectives

- read and interpret everyday statements involving time (Measurement, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)

Activity One

This is a problem-solving activity using time calculations. You may have students work in problem-solving groups of two to four. Encourage them to take a little time to understand the problem. It may help them if they restate the problem in their own words.

If the students come up with 12 minutes as the answer, have them make a chart showing the algebra pattern for an even number of patties:

Number of patties	Total minutes
2	6
4	12
6	18
8	24

Have the students look for a pattern in their table. Ask them how long three patties would take to cook following this pattern. The pattern should suggest 9 minutes. The task is now to prove that 9 minutes is a practical answer.

This is an ideal problem to be acted out. Have the students record the position that each patty needs to be in every 3 minutes:

Patty 1	Patty 2	Patty 3	Minutes
side 1	side 1	not cooking	3
not cooking	side 2	side 1	6
side 2	cooked	side 2	9

Activity Two

This is a variation on the well-known "jugs" problems. The problem is relatively easy if you allow students to guess when the 2 minute timer is halfway through and the damper is put on the fire at that point. They would then just use the 5 minute timer followed by the 2 minute timer again. The problem is more challenging if you insist on exactness and have students look for a method that will work without estimating a halfway point.

Making a chart or acting it out would be good strategies for students to try while they look for a solution. The question asks how Robinson uses both timers to cook his bread. This eliminates the possibility of using the 2 minute timer four times.

Activity Three

Students will need to understand that, because she is using a microwave, Esther must work out the cooking times for each item separately and then add them together.

Page 24: Holey Moley

Achievement Objectives

- perform measuring tasks, using a range of units and scales (Measurement, level 3)
- effectively plan mathematical exploration (Mathematical Processes, problem solving, level 3)

Game

This game should provide motivation for solving length problems. The mathematical learning comes from the course construction and any modifications that are made to the design in order to adjust the par for a hole.

If students wish to continue to play the game more than a few times, have them redesign the course.

You may wish to increase the accuracy of the course designs by getting students to record the measurements in centimetres or millimetres.

Students could design different types of golf courses that have the same total par number for nine holes. A cotton bud or a tiddlywinks golf course may provide some interesting variations.



Acknowledgments

Learning Media would like to thank Geoff Woolford, Education Advisory Service, Auckland College of Education, for developing the teachers' notes. Thanks also to Paulette Holland for reviewing the answers and notes.

The main illustrations on the cover and contents page and the hourglass and stylised fern line art on the cover, contents page, and pages 2 and 8 are by Geoffrey Notman; and the golfer line art on the cover, contents page, and pages 2 and 8 is by Fraser Williamson.

All illustrations are copyright © Crown 2000.

Series Editor: Susan Roche Series Designer: Esther Chua

Published 2000 for the Ministry of Education by Learning Media Limited, Box 3293, Wellington, New Zealand. Website: www.learningmedia.co.nz

Copyright © Crown 2000 All rights reserved. Enquiries should be made to the publisher.

Dewey number 510.76 ISBN 0 478 12689 1 Item number 12689 Students' book: item number 12688